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III

Total No. of Questions – 24
Total No. of Printed Pages – 3

Regd. No.

Part – III

MATHEMATICS, Paper-II(B)

(English Version)

Time : 3 Hours] *[Max. Marks : 75*

Note : This question paper consists of three sections A, B and C.

SECTION – A

$10 \times 2 = 20$

I. Very short answer type questions :

(i) Attempt all questions.

(ii) Each question carries two marks.

1. Find the other end of the diameter of the circle $x^2 + y^2 - 8x - 8y + 27 = 0$ if one end of it is (2, 3).

2. Define chord of contact and find the chord of contact of (1, 1) to the circle $x^2 + y^2 = 9$.

3. Find k if the circles $x^2 + y^2 - 5x - 14y - 34 = 0$ and $x^2 + y^2 + 2x + 4y + k = 0$ are orthogonal.

4. Find the equation of the parabola whose vertex is (3, –2) and focus is (3, 1).

5. If $3x - 4y + k = 0$ is a tangent to the hyperbola $x^2 - 4y^2 = 5$, find the value of k.

6. Evaluate : $\int \frac{\cos x}{(1+\sin x)^2}\, dx$.

7. Evaluate : $\int x \log x \, dx$ on $(0, \infty)$.

8. Evaluate : $\lim_{n \to \infty} \frac{1 + 2^4 + 3^4 + \ldots\ldots + n^4}{n^5}$

9. Find : $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^4 x \, dx$.

10. Solve : $y(1 + x) \, dx + x (1 + y) \, dy = 0$.

SECTION – B **5 × 4 = 20**

II. Short answer type questions :

(i) Attempt any five questions.

(ii) Each question carries four marks.

11. Find the area of the triangle formed by the tangent at $P(x_1, y_1)$ to the circle $x^2 + y^2 = a^2$ with the co-ordinate axes where $x, y, \neq 0$.

12. If the two circles $x^2 + y^2 + 2gx + 2fy = 0$ and $x^2 + y^2 + 2g'x + 2f'y = 0$ touch each other then show that $f'g = fg'$.

13. S and T are the foci of an ellipse and B is one end of the minor axis. If STB is an equilateral triangle, then find the eccentricity of the ellipse.

14. Find the condition for the line

$x \cos \alpha + y \sin \alpha = P$ to be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

15. Find the centre, foci, eccentricity, equation of the directrices of the hyperbola $x^2 - 4y^2 = 4$.

16. Find the area of the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$.

17. Solve : $(x^2 + y^2)dx = 2xy \, dy$.

SECTION – C

$5 \times 7 = 35$

III. Long answer type questions :

(i) Attempt any five questions.

(ii) Each question carries seven marks.

18. Find the equation of the circle which passes through (4, 1), (6, 5) and having the centre on $4x + 3y - 24 = 0$.

19. Find the equation of the circle which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ externally at (5, 5) with radius 5.

20. From an external point P tangents are drawn to the parabola $y^2 = 4ax$ and these tangents make angles θ_1, θ_2 with its axis, such that $\tan\theta_1 + \tan\theta_2$ is a constant b. Then show that P lies on the line $y = bx$.

21. Evaluate : $\int \frac{1}{1 + \sin x + \cos x}\, dx$.

22. If $I_n = \int \cos^n x\, dx$, then show that $I_n = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n} I_{n-2}$. (where $n \geq 2$)

23. Show that : $\int_0^{\pi/2} \frac{x}{\sin x + \cos x}\, dx = \frac{\pi}{2\sqrt{2}} \log(\sqrt{2} + 1)$.

24. Solve : $x \log x \frac{dy}{dx} + y = 2 \log x$.
