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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)
Memory Based Questions & Solutions

Date: 29 January, 2023 (SHIFT-2) | TIME : (3.00 p.m. to 6.00 p.m)
Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. If all words which are formed using all letters of word "TOUGH" are arranged alphabetically then the rank of TOUGH is

(1) 89 (2) 79 (3) 84 (4) 77

Ans. (1) 89

Sol. G,H,O,T,U

No. of words starting from G = 4! = 24

No. of words starting from G = 4! = 24

No. of words starting from H = 4! = 24

No. of words starting from O = 4! = 24

No. of words starting from TG = 3! = 6

No. of words starting from TH = 3! = 6

No. of words starting from TOG = 2! = 2

No. of words starting from TOH = 2! = 2

Next "TOUGH" = 1

Rank of "TOUGH" = 24 + 24+ 24 + 6 + 6 + 2 + 2 +1 = 89 Ans.

2. The value of integral $\int_{\frac{1}{2}}^{2}\frac{\tan^{-1}x}{x}dx$ is

Ans. 2

Sol. Let $I=\int_{\frac{1}{2}}^{2}\frac{\tan^{-1}x}{x}dx$ ____(1)

replace x by $\frac{1}{x}$

$$I=\int_{2}^{\frac{1}{2}}x\left(\tan^{-1}\frac{1}{x}\right)\left(-\frac{1}{x^2}dx\right)$$

$$I=\int_{\frac{1}{2}}^{2}\frac{1}{x}\tan^{-1}\frac{1}{x}dx \quad \text{\_\_\_\_(2)}$$

Adding (1) & (2)

$$2I=\int_{\frac{1}{2}}^{2}\frac{1}{x}\left(\tan^{-1}x+\tan^{-1}\frac{1}{x}\right)dx$$

$$2I=\frac{\pi}{2}\int_{\frac{1}{2}}^{2}\frac{1}{x}dx$$

$$2I=\frac{\pi}{2}(\ln x)_{\frac{1}{2}}^{2}$$

$$I=\frac{\pi}{2}\ln 2$$

3. Shortest distance $\frac{x-1}{2}=\frac{2y-2}{\frac{3}{2}}=\frac{z-3}{1}$ and $\frac{x-2}{3}=\frac{y-1}{2}=\frac{z+2}{4}$

(1) $\frac{13}{\sqrt{165}}$ (2) $\frac{9}{\sqrt{165}}$ (3) $\frac{11}{\sqrt{165}}$ (4) $\frac{10}{\sqrt{165}}$

Ans. (1)

Sol. Shortest distance = $\left|\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}\times\vec{d})}{(\vec{b}\times\vec{d})}\right|$

$$\vec{b}\times\vec{d}=\begin{vmatrix} i & j & k \\ 2 & \frac{3}{2} & 1 \\ 3 & 2 & 4\end{vmatrix}=i\,(6-2)-j\,(8-3)+k\left(4-\frac{9}{2}\right)$$

$\Rightarrow \quad 4\hat{i} - 5\hat{j} - \frac{k}{2}$

$\hat{a}_2 - \hat{a}_1 = \hat{i} - 5\hat{k}$

So $\quad S.D. = \left| \frac{2}{\sqrt{165}} \begin{vmatrix} 1 & 0 & -5 \\ 2 & \frac{3}{2} & 1 \\ 3 & \frac{2}{2} & 4 \end{vmatrix} \right| = \frac{2}{\sqrt{165}}\left(4 - \frac{5}{2}\right) = \frac{13}{\sqrt{165}}$

4. Value of λ for which the equation $\cos^2 2x - 2\sin^4 x - \cos^2 x = \lambda$ has a solution, lies in interval.

(1) $\left[\frac{-9}{8}, 0\right]$ (2) $\left[0, \frac{9}{8}\right]$ (3) $\left[\frac{-9}{8}, \frac{9}{4}\right]$ (4) $\left[0, \frac{9}{4}\right]$

Ans. (1)

Sol. Let $f(x) = \cos^2 2x - 2\sin^4 x - \cos^2 x$

$= \cos^2 2x - \left(\frac{1-\cos^2 x}{2}\right)^2 - \left(\frac{1+\cos^2 x}{2}\right)$

$= \cos^2 2x - \frac{1}{2}\left(1 - 2\cos 2x + \cos^2 2x\right) - \left(\frac{1+\cos 2x}{2}\right)$

$= \frac{1}{2}\left(\cos^2 2x + \cos 2x - 2\right)$

$f(x) = \frac{1}{2}\left(\left(\cos 2x + \frac{1}{2}\right)^2 - \frac{9}{4}\right)$

$\because \quad \left(\cos 2x + \frac{1}{2}\right)^2 \in \left[0, \frac{9}{4}\right]$

