



JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT) Memory Based Questions & Solutions

Date: 30 January, 2023 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. $\lim_{x \rightarrow 0} \frac{48 \int_0^x \frac{t^3}{1+t^6} dt}{x^4}$ is equal to.

(1) 3

(2) 4

(3) 5

(4) 12

Ans. (4)

$\left(\frac{x}{1+t^6} \right)$

Sol. $\lim_{x \rightarrow 0} \left(\frac{48 \int_0^x 1+t^6 dt}{x^4} \right)$ ($\frac{0}{0}$ form)

using Leibnitz theorem we have

$$\lim_{x \rightarrow 0} \frac{48 \frac{x^3}{3}}{4x^3} = \frac{48}{4} = 12$$

2. The coefficient of x^{301} in $(1+x)^{500} + x(x+1)^{499} + x^2(1+x)^{498} + \dots + (1+x)^0 x^{500}$ is.
 (1) ${}^{500}C_{200}$ (2) ${}^{500}C_{300}$ (3) ${}^{501}C_{300}$ (4) ${}^{501}C_{200}$

Ans. (4)

Sol. $(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \dots + (1+x)^0 x^{500}$

$$= \frac{(1+x)^{500} \left[1 - \left(\frac{x}{1+x} \right)^{501} \right]}{1 - \frac{x}{1+x}}$$

$$= \frac{(1+x)^{501} \left[1 - \left(\frac{x}{1+x} \right)^{501} \right]}{1+x-x}$$

$$= (1+x)^{501} - x^{501}$$

Now coefficient of x^{301} in $(1+x)^{501} - x^{501}$

$$= {}^{501}C_{301} = {}^{501}C_{200}$$

3. If $\tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2\alpha$. Then find $\left(\alpha + \frac{1}{\alpha} \right)$

- (1) 2 (2) 3 (3) 4 (4) -4

Ans. (3)

Sol. $\Rightarrow \tan 15^\circ + \tan 15^\circ - \tan 15^\circ + \tan 15^\circ = 2\alpha$

$$\Rightarrow \alpha = \tan 15^\circ$$

$$\text{Now } \alpha + \frac{1}{\alpha} = \tan 15^\circ + \frac{1}{\tan 15^\circ}$$

$$= \frac{1 + \tan^2 15^\circ}{\tan 15^\circ} = \frac{2}{\sin 30^\circ} = 4$$

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4. The number of four digit numbers using the digits 1,2,3,5 (repetition is allowed) and sum of digits is divisible by 15 is

Ans. (12)

Sol. Number should be formed using 5,5,3,2

$$\Rightarrow \frac{4!}{2!} = 12$$

5. Let $R = \{(a,b), (b,c)\}$ be a relation defined on set $A = \{a,b,c\}$. Minimum number of elements should be added in relation R to make this relation symmetric and transitive is

- (1) 2 (2) 3 (3) 5 (4) 7

Ans. (4)

Sol. Required elements in sets for symmetric and transitive (a,a) (b,b) (c,c) (b,a) (c,b) (a,c) (c,a) Minimum no. of elements required = 7

6. If mean and variance of 7 varieties are 8 & 16 respectively. If one data 14 is removed the new mean and variance are a and b respectively then the value of $2a - 3b + 5$ is

- (1) 16 (2) -16 (3) 6 (4) 0

Ans. (2)

$$\text{Sol. } \sum_{i=1}^7 x_i = 7 \times 8 = 56$$

$$\sum x_i^2 = 7(16 + 64) = 560$$

Where 14 is removed then

Now mean $a \Rightarrow 6a = \sum_{i=1}^7 x_i - 14$

$\Rightarrow 6a + 14 = 56$

$\Rightarrow a = 7$

New variance $b \Rightarrow 6b = \left(\sum_{i=1}^7 x_i^2 - 196 \right) - (7)^2 \cdot 6$

