

293

III

Total No. of Questions – 24

Regd.

Total No. of Printed Pages – 3

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Part – III

MATHEMATICS, Paper-II(B)
(English Version)

Time : 3 Hours]

[Max. Marks : 75

Note : This question paper consists of three sections A, B, C.

SECTION – A

10 × 2 = 20

I. Very short answer type questions :

- (i) Attempt **all** questions.
- (ii) Each question carries **two** marks.

1. Find the equation of the circle passing through (3, 4) and having the centre at (-3, 4).
2. If the length of the tangent from (5, 4) to the circle $x^2 + y^2 + 2ky = 0$ is 'l' then find 'k'.
3. Find the angle between the circles $x^2 + y^2 - 12x - 6y + 41 = 0$, $x^2 + y^2 + 4x + 6y - 59 = 0$.
4. If $\left(\frac{1}{2}, 2\right)$ is one extremity of a focal chord of the parabola $y^2 = 8x$, find the co-ordinates of the other extremity.

5. Find the equation of the hyperbola whose foci are $(\pm 5, 0)$, the transverse axis is of length 8.

6. Evaluate : $\int \frac{(3x+1)^2}{2x} dx$ $x \in \mathbb{R} - \{0\}$

7. Evaluate : $\int \frac{\log x}{x^2} dx$ on $(0, \infty)$

8. Evaluate : $\lim_{n \rightarrow \infty} \frac{2^k + 4^k + 6^k + \dots + (2n)^k}{n^{k+1}}$ by using the method of finding definite integral of the limit of a sum.

9. Find the value of $\int_0^{2\pi} \sin^2 x \cos^4 x dx$.

10. Solve $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$.

SECTION - B

5 × 4 = 20

II. Short answer type questions :

(i) Attempt any **five** questions.

(ii) Each questions carries **four** marks.

11. Find the equation of the circle with centre $(-2, 3)$ cutting a chord length 2 units on $3x + 4y + 4 = 0$.

12. Find the equation of the circle which passes through origin and intersects the circles $x^2 + y^2 - 4x + 6y + 10 = 0$, $x^2 + y^2 + 12y + 6 = 0$ orthogonally.

13. Find the length of the latus rectum, eccentricity, centre and foci of the ellipse $4x^2 + y^2 - 8x + 2y + 1 = 0$.

14. A circle of radius 4 is concentric with the ellipse $3x^2 + 13y^2 = 78$. Prove that a common tangent is inclined to the major axis at an angle $\frac{\pi}{4}$.

15. Tangents to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ make angles θ_1, θ_2 with transverse axis of a hyperbola.

Show that the point of intersection of these tangents lies on the curve $2xy = k(x^2 - a^2)$ when $\tan \theta_1 + \tan \theta_2 = k$.

16. Evaluate : $\int_0^{\pi/4} \log(1 + \tan x) dx$.

17. Solve : $\frac{1}{x} \frac{dy}{dx} + y e^x = e^{(1-x)e^x}$

SECTION - C

5 × 7 = 35

III. Long answer type questions :

- (i) Attempt any **five** questions.
 (ii) Each question carries **Seven** marks.

18. Find the equation of the circle passing through (2, 1), (5, 5) and (-6, 7).

19. Show that $x^2 + y^2 - 6x - 9y + 13 = 0$, $x^2 + y^2 - 2x - 16y = 0$ circles touch each other. Find the point of contact and the equation of common tangent at their point of contact.

20. Find the equation of the parabola whose axis is parallel to x-axis and which passes through the points (-2, 1), (1, 2) and (-1, 3).

21. Evaluate : $\int (6x + 5) \sqrt{6 - 2x^2 + x} dx$

22. If $I_n = \int \cos^n x dx$ then show that $I_n = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} I_{n-2}$, n being a positive integer $n \geq 2$ and deduce the value of $\int \cos^5 x dx$.

23. Evaluate : $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$.

24. Solve $(x^3 - 3xy^2) dx + (3x^2y - y^3) dy = 0$.