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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2023

COMPUTER BASED TEST (CBT)

Memory Based Questions & Solutions

Date: 31 January, 2023 (SHIFT-2) | TIME : (3.00 p.m. to 6.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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1. The range of $y = \frac{x^2+2x+1}{x^2-8x+12}$ is, for $x \in (2, 6)$

(1) $(-\infty, \infty)$ (2) $\left(-\infty, \frac{21}{4}\right]$ (3) $(1, \infty)$ (4) $\left[\frac{21}{4}, \infty\right)$

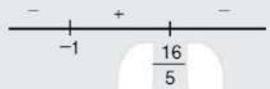
Ans. (2)

Sol. $y = \frac{x^2+2x+1}{x^2-8x+12} = \frac{(x+1)^2}{(x-2)(x-6)}$ ... (1)

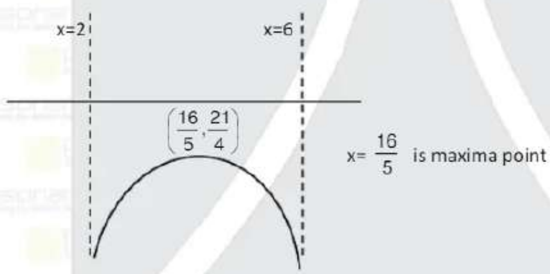
$\frac{dy}{dx} = \frac{-2(x+1)(5x-16)}{(x-2)^2(x-6)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{2(x+1)(5x-16)}{(x-2)^2(x-6)^2}$

Number line of $\frac{dy}{dx}$ =



So Graph of $y = \frac{(x+1)^2}{(x-2(x-6)}$ for $x \in (2, 6)$



at $x = \frac{16}{5}$ Þ $y = \frac{21}{4}$ so range = $\left(-\infty, \frac{21}{4}\right]$

2. If the foci of hyperbola with eccentricity $\sqrt{2}$ are at $(1 \pm \sqrt{2}, 0)$ then the length of latusrectum is

(1) $\sqrt{2}$ (2) 2 (3) $2\sqrt{2}$ (4) 4

Ans. (2)

Sol. $S(1 + \sqrt{2}, 0)$ $S'(1 - \sqrt{2}, 0)$

$SS' = 2ae = 2\sqrt{2} \Rightarrow ae = \sqrt{2} \Rightarrow a\sqrt{2} = \sqrt{2} \Rightarrow a = 1$

$L.R. = \frac{2b^2}{a} = \frac{2a^2(e^2-1)}{a} \Rightarrow 2a(e^2-1) = 2$

3. If $\int_0^a \frac{x}{\sqrt{a+x}-\sqrt{x}}dx = \frac{16+20\sqrt{2}}{15}$, then the value of a is

(1) 1 (2) 2 (3) 3 (4) 4

Ans. (2)

$\int_0^a \frac{x\left(\sqrt{a+x}+\sqrt{x}\right)}{a}dx = \frac{1}{a}\left[\int_0^a x\sqrt{a+x}\,dx + \int_0^a x^{3/2}dx\right]$

Put $a + x = t^2 \Rightarrow dx = 2tdt$

$\frac{1}{a}\int_{\sqrt{a}}^{\sqrt{2a}} (t^2 - a)2t^2dt + \frac{2a^{3/2}}{5}$

$\frac{1}{a}\left[\frac{2t^5}{5} - \frac{2at^3}{3}\right]_{\sqrt{a}}^{\sqrt{2a}} + \frac{2a^{3/2}}{5}$

$\frac{4a\sqrt{2a}+10a\sqrt{a}}{15} = \frac{16+20\sqrt{2}}{15} \Rightarrow a = 2$

4. If a_n is n^{th} term and s_n is sum of n terms of an A.P. such that $a_7 = 6$, $S_n = 0$ and a_1a_4 is minimum then the value of $n! + a_n(n+2)$ is

(1) 140 (2) 139 (3) 145 (4) 141

Ans. (4)

Sol. $a_7 = 6$

$\Rightarrow a + 6d = 6$

$\Rightarrow a + 6d = 0$
$\Rightarrow a = 6 - 6d$
$a_1a_4 = a(a + 3d)$
$= (6 - 6d)(6 - 6d + 3d)$
$= 18(d - 1)(d - 2)$
$= 18(d^2 - 3d + 2)$
Let $f(d) = 18(d^2 - 3d + 2)$
$f'(d) = 18(2d - 3)$
at $d = \frac{3}{2}$, a_1a_4 is minimum
So, $a = 6 - 6d = -3$
$S_n = 0$
$\Rightarrow \frac{n}{2}[2a + (n - 1)d] = 0$
$\Rightarrow \frac{n}{2}\left[-6 + (n-1)\frac{3}{2}\right] = 0]$
$\Rightarrow n = 5$
Now, $a_5 = a + 4d = 3$
Now $n! + a_n (n + 2) = 5! + 3 \times 7$
$= 120 + 21 = 141$

