166 **III**

Total No. of Questions – 24 **Regd. No.**

Total No. of Printed Pages - 4

Part – III
MATHEMATICS, Paper – I(A)
(English Version)

Time : 3 Hours] *[Max. Marks : 75*

Note : This question paper consists of **three** Sections – **A**, **B** and **C**.

SECTION – A $10 \times 2 = 20$

I. Very Short Answer Type questions :

(i) Answer **all** the questions.

(ii) Each question carries **two** marks.

1. If $A = \{-2, -1, 0, 1, 2\}$ and $f : A \to B$ is a surjection defined by $f(x) = x^2 + x + 1$, then find B.

2. If $f(x) = 2x - 1$, $g(x) = \frac{x+1}{2}$ for all $x \in \mathbb{R}$, then find (gof) (x).

3. If $\begin{bmatrix} x-3 & 2y-8 \\ z+2 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -2 & a-4 \end{bmatrix}$, then find the values of x, y, z and a.

4. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ find the rank of this matrix.

5. Let $\bar{a} = 2\bar{i} + 4\bar{j} - 5\bar{k}$, $\bar{b} = \bar{i} + \bar{j} + \bar{k}$ and $\bar{c} = \bar{j} + 2\bar{k}$. Find the unit vector in the opposite direction of $\bar{a} + \bar{b} + \bar{c}$.

6. Find the vector equation of the plane passing through the points

$\bar{i} - 2\bar{j} + 5\bar{k}$, $-5\bar{j} - \bar{k}$ and $-3\bar{i} + 5\bar{j}$.

7. If the vectors $\lambda\bar{i} - 3\bar{j} + 5\bar{k}$ and $2\lambda\bar{i} - \lambda\bar{j} - \bar{k}$ are perpendicular to each other, find λ.

8. If $\sin\theta = \frac{4}{5}$ and θ is not in the first quadrant, find the value of $\cos\theta$.

9. If θ is not an integral multiple of $\frac{\pi}{2}$, prove that

$\tan\theta + 2\tan 2\theta + 4\tan 4\theta + 8\cot 8\theta = \cot\theta$.

10. Show that $\tanh^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}\log_e 3$.

SECTION – B

$5 \times 4 = 20$

II. Short Answer Type questions :

(i) Answer any **five** questions.

(ii) Each question carries **four** marks.

11. If $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, then show that the adjoint of A is $3A'$. Find A^{-1}.

12. Show that the line joining the pair of points $6\bar{a} - 4\bar{b} + 4\bar{c}$, $-4\bar{c}$ and the line joining the pair of points $-\bar{a} - 2\bar{b} - 3\bar{c}$, $\bar{a} + 2\bar{b} - 5\bar{c}$, intersect at the point $-4\bar{c}$, when $\bar{a}$, $\bar{b}$, $\bar{c}$ are non-coplanar vectors.

13. If $\bar{a} = 2\bar{i} + \bar{j} - \bar{k}$, $\bar{b} = -\bar{i} + 2\bar{j} - 4\bar{k}$ and $\bar{c} = \bar{i} + \bar{j} + \bar{k}$, then find $(\bar{a} \times \bar{b}) \cdot (\bar{b} \times \bar{c})$.

14. Prove that $\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right) = \frac{1}{16}$.

15. Give $p \neq \pm q$. show that the solutions of $\cos p\theta + \cos q\theta = 0$ form two series each of which is in A.P. Also, find the common difference of each A.P.

16. Prove that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

17. In ΔABC if $a = (b + c)\cos\theta$, then prove that $\sin\theta = \frac{2\sqrt{bc}}{b+c}\cos\frac{A}{2}$.

SECTION – C

$5 \times 7 = 35$

III. Long Answer Type questions :

(i) Answer any **five** questions.

(ii) Each question carries **seven** marks.

18. Let $f : A \to B$, $g : B \to C$ be bijections, then prove that $(gof)^{-1} = f^{-1}og^{-1}$.

19. Using mathematical induction, prove that for all $n \in N$,

$a + ar + ar^2 + \ldots$ upto n terms $= \frac{a(r^n - 1)}{(r-1)}$, $r \neq 1$.

20. Show that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ac - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix} = (a^3 + b^3 + c^3 - 3abc)^2$.

21. $x - y + 3z = 5$, $4x + 2y - z = 0$, $-x + 3y + z = 5$, solve the system of equations by using Cramer's rule.

22. Show that the volume of a tetrahedron with $\bar{a}$, $\bar{b}$ and $\bar{c}$ as coterminous edges is $\frac{1}{6}|[\bar{a}\ \bar{b}\ \bar{c}]|$.

23. If $A + B + C = 0$, then prove that $\sin 2A + \sin 2B + \sin 2C = -4\sin A \sin B \sin C$.

24. In a ΔABC, if $a = 13$, $b = 14$, $c = 15$, show that $R = \frac{65}{8}$, $r = 4$, $r_1 = \frac{21}{2}$, $r_2 = 12$ and $r_3 = 14$.