$\Rightarrow \quad \left(\cos 2x + \frac{1}{2}\right)^2 - \frac{9}{4} \in \left[\frac{-9}{4}, 0\right]$

$f(x) \in \left[\frac{-9}{4}, 0\right]$

5. The 3 digit numbers which are divisible by either 3 or 4 but not divisible by 48.

Ans. 432

Sol. no. divisible by 3 of 3 digits = 102, 105, ……. , 999

= 300

no. divisible by 4 of 3 digits = 100, 104, …. , 996

= 225

now no. divisible by 12

= 75

So numbers divisible by 3 or 4

= 300 + 225 – 75 = 450

now numbers divisible by 48 are = 18

So total numbers = 450 – 18

= 432

6. The relation R = { (a, b) : 2a + 3b is divisible by 5 and a, b $\in$ N} is

(1) Transitive but not symmetric (2) symmetric but no transitive

(3) Equivalence (4) Not equivalence

Ans. (3)

Sol. for (a, a) $\Rightarrow$ 2a + 3b = 2a + 3b = 5a $\Rightarrow$ divisible by 5

$\Rightarrow (a, a) \in R \ \forall a \in N \Rightarrow$ reflexive

Let $(a, b) \in R \Rightarrow 2a + 3b = 5k_1$

and $5a + 5b = 5k_2$ then

$5a + 5b - 20 - 3b = 5(k_2 - k_1) \Rightarrow 2b + 3a = 5k$

$\Rightarrow (b, a) \in R \Rightarrow$ symmetric

Let (a, b) & (b, c) both belong to R

$\Rightarrow 2a + 3b = 5k_1$ & $2b + 3c = 5k_2$

then $2a + 3b + 2b + 3c = 5(k_1 + k_2) \Rightarrow 2a + 3c = 5k - 5b = 5\lambda$

$\Rightarrow (a, c) \in R \Rightarrow$ transitive

7. If $f(1) + 2f(2) + 3f(3)$ .... $nf(n) = n(n+1)f(n)$ is true $\forall\ n \geq 2$ and $f(1) = 1$ then the value of $\frac{1}{f(2022)} + \frac{1}{f(2028)}$ is

Ans. 8100

Sol. $f(1) + 2f(2) + 3f(3)$ .... $n f(n) = n(n+1) f(n)$ ....(1)

replace in by $n + 1$

$f(1) + 2f(2) + 3f(3)$ .... $nf(n) + (n+1) f(n+1) = (n+1)(n+2) f(n+1)$ ....(2)

Subtracting (1) from (2)

$(n+1) f(n+1) = (n+1)(n+2) f(n+1) - n(n+1) f(n)$ or $(n+1)f(n+1) - nf(n) = 0$ ....(3)

put n = 2, 3, 4 .... n + 1 in eq. (3)

$3f(3) - 2f(2) = 0$

$4f(4) - 3f(3) = 0$

,,

$(n+1) f(n+1) - n f(n) = 0$

Add $\Rightarrow (n+1) f(n+1) = 2f(2)$

$\Rightarrow \quad f(n+1) = \frac{2f(2)}{n+1}$ ....(4)

again put n = 2 in eq. (1)

$f(1) + 2f(2) = 6f(2)$

$\Rightarrow 1 = 4f(2) \Rightarrow f(2) = \frac{1}{4}$

Now by (4) $f(n) = \frac{1}{2n}$

$\Rightarrow \frac{1}{f(2022)} + \frac{1}{f(2028)} = 4044 + 4056 = 8100$

8. The proposition $(\sim A) \vee B$ is equivalent to.

(1) $a \leftrightarrow b$ (2) $A \to B$ (3) $B \to A$ (4) $A \to \sim B$

Ans. (2)

Sol. Obvious

9. If z is a non-zero complex number such that $\left|z - \frac{1}{z}\right| = 2$ then the maximum value of |z|.

(1) $\sqrt{2} - 1$ (2) $\sqrt{2}$ **(3)** $\sqrt{2} + 1$ (4) $2\sqrt{2} + 1$

Ans. (3)

Sol. $\therefore \left||z| - \frac{1}{|z|}\right| \leq \left|z - \frac{1}{z}\right| \leq |z| + \frac{1}{|z|}$

$\Rightarrow \left| |z| - \frac{1}{|z|} \right| \le 2$ and $2 \le |z| + \frac{1}{|z|}$

$\Rightarrow -2 \le |z| - \frac{1}{|z|} \le 2$ $\Rightarrow |z|^2 - 2.|z| + 1 \ge 0$

$\Rightarrow (|z| - 1)^2 \ge 0$ always true

$\Rightarrow |z| - \frac{1}{|z|} \ge -2$ &

$\Rightarrow |z|^2 + 2.|z| - 1 \ge 0$

$\Rightarrow |z| \in (-\infty, -1-\sqrt{2}] \cup [-1+\sqrt{2}, \infty)$

$\Rightarrow |z| \in [\sqrt{2}-1, \infty)$ .... (1)

