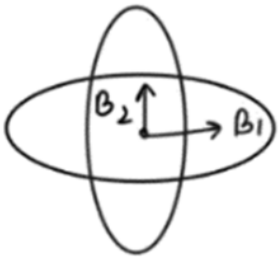
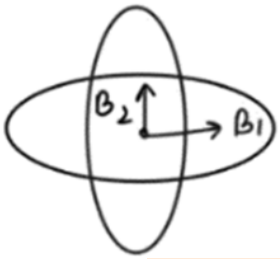
# JEE-Mains-06-04-2023 [Memory Based] [Morning Shift]

## Physics

**Question:** Current $\sqrt{2I}$ in both rings, find resultant B?




**Solution:**




Current $=\sqrt{2}I$

$$B_1 = B_2 = \frac{\mu_0\left(\sqrt{2}I\right)}{2a}$$

So,

$$B_N = \sqrt{B_1^2 + B_2^2}$$

$$= \frac{\mu_0}{2a}\left(\sqrt{2}I\right)\sqrt{2} = \left(\frac{\mu_0 I}{a}\right)$$

**Question:** If rate of heat supplied to the system is 1000 watt and the rate of work done by the system is 200 watt. Find rate of change of internal energy.

**Answer: 800.00**

**Solution:**

$\frac{dQ}{dt} = +1000 \text{ watt}$

$\frac{dw}{dt} = +200 \text{ watt}$

$\frac{dQ}{dt} = \frac{dw}{dt} + \frac{du}{dt}$

$+1000 = +200 + \frac{dw}{dt}$

$\frac{dw}{dt} = +800 \text{ watt}$

**Question:** Find the ratio of energy density of E and B in EM waves.
**Options:**
(a) 1 : 1
(b) 1 : 2
(c) 2 : 1
(d) None of these
**Answer: (a)**
**Solution:**

Average electric field energy density = $\frac{1}{2}\varepsilon_0 E^2$

Average magnetic field energy density = $\frac{B^2}{2\mu_0}$

As both are equal

$\frac{\frac{1}{2}\varepsilon_0 E^2}{\frac{B^2}{2\mu_0}} = 1$

**Question:** Percentage error in equivalent resistance if connected in parallel (10 ± 0.5) ohm and (15 ± 0.5) ohm
**Options:**
(a) 13 %
(b) 3 %
(c) 13/5 %
(d) 13/3 %
**Answer: (d)**
**Solution:**

$$\frac{\Delta R_{eq}}{R_{eq}^2}=\frac{\Delta R_1}{\Delta R_1^2}+\frac{\Delta R_2}{\Delta R_2^2}$$

$$\frac{\Delta R_{eq}}{R_{eq}}=\left(\frac{0.5}{10^2}+\frac{0.5}{15^2}\right)\times\frac{15\times10}{15+10}$$

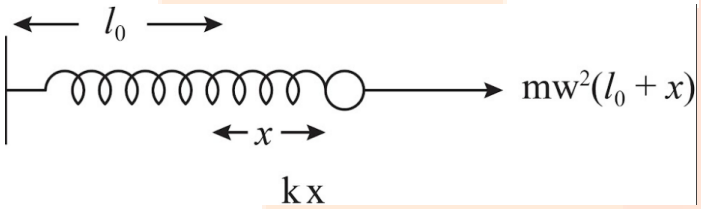
$$=0.5\left(\frac{1}{10^2}+\frac{1}{15^2}\right)\times\frac{150}{25}$$

$$=3\left(\frac{1}{100}+\frac{1}{225}\right)=3\left(\frac{225+100}{225\times100}\right)$$

$$\frac{\Delta R}{R}\times100=\frac{3\times325}{225\times100}\times100=\frac{13}{3}\%$$

**Question:** Assertion: Earth has atmosphere while moon does not.
Reason: Escape velocity in moon is very small than earth.
**Options:**
(a) A correct, R correct & R is correct explanation
(b) A correct, R correct but not correct explanation
(c) A correct, R false
(d) A false, R false
**Answer: (a)**
**Solution:** A correct, R correct & R is correct explanation

**Question:** A mass of 100 g is rotated with a spring of natural length 20 cm, with angular velocity 5 $rads^{-1}$. Find tension in spring [R = spring constant 7.5 $nm^{-1}$]
**Answer:** 0.75
**Solution:**




$$T=kx=m\omega^2(l_0+x)$$

$$7.5x=\frac{1}{10}\times25(0.2+x)$$

$$3x=0.2+x$$

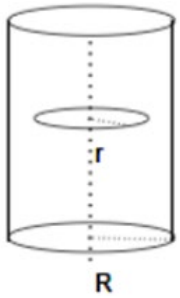
$$x=0.1$$

so $T=kx=7.5\times0.1=0.75\,\text{N}$

**Question:** A solid infinite cylindrical wire with radius a is carrying current I find the graph of magnetic field inside & outside the wire.