$\Rightarrow 6b = 560 - 196 - 294$

$\Rightarrow 6b = 560 - 490 = 70$

$\Rightarrow 3b = 35$

Hence $2a - 3b + 5 = 14 - 35 + 5$

$= -16$

7. Let a line meets the OX & OY axes at A and B respectively in first quadrant such that $OA = a$, $OB = b$ and perpendicular from origin on it makes an angle $\frac{\pi}{6}$ with y-axis. If area of triangle OAB is $\frac{98\sqrt{3}}{3}$ then the value of $a^2 - b^2$ is

(1) $\frac{392}{3}$

(2) $\frac{395}{3}$

(3) $\frac{389}{3}$

(4) $\frac{403}{3}$

Ans. (1)

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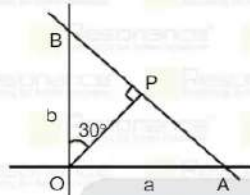
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Sol. Area of $\Delta OAB = \frac{98\sqrt{3}}{3}$



$\Rightarrow \frac{1}{2} ab = \frac{98\sqrt{3}}{3} \Rightarrow \sqrt{3} ab = 196 \dots\dots\dots(1)$

Now $OP = OB \cos 30^\circ = OA \cos 60^\circ$

$\Rightarrow b \cdot \frac{\sqrt{3}}{2} = a \cdot \frac{1}{2}$

$\Rightarrow \sqrt{3} b = a \dots\dots\dots(2)$

by (1) and (2)

$a^2 = 196$

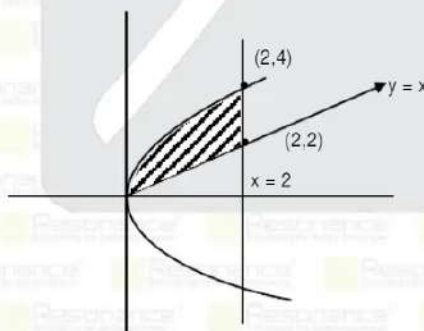
and $b^2 = \frac{a^2}{3}$

so $a^2 - b^2 = a^2 - \frac{a^2}{3} = \frac{2a^2}{3} = \frac{2}{3} \times 196 = \frac{392}{3}$

8. If the area enclosed in the region $\{(x,y) : y^2 \leq 8x, x \leq 2 \text{ and } y \geq x\}$ is α then the value of 3α is

Ans. (10.00)

Sol. Area = $\int_0^2 2\sqrt{2} x^{1/2} dx - \frac{1}{2}(2)(2)$



$$\Rightarrow \alpha = \frac{4\sqrt{2}}{3} x^{3/2} \Big|_0^2 - 2$$

$$\Rightarrow \alpha = \frac{16}{3} - 0 - 2$$

$$\Rightarrow \alpha = \frac{10}{3} \quad \Rightarrow 3\alpha = 10$$

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9. If $z = 1 + i$ and $z_1 = \frac{i+z(1-i)}{z(1-z)}$ then the value of $\frac{12}{\pi} \arg(z_1)$ is

Ans. (3)

$$\begin{aligned} \text{Sol. } z_1 &= \frac{i+(1-i)^2}{z(-i)} = \frac{i+1+i^2-2i}{z(-i)} \\ &= \frac{-i}{z(-i)} = \frac{1}{z} = \frac{z}{|z|^2} = \frac{z}{2} \end{aligned}$$

$$\arg z_1 = \arg z = \frac{\pi}{4}$$

$$\Rightarrow \frac{12}{\pi} \arg(z_1) = \frac{12}{\pi} \times \frac{\pi}{4} = 3$$

10. Let $P(h,k)$ be any two points on $x^2 = 4y$ which is at shortest distance from $Q_1(0,33)$ then difference of distance of $P(h,k)$ from directrix of $y^2 = 4(x+y)$ is ?

(1) 2

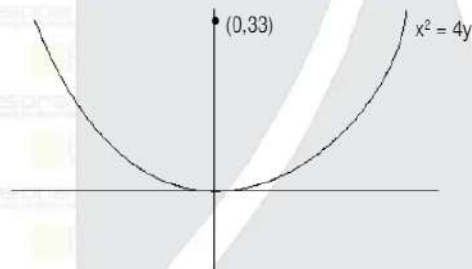
(2) 4

(3) 6

(4) 8

Ans. (2)

Sol.