$|z| - \frac{1}{|z|} \le 2$

$\Rightarrow |z|^2 - 2.|z| - 1 \le 0$

$\Rightarrow |z| \in [1-\sqrt{2}, 1+\sqrt{2}]$

$\Rightarrow |z| \in [0, 1+\sqrt{2}]$ .... (2)

(1) and (2) $|z| \in [\sqrt{2} - 1, \sqrt{2} + 1]$

10. Number of four digit integers whose greatest common divisor with 54 is 2, is

Ans. 3000

Sol. Four digit numbers which are divisible by 2 $\Rightarrow$ 1000 to 9998 Þ 4500

Four digit numbers divisible by 6

$\Rightarrow$ 1002 to 9996 $\Rightarrow$ 1500

Four digit integer divisible by 2 but not divisible by 3 $\Rightarrow$ 4500 – 1500 = 3000

11. The value of $\int_1^2 \frac{t^4+1}{t^6+1} dt$ is

(1) $\frac{1}{3}\left[\tan^{-1}8 - \frac{\pi}{4}\right]$ (2) $\frac{1}{2}\left[\tan^{-1}8 - \frac{\pi}{2}\right]$ (3) $\tan^{-1}8 - \frac{\pi}{2}$ (4) $\frac{1}{3}\tan^{-1}8 - \frac{\pi}{6}$

Ans. (1)

Sol $= \int_1^2 \frac{(t^4+1)}{(t^6+1)} \frac{(t^2+1)}{(t^2+1)} dt$

$= \int_1^2 \frac{t^6+t^4}{(t^6+1)} \frac{t^2+1}{(t^2+1)} dt$

$= \int_1^2 \frac{t^6+1}{(t^6+1)(t^2+1)} dt + \int_1^2 \frac{t^4+t^2}{(t^6+1)(t^2+1)} dt$

$= \int_1^2 \frac{1}{t^2+1} dt + \int_1^2 \frac{t^2}{t^6+1} dt,$

$I = [\tan^{-1}t]_1^2 + I_1$

$I_1 = \int_1^2 \frac{t^2}{t^6+1} dt$

put $t^3 = \theta$. $3t^2 dt = d\theta$

$t = 1$ then $\theta = 1$

$t = 2$ then $\theta = 8$

1 $\int$ dθ

$= \frac{1}{3}\int \frac{d\theta}{\theta^2+1}$

$= \frac{1}{3}\left[\tan^{-1}(\theta)\right]_1^8$

$= \frac{1}{3}\left[\tan^{-1}8 - \frac{\pi}{4}\right]$

12. Let f (x) = $x^2 + 2x + 5$ and α , β are the roots of $f\left(\frac{1}{t}\right) = 0$, then $\alpha + \beta$ is equal to

(1) – 2/5 (2) –2 (3) 5/2 (4) –5/2

Ans. (1)

Sol. $f(x) = x^2 + 2x + 5$

$f\left(\frac{1}{t}\right) = \frac{1}{t^2} + \frac{2}{t} + 5$

$f\left(\frac{1}{t}\right) = 0$

$5t^2 + 2t + 1 = 0$

α & β are the roots then $\alpha + \beta = -2/5$

13. If y = y(x) is this solution of equation $x\ \ell n\, x \frac{dy}{dx} + y = x^2\ \ell nx$ and y (2) = 2 then y (e) is equal to

(1) $1 + \frac{e^2}{4}$ (2) $1 - \frac{e^2}{4}$ (3) $2 + \frac{e^2}{4}$ (4) $2 - \frac{e^2}{4}$

Ans. (1)

Sol. $x\ \ell n\, x \frac{dy}{dx} + y = x^2\ \ell nx$

$\frac{dy}{dx} + \frac{y}{x\ell nx} = x$

I.F. $= e^{\int \frac{dx}{x\ell nx}} = e^{\ell n(\ell nx)} = \ell nx$

now solution is

$y\ell nx = \int x\ell nx\, dx$

$y\ell nx = (\ell nx)\frac{x^2}{2} - \frac{x^2}{4} + c$

$y = \frac{x^2}{2} - \frac{x^2}{4\ell nx} + \frac{c}{\ell nx}$

$y(2) = \frac{4}{2} - \frac{4}{4\ell n2} + \frac{c}{\ell n2}$

$2 = 2 - \frac{1}{\ell n2} + \frac{c}{\ell n2}$

$c = 1$

$y = \frac{x^2}{2} - \frac{x^2}{4\ell nx} + \frac{1}{\ell nx}$

$y(e) = \frac{e^2}{2} - \frac{e^2}{4} + 1$

$y(e) = \frac{e^2}{4} + 1$

14. The area bounded by the region $\left\{(x, y) : \lfloor \cos x - \sin x \rfloor \le y \le \sin x : x \in \left[0, \frac{\pi}{2}\right]\right\}$ is