**Answer:**

**Solution:**

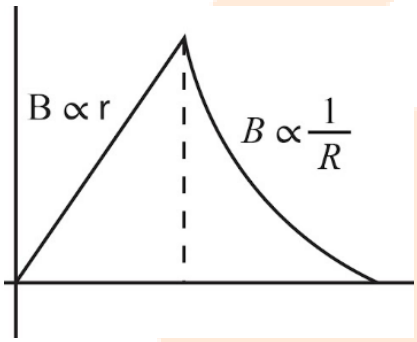



Dependency of magnetic fixed in solid current carrying wire

$$B_{out} = \frac{\mu_0 i}{2\pi r}$$

$$B_{surface} = \frac{\mu_o i}{2\pi R}$$

$B_{inside}$




Current through h loop

$$i_1 = \frac{i}{\pi R^2} \times \pi r^2$$

$$\oint B \cdot dl = \mu_o i$$

$$B.2\pi r = \mu_o \frac{i \times \pi r^2}{\pi R^2}$$

$$B = \frac{\mu_o i}{2\pi R^2} \times r$$

$$B \propto r$$

**Question:** A: range is max at θ = 45°
R : range is max when sin 2θ = 1

**Options:**

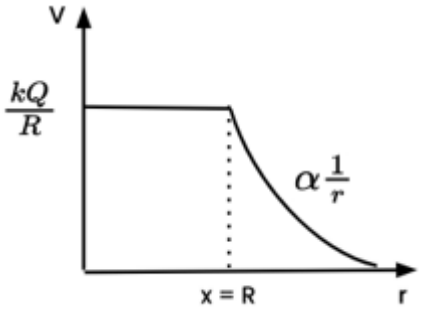
(a) R : true A : False
(b) R : true A : True
(c) R : False A : False
(d) R : False A : True

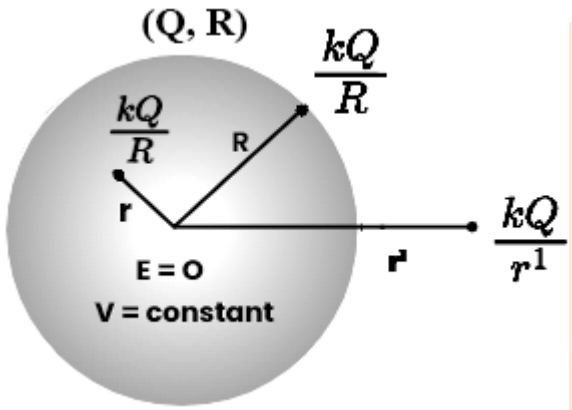
**Answer: (b)**

**Question:** Graph of electric potential inside conducting solid sphere is

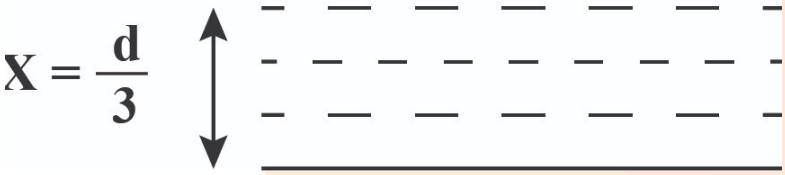
**Solution:**

$$v_{\text{inside}} = v_{\text{surface}} = \frac{kQ}{R} = \text{constant}$$



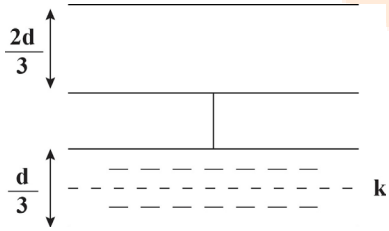




**Question:** In a capacitor when liquid of dielectric constant 'k' is filled upto height d/3 then capacitance is 2μF. Find capacitance when it is filled till $x = 2d/3$ Take $k = 2$