For line of $-\frac{2}{h}(X-h)$ shortest distance from $(0,33)$ the line must be normal to the parabola $x^2 = 4y$ at

$P(h,k)$ Equation normal at $P(h,k)$ is

$$y - k = -\frac{2}{h}(X - h)$$

$$\text{passes } (0,33) \Rightarrow 33 - k = -\frac{2}{h}(-h)$$

$$\Rightarrow 33 - k = 2$$

$$\text{and } h = \pm 2\sqrt{31}$$

Now directrix of $y^2 = 4x + 4y$ is $x + 2 = 0$

Distance of directrix from $(2\sqrt{31}, 31)$ is $d_1 = 2 + 2\sqrt{31}$

distance of directrix from $(-2\sqrt{31}, 31)$ is $d_2 = 2\sqrt{31} - 2$

$$|d_1 - d_2| = 4$$

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11. If $a_n = \frac{-2}{4n^2 - 16n + 15}$, and $a_1 + a_2 + \dots + a_{25} = \frac{m}{n}$ where m and n are co-prime, then the value of

$m + n = \dots$

Ans. (191)

Sol. $a_1 + a_2 + \dots + a_{25} = \sum_{n=1}^{25} a_n$

$$= \sum_{n=1}^{25} \frac{-2}{4n^2 - 16n + 15} = \sum_{n=1}^{25} \frac{-2}{(2n-5)(2n-3)}$$

$$= \sum_{n=1}^{25} \left(\frac{1}{2n-3} - \frac{1}{2n-5} \right)$$

$$= \left[\left(\frac{1}{-1} - \frac{1}{-3} \right) + \left(\frac{1}{1} - \frac{1}{-1} \right) + \left(\frac{1}{3} - \frac{1}{1} \right) + \dots \right]$$

$$= \frac{1}{2(25)-3} + \frac{1}{3} = \frac{50}{141}$$

$\Rightarrow m = 50, n = 141$
 $\Rightarrow m + n = 191$

12. If $S = \{1, 2, 3, 4, 5\}$ then the number of one-one function $f : S \rightarrow P(S)$, where $P(S)$ is the power set of S , is

- (1) $^{30}P_5$ (2) $^{32}P_5$ (3) $^{31}C_5$ (4) $^{30}C_5$

Ans. (2)

Sol. $S = \{1, 2, 3, 4, 5\}$
 $n(P(S)) = 2^5$
 Total number of one-one function from S to $P(S)$
 $= ^{32}P_5$

13. Numbers on the faces of an unbiased die are the elements of set $\{-2, -1, 0, 1, 2, 3\}$. If the die is rolled 5 times then the probability that the product of numbers appearing on die is positive, is

- (1) $\frac{5}{27}$ (2) $\frac{11}{54}$ (3) $\frac{65}{324}$ (4) $\frac{67}{324}$

Ans. (3)

Sol. All positive or 3 positive and 2 negative or 1 positive and 4 negative

$$\Rightarrow P = \binom{3}{6}^5 + {}^5C_2 \left(\frac{2}{6} \right)^2 \left(\frac{3}{6} \right)^3 + {}^5C_1 \left(\frac{3}{6} \right) \left(\frac{2}{6} \right)^4$$

$$= \frac{3^5}{6^5} + \frac{10 \cdot 2^2 \cdot 3^3}{6^5} + \frac{5 \cdot 3 \cdot 2^4}{6^5}$$

$$= \frac{243 + 1080 + 240}{6^5} = \frac{1560}{6^5} = \frac{65}{324}$$

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14. If coefficient of x^{15} in expansion of $\left(ax^3 + \frac{1}{bx^3} \right)^{15}$ is equal to coefficient of x^{-15} in expansion of

$\left(ax^3 + \frac{1}{bx^3} \right)^{15}$ then $|ab - 5|$ is equal to

Ans. (4)

