166 (I)

Total No. of Questions—**24**

Total No. of Printed Pages—**4** **Regd. No.** | | | | | | | | | | |

Part III

MATHEMATICS

Paper I-A

(English Version)

Time : 3 Hours ***Max. Marks : 75***

***Note* :—** This question paper consists of THREE Sections A, B and C.

SECTION A 10×2=20

(I) Very short answer type questions :

(*i*) Answer ALL questions.

(*ii*) Each question carries TWO marks.

1. If $A = \{-2, -1, 0, 1, 2\}$ and $f : A \rightarrow B$ is a surjection defined by $f(x) = x^2 + x + 1$, then find B.

2. Find the domain of the real valued function $f(x) = \sqrt{4x - x^2}$.

3. If $A = \begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}$, then find $A + A'$ and AA'.

4. Find the rank of the matrix $\begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$.

5. If the position vectors of the points A, B and C are $-2\bar{i} + \bar{j} - \bar{k}$, $-4\bar{i} + 2\bar{j} + 2\bar{k}$ and $6\bar{i} - 3\bar{j} - 13\bar{k}$ respectively and $\overline{AB} = \lambda \overline{AC}$, then find the value of λ.

6. Find the vector equation of the plane passing through the points $\bar{i} - 2\bar{j} + 5\bar{k}$, $-5\bar{j} - \bar{k}$ and $-3\bar{i} + 5\bar{j}$.

7. $\bar{a} = 2\bar{i} - \bar{j} + \bar{k}$, $\bar{b} = \bar{i} - 3\bar{j} - 5\bar{k}$. Find the vector $\bar{c}$ such that $\bar{a}, \bar{b}$ and $\bar{c}$ form the sides of a triangle.

8. Sketch the graph of the function $\sin 2x$ in the interval $(0, \pi)$.

9. Evaluate $\cos^2 52\frac{1^\circ}{2} - \sin^2 22\frac{1^\circ}{2}$.

10. If $\sinh x = 3$, then show that $x = \log_e\left(3 + \sqrt{10}\right)$.

SECTION B

5×4=20

(II) Short answer type questions :

(*i*) Answer ANY FIVE questions.

(*ii*) Each question carries FOUR marks.

11. If $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is non-singular matrix, then A is invertible and prove that $A^{-1} = \frac{\text{Adj A}}{\det A}$.

12. If $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors, prove that the following four points are co-planar $6\bar{a} + 2\bar{b} - \bar{c}$, $2\bar{a} - \bar{b} + 3\bar{c}$, $-\bar{a} + 2\bar{b} - 4\bar{c}$, $-12\bar{a} - \bar{b} - 3\bar{c}$.

13. If $\bar{a}+\bar{b}+\bar{c}=0$, $|\bar{a}|=3$, $|\bar{b}|=5$ and $|\bar{c}|=7$, then find the angle between $\bar{a}$ and $\bar{b}$.

14. If a, b, c are non-zero real numbers and α, β are solutions of the equation $a\cos\theta + b\sin\theta = c$, then show that :

(i) $\sin\alpha + \sin\beta = \dfrac{2bc}{a^2+b^2}$

(ii) $\sin\alpha \cdot \sin\beta = \dfrac{c^2-a^2}{a^2+b^2}$.

15. Solve the equation $\cot^2 x - (\sqrt{3}+1)\cot x + \sqrt{3} = 0;\ 0 < x < \frac{\pi}{2}$.

16. Prove that :

$$\tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} - \tan^{-1}\frac{2}{9} = 0.$$

17. Prove that :

$$\frac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B} = \frac{a^2+b^2}{a^2+c^2}.$$

SECTION C

5×7=35

(III) Long answer type questions :

(i) Answer ANY FIVE questions.

(ii) Each question carries SEVEN marks.

18. Let $f : A \to B$, $g : B \to C$ be bijections, then prove that $(g\,o\,f)^{-1} = f^{-1}\,o\,g^{-1}$.

19. Using mathematical induction, prove that :

$$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\cdots\cdots\left(1+\frac{2n+1}{n^2}\right) = (n+1)^2.$$

20. Find the value of x if $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0.$

21. Examine whether the equations are consistent or inconsistent and if consistent find the complete solution :

$$x + y + z = 6$$

$$x - y + z = 2$$

$$2x - y + 3z = 9.$$

22. $\bar{a}, \bar{b}, \bar{c}$ are three vectors, then prove that :

$$(\bar{a} \times \bar{b}) \times \bar{c} = (\bar{a} \cdot \bar{c})\,\bar{b} - (\bar{b} \cdot \bar{c})\,\bar{a}.$$

23. If A, B, C are angles in a triangle, then prove that :

$$\sin^2 A + \sin^2 B - \sin^2 C = 2 \sin A \sin B \cos C.$$

24. In ΔABC, show that :

$$(r_1 + r_2)\sec^2\frac{C}{2} = (r_2 + r_3)\sec^2\frac{A}{2} = (r_3 + r_1)\sec^2\frac{B}{2}.$$