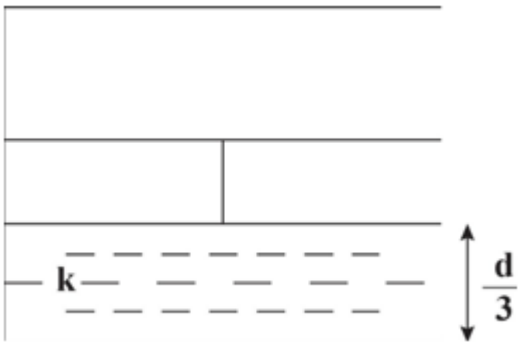

**Answer: 2.5**

**Solution:**




$$C_{eq} = \frac{\frac{3Ak\varepsilon}{d} \cdot \frac{3A\varepsilon_0}{2d}}{3\frac{A\varepsilon_0}{d}\left[k + \frac{1}{2}\right]}$$

$$2 = \frac{3K}{\left(K + \frac{1}{2}\right)} \frac{A\varepsilon_0}{2d} \Rightarrow 2 = \frac{3 \times 2}{\frac{5}{2}} \frac{A\varepsilon_0}{2d} \Rightarrow \frac{A\varepsilon_0}{d} = \frac{10}{6}$$




$$C_{eq}=\frac{\frac{3AK\varepsilon_0}{2d}\cdot\frac{3A\varepsilon_0}{d}}{\frac{3A\varepsilon_0}{d}\left[\frac{k}{2}+1\right]}=\frac{3A\varepsilon_0}{2d}\frac{k}{\left[\frac{k}{2}+1\right]}$$

$$C_{eq}=\frac{3}{2}\left(\frac{10}{6}\right)\left[\frac{2}{2}\right]=2.5F$$

**Question: I**f retardation of a body of mass 10 gram is given as $2x$, where $x$ is the position of the particle starting from origin at rest. If loss of kinetic energy is $\left[\frac{10}{x}\right]^{-n}$ find n .

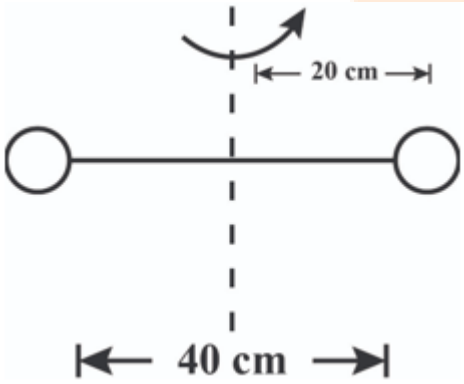
**Answer: 2.00**
**Solution:**

$$\Delta KE=W=\int_0^x mad\,x$$

$$\Delta KE=\frac{10}{100}\int_0^x(-2x)\,dx=-\frac{1}{100}.x^2=-\left[\frac{x^2}{100}\right]=\left(\frac{10}{x}\right)^{-2}$$

So n = 2.

**Question:** Two spheres of mass 2 kg each placed on the ends of a light rod and r = 10 cm and dist b/w the centres = 40 cm find MOI about centre of the rod perpendicular to the line joining centres.
**Answer: 0.17**
**Solution:**



$$I = I_{cm} + ml^2$$

$$= \frac{2}{5}MR^2 + Ml^2$$

$$= \frac{2}{5} \times 2 \times (0.1)^2 + 2(0.2)^2$$

$$I = \frac{4}{500} + \frac{8}{100}$$

$$I = \frac{4+40}{500} = \frac{44}{500} \text{ kg m}^2$$

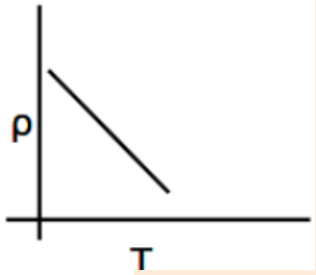
For 2 spheres

$$I_{final} = 2 \times I = 2 \times \frac{44}{500} = 0.176 \text{ kg m}^2$$

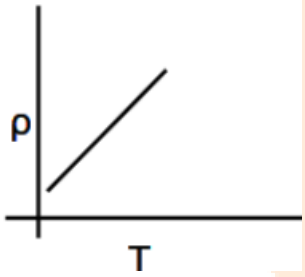
**Question:** Resistivity of semiconductor changes with temp according to which graph