5. The coefficient of x^{-6} in the expansion of $\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9$ is

(1) 5140 (2) 5240 (3) 5040 (4) 4940

Ans. **(3)**

Sol. $T_{r+1} = {}^9C_r \left(\frac{5}{2x^2}\right)^r \cdot \left(\frac{4x}{5}\right)^{9-r}$

$= {}^9C_r \frac{2^{18-3r}}{5^{9-2r}} x^{9-3r}$

for coefficient of $x^{-6} \Rightarrow 9 - 3r = -6 \Rightarrow r = 5$

coff. of $x^{-6} = \frac{{}^9C_5 \cdot 2^3}{5^{-1}}$

$= \frac{9.8.7.5}{4.3.2.1} .(8 \times 5)$

$= 5040$

6. If $\frac{{}^{2n+1}P_{n-1}}{{}^{2n-1}P_n} = \frac{11}{21}$ then the value of $n^2 + n + 15$ is.

(1) 45 (2) 50 (3) 35 (4) 57

Ans. **(1)**

Sol. $\frac{\left(\frac{(2n+1)!}{(n+2)!}\right)}{\frac{(2n-1)!}{(n-1)!}} = \frac{11}{21}$

$\Rightarrow \frac{(2n+1)(2n)}{(n+2)(n+1)n} = \frac{11}{21}$

$\Rightarrow 42(2n + 1) = 11 (n + 1) (n + 2)$
$\Rightarrow 11n^2 - 51n - 20 = 0$
$\Rightarrow (n - 5) (11n + 4) = 0$
$\Rightarrow n = 5$
$\Rightarrow n^2 + n + 15 = 45$

7. If $z = \frac{i - 1}{\cos\pi/6 + i\sin\pi/6}$, then z is equal to

(1) $\sqrt{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$ (2) $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

(3) $\sqrt{2}\left(\cos\frac{7\pi}{12} - i\sin\frac{7\pi}{12}\right)$ (4) $\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$

Ans. **(1)**

Ans. (1)

Sol. $Z = \dfrac{i-1}{\cos\pi/6 + i\sin\pi/6} = \dfrac{\sqrt{2}\,e^{i3\pi/4}}{e^{i\pi/6}} = \sqrt{2}\,e^{i7\pi/12}$

$= \sqrt{2}\left(\cos\dfrac{7\pi}{12} + i\sin\dfrac{7\pi}{12}\right)$

8. Sum of series $1^2 - 2.3^2 + 3.5^2 - 4.7^2 + \ldots\ldots + 15.29^2$ is

Ans. **(6952)**

Sol. $1^2 - 2.3^2 + 3.5^2 - 4.7^2 + \ldots\ldots + 15.29^2 = !!\ 5952$

$\sum_{r=1}^{15}(2r-1)^2 - \sum 2r(r-1)^2$

$1 + \sum_{r=1}^{7}(2r+1)\,(4r+1)^2 - 2r(4r-1)^2$

$1 + \sum_{r=1}^{7} 2r(16r + (4r+1)^2$

$1 + 48\sum_{r=1}^{7} r^2 + 8\sum r + \sum 1$

$1 + 48\times\dfrac{7\times8\times15}{6} + 8\times\dfrac{7\times8}{2} + 7$

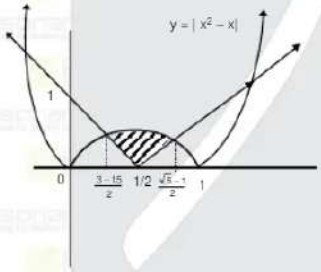
$1 + 56 \times 120 + 56 \times 4 + 7$

$8 + 56 \times 124 = 6944 + 8 = 6952$

9. If area of curve $|2x-1| \le y \le |x^2 - x|$ is A then value of $(6A + 11)^2$ is.

Ans. **125**

Sol. $y \ge |2x-1|$ $\qquad$ $y \le |x^2 - x|$



Both curve are symmetric about $x = \dfrac{1}{2}$ Hence

$A = 2\int_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}} \left((x - x^2) - (1-2x)\right)dx$

$A = 2\int_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}} \left(-x^2 + 3x - 1\right)dx = 2\left(\dfrac{-x^3}{3} + \dfrac{3}{2}x^2 - x\right)_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}}$

on solving $\quad 6A + = 5\sqrt{5}$

$(6A + 11)^2 = 125$ Ans.

