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II

Total No. of Questions – 24

Total No. of Printed Pages - 3

Regd. No. | | | | | | | | | | |

Part – III
MATHEMATICS, Paper – I(A)
(English Version)

Time : 3 Hours] **[Max. Marks : 75**

Note : This question paper consists of **three** Sections – **A**, **B** and **C**.

SECTION – A

$10 \times 2 = 20$

I. Very Short Answer Type questions :

(i) Answer **all** the questions.

(ii) Each question carries **two** marks.

1. If $f : R - \{\pm 1\} \to R$ is defined by $f(x) = \log \left|\frac{1+x}{1-x}\right|$, then show that $f\left(\frac{2x}{1+x^2}\right) = 2\, f(x)$.

2. Find the domain of the real valued function $f(x) = \sqrt{x^2 - 25}$.

3. If $A = \begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}$, then find $A + A^T$ and AA^T.

4. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

5. Let $\bar{a} = \bar{i} + 2\bar{j} + 3\bar{k}$ and $\bar{b} = 3\bar{i} + \bar{j}$. Find the unit vector in the direction of $\bar{a} + \bar{b}$.

6. Find the vector equation of the plane passing through the points (0, 0, 0), (0, 5, 0) and (2, 0, 1).

7. If $\bar{a} = \bar{i} + 2\bar{j} - 3\bar{k}$ and $\bar{b} = 3\bar{i} - \bar{j} + 2\bar{k}$, then show that $\bar{a} + \bar{b}$ and $\bar{a} - \bar{b}$ are perpendicular to each other.

8. If $\sec\theta + \tan\theta = \frac{2}{3}$, find the value of $\sin\theta$.

9. If A is not an integral multiple of $\frac{\pi}{2}$, prove that $\tan A + \cot A = 2 \operatorname{cosec} 2A$.

10. If $\cosh x = \frac{5}{2}$, find the values of (i) $\cosh(2x)$ and (ii) $\sinh(2x)$.

SECTION – B $5 \times 4 = 20$

II. Short Answer Type questions :

(i) Answer any **five** questions.

(ii) Each question carries **four** marks.

11. If $\theta - \phi = \frac{\pi}{2}$, then show that

$$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix} = 0.$$

12. Let ABCDEF be a regular hexagon with centre 'O'. Then show that $\overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} = 3\overline{AD} = 6\overline{AO}$.

13. Let $\bar{a} = 4\bar{i} + 5\bar{j} - \bar{k}$, $\bar{b} = \bar{i} - 4\bar{j} + 5\bar{k}$ and $\bar{c} = 3\bar{i} + \bar{j} - \bar{k}$. Find vector $\bar{\alpha}$ which is perpendicular to both $\bar{a}$ and $\bar{b}$ and $\bar{\alpha} \cdot \bar{c} = 21$.

14. Prove that $\cos^2 76° + \cos^2 16° - \cos 76° \cos 16° = \frac{3}{4}$.

15. Solve $\sqrt{2}(\sin x + \cos x) = \sqrt{3}$.

16. Prove that $\cos\left(2\tan^{-1}\frac{1}{7}\right) = \sin\left(2\tan^{-1}\frac{3}{4}\right)$.

17. Show that $a^2 \cot A + b^2 \cot B + c^2 \cot C = \frac{abc}{R}$ in a triangle.

SECTION – C

$5 \times 7 = 35$

III. Long Answer Type questions :

(i) Answer any **five** questions.

(ii) Each question carries **seven** marks.

18. Let $f : A \rightarrow B$ be a function. Then f is a bijection if and only if there exists a function $g : B \rightarrow A$ such that $fog = I_B$ and $gof = I_A$ and, in this case, $g = f^{-1}$.

19. Using mathematical induction, prove that :

$3.5^{2n+1} + 2^{3n+1}$ is divisible by 17, for all $n \in N$.

20. Show that

$$\det\begin{bmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{bmatrix} = (a+b+c)^3.$$

21. Solve the following equations by Gauss-Jordan Method :

$x + y + z = 1$, $2x + 2y + 3z = 6$ and $x + 4y + 9z = 3$.

22. If $\bar{a} = \bar{i} - 2\bar{j} - 3\bar{k}$, $\bar{b} = 2\bar{i} + \bar{j} - \bar{k}$ and $\bar{c} = \bar{i} + 3\bar{j} - 2\bar{k}$, verify that $\bar{a} \times (\bar{b} \times \bar{c}) \neq (\bar{a} \times \bar{b}) \times \bar{c}$.

23. If A, B, C are angles of a triangle, then prove that

$$\sin^2\frac{A}{2} + \sin^2\frac{B}{2} - \sin^2\frac{C}{2} = 1 - 2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}.$$

24. Show that $r + r_3 + r_1 - r_2 = 4R\cos B$, in ΔABC.