**Options:**

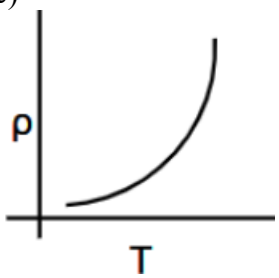
(a)



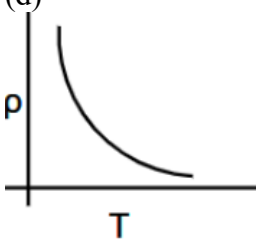

(b)




(c)



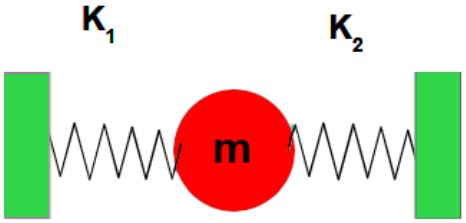

(d)




**Answer: (d)**

**Question:** Alpha, electron, proton has KE is such that $K_\alpha = 4K$, $K_e = 2K$, $K_p = K$ write order of de broglie wave
**Solution:** $\lambda_e > \lambda_P > \lambda_\alpha$

**Question:** For the oscillations exhibited by the spring block system on the smooth surface along the spring, the time period is equal to




**Options:**

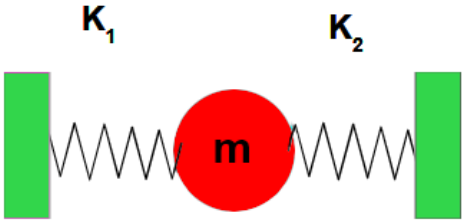
(a) $2\pi\sqrt{\frac{m(K_1+K_2)}{K_1K_2}}$

(b) $2\pi\sqrt{\frac{m(K_1+K_2)}{2K_1K_2}}$

(c) $2\pi\sqrt{\frac{m}{K_1+K_2}}$

(d) $\pi\sqrt{\frac{m}{K_1+K_2}}$

**Answer: (c)**
**Solution:**

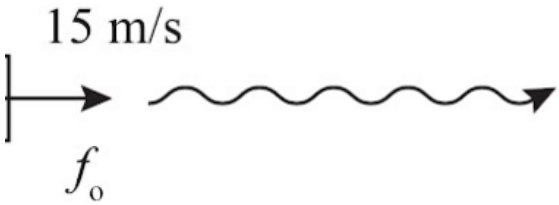



Since the springs are in parallel connection

$k_{eq} = k_1 + k_2$

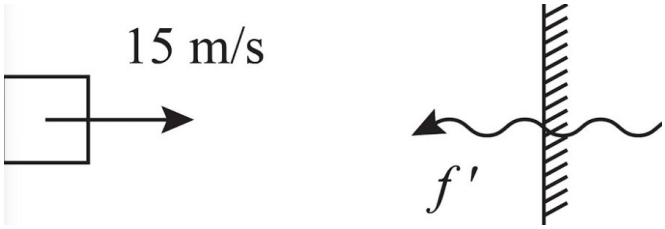
$T = 2\pi\sqrt{\frac{M}{K_1+K_2}}$

**Question:** A car is moving with speed of 15 m/s towards a stationary wall. A person in the car press the horn and experience the change in frequency of 40 Hz due to reflection from the stationary wall. Find the frequency of horn. (Use $v_{sound}$ = 330 m/s)
**Answer: 420.00**
**Solution:**



$$f' = f_0\left[\frac{V}{V - V_C}\right]$$

$$f'' = f'\left[\frac{V + V_C}{V}\right]$$




$$f'' = f_0\left[\frac{V}{V - V_C}\right]\left[\frac{V + V_C}{V}\right]$$

$$f'' = f_0\left[\frac{V + V_C}{V - V_C}\right]$$

$$f'' - f_0 = 40$$

$$f_0\left(\frac{330 + 15}{330 - 15}\right) - f_0 = 40$$

$$f_0\left[\frac{23}{21} - 1\right] = 40$$

$$f_0 = 420$$

**Question:** Communication system
Height of the tower increased 21% percentage increase in range.
**Options:**
(a) 10
(b) 12
(c) 14
(d) 15
**Answer: (a)**
**Solution:**
Range = $\sqrt{2R_E h}$