10. The equation $e^{4x} + 3e^{3x} + 13e^{2x} - 8e^{x} + 1 = 0$ has

(1) Two real root both are positive (2) No real roots

(3) Four real roots of which two are negative (4) Two real roots both are negative

Ans. **(4)**

Sol. Let $e^x = t$

$f(t) = t^4 + 3t^3 + 13t^2 - 8t + 1$

$f'(t) = 4t^3 + 9t^2 + 26t - 8$

$f''(t) = 12t^2 + 18t + 26 > 0 \ \forall \ t$

$\Rightarrow f'(t) = 0$ has at most one real root

$\because f'(0) < 0$ and $f'\left(\frac{1}{2}\right) > 0$

then real root of $f'(t) = 0$ lies between 0 and $\frac{1}{2}$

$\because f\left(\frac{1}{4}\right) < 0$ and $f\left(\frac{1}{2}\right) > 0$ and $f(0) > 0$

$\Rightarrow f(t) = 0$ has at most two real roots

Let α, β be real roots of $f(t) = 0$

for roots of given equation $\Rightarrow e^x = \alpha, \beta$

$\Rightarrow x = \ell n\alpha, \ \ell n\beta < 0$

given equation has two negative real roots

11. If $a, b \in I$ and relation R_1 is defined as $a^2 - b^2 \in I$ and relation R_2 is defined as $2 + \frac{a}{b} > 0$ then.

(1) R_1 is symmetric but R_2 is not (2) R_2 is symmetric but R_1 is not

(3) R_1 and R_2 are both symmetric (4) R_1 and R_2 are both transitive

Ans. **(1)**

Sol. $a \ R_1 \ b \Rightarrow a^2 - b^2 \in I$ always true Hence equivalence

$a \ R_2 \ b \Rightarrow 2 + \frac{a}{b} > 0$ not symmetric when $a = 2, b = 1$

When $a = -8, b = 6, c = 3$

Clearly $(-8, 6) \in R_2, (6, 3) \in R_2$ but $(-8, 3) \notin R_2$

Hence not transitive.

12. Let A be an event that the absolute difference of two real number lying between 0 and 60 is less than a.

If the probability of happening of event A is $\frac{11}{36}$ then a is equal to

Ans. **10**

Sol. $|x - y| < a \Rightarrow -a < x - y < a$

$\Rightarrow \quad x - y < a$ and $x - y > -a$

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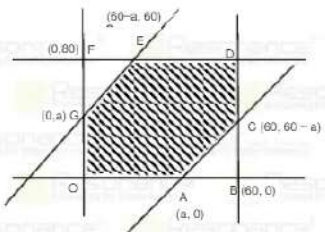

$$P(A)=\frac{ar(OACDEG)}{(OBDF)}$$

$$=\frac{ar(OBDF)-ar(ABC)-ar(EFG)}{ar(OBDF)}$$

$$\Rightarrow \frac{11}{36}=\frac{(60)^2-\frac{1}{2}(60-a)^2-\frac{1}{2}(60-a)^2}{3600}$$

$$\Rightarrow 1100=3600-(60-a)^2$$

$$\Rightarrow (60-a)^2=2500 \Rightarrow 60-a=50$$

$$\Rightarrow a=10$$

13. If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+3\hat{k}$, $\vec{c}=\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{r}$ is a vector such that $\vec{r}\times\vec{b}=\vec{b}\times\vec{c}$ & $\vec{r}.\vec{a}=0$

then the value of $25.|\vec{r}|^2$

Ans. 202.75

Sol. $\vec{r}\times\vec{b}=\vec{b}\times\vec{c}=-\vec{c}\times\vec{b}$

$$\Rightarrow \left(\vec{r}+\vec{c}\right)\times\vec{b}=0 \Rightarrow \vec{r}+\vec{c}=\lambda\vec{b} \Rightarrow \vec{r}=\lambda\vec{b}-\vec{c}$$

$$\vec{r}.\vec{a}=\lambda\left(\vec{b}.\vec{a}\right)-\vec{c}.\vec{a}=0$$

$$\lambda=\frac{\vec{c}.\vec{a}}{\vec{b}.\vec{a}}=\frac{-1}{-10}=\frac{1}{10}$$

$$\vec{r}=\frac{1}{10}\vec{b}-\vec{c}=\frac{1}{10}\left(\hat{i}-\hat{j}+3\hat{k}\right)-\left(\hat{i}+2\hat{j}+2\hat{k}\right)$$

$$\vec{r}=\frac{-9}{10}\hat{i}-\frac{21}{10}\hat{j}-\frac{17}{10}\hat{k}$$

$$10\vec{r}=-9\hat{i}-21\hat{j}-17\hat{k}$$

$$100\,|\vec{r}|^2=811$$

$$25\,|\vec{r}|^2=\frac{811}{4}=202.75$$


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