(1) $\sqrt{5} + 1 - 2\sqrt{2}$ (2) $\sqrt{5} - 2\sqrt{2}$

(3) $\sqrt{5} + 1 - \sqrt{2}$ (4) $\sqrt{5} - \sqrt{2}$

(3) $\sqrt{5}+1-\sqrt{2}$ (4) $\sqrt{5}-\sqrt{2}$

Ans. (1)

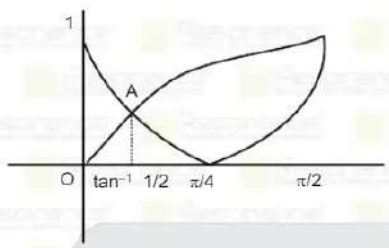
Sol. $y = \begin{cases} \cos x - \sin x & : 0 \le x \le \frac{\pi}{4} \\ \sin x - \cos x & : \frac{\pi}{4} 2x < x \le \frac{\pi}{2} \end{cases}$

$y^1 = \begin{cases} -\sin x - \cos x & : 0 \le x \le \frac{\pi}{4} \\ \cos x + \sin x & : \frac{\pi}{4} < x < x \le \frac{\pi}{2} \end{cases}$

For pt. A

$\cos x - \sin x = \sin x$

$\tan x = \frac{1}{2}$



Let $x = \tan^{-1}\frac{1}{2}$

Area = $\int_{\tan^{-1}1/2}^{\pi/4} (\sin x - (\cos x - \sin x))dx$

$+ \int_{\pi/4}^{\pi/2} (\sin x - \sin x + \cos x)\,dx$

$= (-2\cos x - \sin x)_x^{\pi/4} + (\sin x)_{\pi/4}^{\pi/2}$

$= \left(\frac{-2}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - (-2\cos x - \sin x) + \left(1 - \frac{1}{\sqrt{2}}\right)$

$= -\sqrt{2} - \frac{1}{\sqrt{2}} + 2\cos x + \sin x + 1 - \frac{1}{\sqrt{2}}$

$= -2\sqrt{2} + 1 + \frac{2 \times 2}{\sqrt{5}} + \frac{1}{\sqrt{5}}$

$= \sqrt{5} + 1 - 2\sqrt{2}$

15. Two matrices A and B are such that $|A| = 2$, $B = \begin{bmatrix} 2 & 1 \\ 3 & 3/2 \end{bmatrix}$. $BA = \begin{bmatrix} 2 & 1 \\ \alpha & \beta \end{bmatrix}$; $A^T = A$ and $Tr(A) = S$

then $\frac{\beta S}{\alpha}$ is

(1) $\frac{3}{2}$ (2) 3 (3) $\frac{5}{2}$ (4) 1

Ans. (1)

Sol. $A^T = A$ b So $A = \begin{bmatrix} a & b \end{bmatrix}$

$\quad\quad b \quad d$

now $ad = b^2 = 2$ ....(1)

also $\quad BA = \begin{bmatrix} 2 & 1 \\ 3 & 3/2 \end{bmatrix}\begin{bmatrix} a & b \\ b & d \end{bmatrix}$

$\Rightarrow 2a + b = 2$ ....(2)

$3a + \frac{3b}{2} = \alpha$ .... (3)

$3b + \frac{3d}{2} = \beta$ .... (4)

$2b + d = 1$ .... (5)

Solving $a = 6/5$, $\alpha = 3$, $b = \frac{-2}{5}$

$d = \frac{9}{5}, \beta = \frac{3}{2}$

So $\quad \frac{\beta \times S}{\alpha} = \frac{3}{2}$

16. The sum of coefficients of odd powers of x in the expansion of $(1+x)^{99}$ is k. The middle term in the expansion of $\left(2+\frac{1}{\sqrt{2}}\right)^{200}$ is α and $\frac{{}^{200}C_{99} \times k}{\alpha} = 2^t \times \frac{m}{n}$; m,n are odd, then (t, n) = ?

(1) (50, 101) (2) (49, 101) (3) (50, 102) (4) (50, 103)

Ans. **(50, 101)**

Sol. Sum of odd coeff. of $x = \frac{2^{99} - 0}{2} = k$

$k = 2^{98}$

Middle term $\alpha = T101 = {}^{200}C_{100} \frac{(2)^{100}}{2^{50}}$

$= {}^{200}C_{100}\ 2^{50}$

Now $\quad \frac{{}^{200}C^{99} \times k}{\alpha} = \frac{200!}{100!\ 99!} \times \frac{2^{98}}{{}^{200}C_{100} \times 2^{50}}$

$= \frac{100}{101} \times 2^{48}$

$= \frac{25}{101} \times 2^{50}$

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