$$R_1 = \sqrt{2R_E \cdot h} = \sqrt{2R_E \cdot h}$$

$$R_2 = \sqrt{2R_E\left[h + \frac{21}{100} \cdot h\right]} = \sqrt{2R_E(1.21h)}$$

$$\frac{R_1}{R_2} = \frac{\sqrt{2R_E h}}{\sqrt{2R_E(1.21h)}} = \frac{1}{\sqrt{1.21}} = \frac{1}{1.1}$$

$$\Rightarrow R_2 = 1.1R_1$$

$$\% \text{ change in R} = \frac{(R_2 - R_1)}{R_1} \times 100$$

$$= \frac{1.1R_1 - R_1}{R_1} \times 100$$

$$\frac{1.1-1}{1} \times 100 = 10\%$$

**Question:** If length of wire is increased 20% and area is increased 4% the % change in resistance is

**Answer: 15.00**

**Solution:**

$$R = \frac{\rho l}{A}$$

$$R' = \frac{\rho(1.2\ell)}{1.04A} \Rightarrow R' = \frac{12}{1.04} R = 1.15R$$

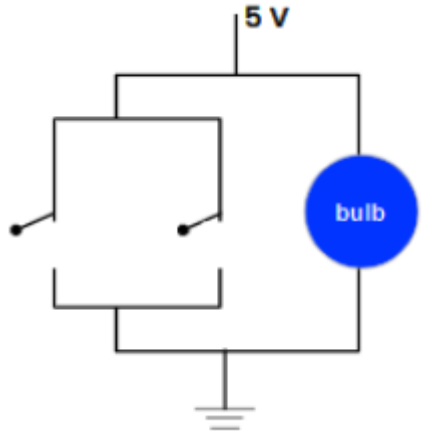
$$\Rightarrow \uparrow 15\%$$

**Question:** A Body has mass m and moving with const vel in viscous fluid having coiff. of viscosity $\eta$ density is ρb liquid density ρL find vel v

**Solution:**

$$\frac{mg\left(1 - \frac{\rho}{S}\right)}{6\pi\eta r} = v$$

**Question:** Which gate is this




**Options:**

(a) NOR

(b) OR

(c) AND

(d) NOT

**Answer:** (a)

# JEE-Mains-06-04-2023 [Memory Based] [Morning Shift]

## Chemistry

**Question:** Polymer which is named as orlon is?
**Options:**
(a) Polyacrylonitrile
(b) Polycarbonate
(c) Polyethene
(d) Polyamide
**Answer:** (a)
**Solution:** Orlon is also called Acrilan or Polyacrylonitrile

**Question:** The correct set of strong oxidising and reducing agent
$Ce^{4+}$, $Yb^{2+}$, $Tb^{4+}$ and $Eu^{2+}$
**Options:**
(a) $Ce^{4+}$, $Tb^{4+}$, $Yb^{2+}$, $Eu^{2+}$
(b) $Tb^{4+}$, $Yb^{2+}$, $Ce^{4+}$, $Eu^{2+}$
(c) $Tb^{4+}$, $Eu^{2+}$, $Yb^{2+}$, $Ce^{4+}$
(d) $Yb^{2+}$, $Eu^{2+}$, $Tb^{4+}$, $Ce^{4+}$
**Answer:** (a)
**Solution:** $Ce^{4+}$, $Tb^{4+}$ act as oxidising agent and $Yb^{2+}$, $Eu^{2+}$ act as reducing agent

**Question:** Match column I (Deficiency) with column II (Disease)

| **Vitamins Deficiency** | **Disease** |
|---|---|
| (P) Vitamin A | (1) Scurvy |
| (Q) Vitamin C | (2) Xeropthalmia |
| (R) Vitamin $B_1$ | (3) Cheilosis |
| (S) Vitamin $B_2$ | (4) Beri-Beri |

**Options:**
(a) P-2, Q-1, R-4, S-3
(b) P-2, Q-4, R-3, S-1
(c) P-4, Q-2, R-4, S-1
(d) P-3, Q-2, R-4, S-1
**Answer:** (a)
**Solution:** Fact based