Sol. $(r+1)^{\text{th}}$ term in the expansion of $\left(ax^{\frac{1}{3}} + \frac{1}{bx^3}\right)^{15}$

$$T_{r+1} = {}^{15}C_r (ax^{\frac{1}{3}})^{15-r} \left(\frac{1}{bx^3}\right)^r$$

$$= {}^{15}C_r a^{15-r} x^{45-3r-\frac{r}{3}} b^{-r}$$

for coefficient of x^{15}

$$45 - 3r - \frac{r}{3} = 15$$

$$30 = \frac{10r}{3}$$

$$r = 9$$

and $(r+1)^{\text{th}}$ term in $\left(ax^{\frac{1}{3}} + \frac{1}{bx^3}\right)^{15}$

$$T_{r+1} = {}^{15}C_r \left(ax^{\frac{1}{3}}\right)^{15-r} \left(\frac{1}{bx^3}\right)^r$$

$${}^{15}C_r a^{15-r} b^{-r} x^{\frac{15-r}{3}-3r}$$

for coefficient of x^{-15}

$$\frac{15-r}{3} - 3r = -15$$

$$15 - r - 9r = -45$$

$$60 = 10r \Rightarrow r = 6$$

$$\text{Now } {}^{15}C_9 a^6 b^9 = {}^{15}C_6 a^9 b^6$$

$$1 = a^3 b^3$$

$$ab = 1$$

$$\text{Now } |ab - 5| = |1 - 5| = 4$$

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15. If $Y = Y(x)$ satisfies the differential equation

$$\frac{dy}{dx} - \frac{(3x^5 \tan^{-1} x^3)}{(1+x^6)^{3/2}} y = 2x \exp \left\{ \frac{x^3 - \tan^{-1} x^3}{\sqrt{1+x^6}} \right\} \text{ and } y(0) = 0 \text{ then value of } y(1) \text{ is}$$

(1) $e^{\frac{1}{\sqrt{2}} \left(\frac{\pi}{4} - 1\right)}$

(2) $e^{\frac{1}{\sqrt{2}} \left(1 - \frac{\pi}{4}\right)}$

(3) 1

(4) $e^{\frac{1}{2} \left(\frac{\pi}{4}\right)}$

Ans. (2)

$$\text{Sol. } \frac{dy}{dx} - \frac{3x^5 \tan^{-1} x^3}{(1+x^6)^{3/2}} y = 2x \exp \left\{ \frac{x^3 - \tan^{-1} x^3}{\sqrt{1+x^6}} \right\}$$

$$\text{I.f.} = e^{-\int \frac{3x^5 \tan^{-1} x^3}{(1+x^6)^{3/2}} dx}$$

$$= e^{-\int \frac{3x^2 \cdot 3 \tan^{-1} x^3 - 3}{(1+x^6)^{3/2}} dx}$$

$$\text{Let } \tan^{-1} x^3 = t \Rightarrow \frac{3x^2}{1+x^6} dx = dt$$

$$= e^{-\int \frac{x^3}{\sqrt{1+x^6}} \cdot \frac{3x^2 \tan^{-1} x^3}{(1+x^6)} dx}$$

$$= e^{-\int \frac{\tan t}{\sec t} dt}$$

$$= e^{-\int t \sin t dt} = e^{-(t \cos t - \sin t)}$$

$$\left[\frac{\tan^{-1} x^3}{\sqrt{1+x^6}} - \frac{x^3}{\sqrt{1+x^6}} \right]$$

Hence solutions of the differential equation is

$$y \cdot e^{\frac{\tan^{-1}x^3}{\sqrt{1+x^6}} - \frac{x^3}{\sqrt{1+x^6}}} = \int 2x e^{\frac{y^3 - \tan^{-1}x^3}{\sqrt{1+x^6}}} \cdot e^{\frac{\tan^{-1}x^3 - x^3}{\sqrt{1+x^6}}} \cdot dx + C$$

$$= x^2 + C$$

Passes through (0,0) $\Rightarrow C = 0$

Hence $y(1) = e^{\frac{1}{\sqrt{2}} \left(1 - \frac{\pi}{4}\right)}$

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