**Question:** Y form FCC lattice in which X occupies 1/3 of tetrahedral Voids. Then formula of the compound will be
**Options:**
(a) $X_3Y_2$
(b) $XY_3$

(c) $X_2Y_3$
(d) $X_3Y$
**Answer:** (c)
**Solution:** tetrahedral voids are 8 in count in FCC thus X is 8/3 and Y = 4 hence the formula

**Question:** Which of the following have highest electron gain enthalpy difference?
**Options:**
(a) F, Ne
(b) Ar, F
(c) Ne, Cl
(d) Ar, Cl
**Answer:** (a)
**Solution:** Fact based

EA values are F = –333, Cl = -349, Ne = 116, Ar = 96

**Question:** Name reactions Matching

| **Name Reaction** | **Reagents** |
|---|---|
| (P) Etard Reaction | (1) NaOI |
| (Q) Iodoform | (2) CO/HCl, Anh. $AlCl_3$ |
| (R) Gatterman aldehyde | (3) $CrO_2Cl_2$, $CS_2$, $H_3O^+$ |
| (S) HVZ | (4) $X_2$/red P, $H_2O$ |

**Options:**
(a) P-3, Q-1, R-2, S-4
(b) P-3, Q-2, R-1, S-4
(c) P-3, Q-4, R-2, S-1
(d) P-1, Q-3, R-2, S-4
**Answer:** (a)
**Solution:** Fact based

**Question:** Match column I (Compound) with column II (Type of Bond)

| **Nitrogen oxides** | **Type of Bonds** |
|---|---|
| (P) $N_2O$ | (1) N-N bond |
| (Q) $N_2O_5$ | (2) N-O-N bond |
| (R) $NO_2$ | (3) N=N or N triple bond N |
| (S) $N_2O_4$ | (4) N=O |

**Options:**
(a) P-1, Q-4, R-2, S-3
(b) P-3, Q-2, R-4, S-1
(c) P-1, Q-2, R-4, S-3
(d) P-1, Q-3, R-2, S-4
**Answer:** (b)
**Solution:** structure-based question

**Question:** Photochemical smog is maximum in
**Options:**

(a) Himalayan Region
(b) Green Healthy vegetation
(c) Marshy Lands
(d) Industrial Region
**Answer:** (d)
**Solution:** Hydrocarbons and nitrogen oxides produced by automobiles and factories.

**Question:** Which of the reaction is correct among the following with appropriate enzyme?
**Options:**
(a) Sucrose → Glucose + fructose : Enzyme – Invertase
(b) Glucose → $CO_2$ + Ethanol : Enzyme : Maltase
(c) Protein → Amino acid : Enzyme : Zymase
(d) Starch → Maltose : Enzyme : Pepsin
**Answer:** (a)
**Solution:** Sucrose → Glucose + fructose : Enzyme – Invertase

**Question:** Which of the following is used for settling of cement?
**Options:**
(a) Gypsum
(b) Limestone
(c) Clay
(d) Silica
**Answer: (a)**
**Solution:** Setting of cement: When mixed with water, the setting of cement takes place to give a hard mass. This is due to the hydration of the molecules of the constituents and their rearrangement.

**Question:** which of the following is having square Pyramidal shape
**Options:**
(a) $XeOF_4$
(b) $BrF_5$
(c) $IF_5$
(d) $ICl_4^-$
**Answer:** (a)
**Solution:** $XeOF_4$ has geometry of $Sp^3d^2$ and shape of square pyramidal

**Question: Assertion:** Loss of the electron from hydrogen atom results in nucleus ($H^+$) of ~ $1.5 \times 10^{-3}$ pm size.
**Reason:** $H^+$ does not exist freely and is always associated with other atoms or molecules.
**Options:**
(a) Both assertion and reason are correct but reason is not correct explanation
(b) Both assertion and reason are correct but reason is correct explanation
(c) Both assertion and reason are incorrect
(d) Assertion is correct and reason is incorrect
**Answer:** (b)

**Solution:** Loss of the electron from hydrogen atom results in nucleus ($H^+$) of ~$1.5 \times 10^{-3}$ pm size. This is extremely small as compared to normal atomic and ionic sizes of 50 to 200pm. As a consequence, $H^+$ does not exist freely and is always associated with other atoms or molecules. Thus, it is unique in behavior.

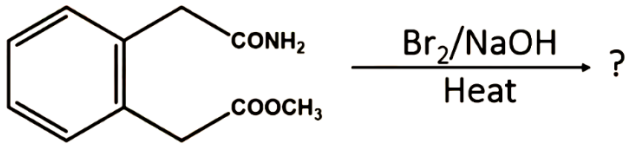
**Question: Assertion:** The magnetic Moment of $[Fe(H_2O)_6]^{3+}$ and $[Fe(CN)_6]^{3-}$ are 5.92 BM and 1.74 BM respectively.
**Reason:** The oxidation state Fe is +3.

**Options:**
(a) Both assertion and reason are correct but reason is not correct explanation
(b) Both assertion and reason are correct but reason is correct explanation
(c) Both assertion and reason are incorrect
(d) Assertion is correct and reason is incorrect
**Answer:** (a)
**Solution:** water as ligand do not cause pairing in complex but CN- does

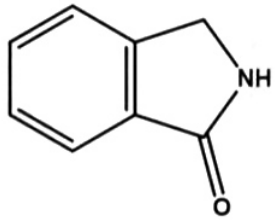
**Question:** If radius of ground state hydrogen is 51 pm, find out the radius of $5^{th}$ orbit of $Li^{2+}$ (closest integer)

**Options:**
(a) 170 pm
(b) 180 pm
(c) 120 pm
(d) 425 pm
**Answer: (d)**
**Solution:** Apply r = 51*5*5/3

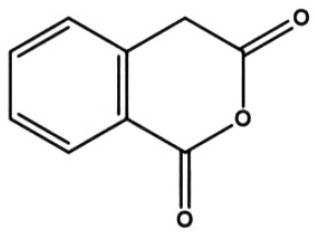
**Question:** Identify the product formed in the following reaction.



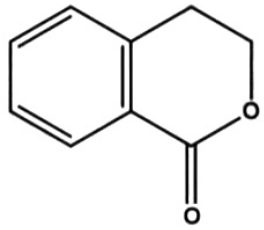

**Options:**
(a)



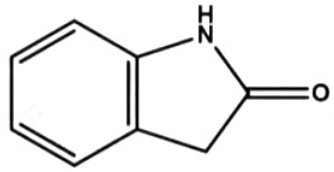

(b)



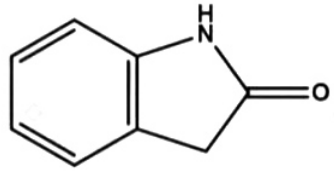
(c)



(d)



**Answer: (d)**
**Solution:**



**Question:** Matrix match for detection of element

| **Column-I** | **Column-II** |
|---|---|
| (A) Nitrogen | (P) AgX |
| (B) Sulphur | (Q) $(NH_4)_3PO_4.12MoO_3$ |
| (C) Phosphorous | (R) $Fe(SCN)_3$ |
| (D) Halogens | (S) $Fe_4[Fe(CN)_6]_3$ |

**Options:**
(a) A-P, B-R, C-Q, D-S
(b) A-R, Q, B-P, C-Q, D-S
(c) A-S, B-R, C-Q, D-P
(d) A-Q, B-R, C-P, D-S
**Answer:** (c)
**Solution:** A-S, B-R, C-Q, D-P

**Question:** Consider the following reaction.

$A_2B_3(g) \rightleftharpoons 2A(g) + 3B(g)$

If the initial concentration of $A_2B_3(g)$ is c, find the value of α

**Options:**

(a) $\left(\frac{K_{eq}}{27c^4}\right)^{\frac{1}{5}}$

(b) $\left(\frac{K_{eq}}{c^4}\right)^{\frac{1}{5}}$

(c) $\left(\frac{K_{eq}}{108c^4}\right)^{\frac{1}{5}}$

(d) $\left(\frac{K_{eq}}{4c^4}\right)^{\frac{1}{5}}$

**Answer:** (c)

**Solution:**

$\left(\frac{K_{eq}}{108c^4}\right)^{\frac{1}{5}}$

# JEE-Mains-06-04-2023 [Memory Based] [Morning Shift]

## Mathematics

**Question:** $\int \frac{x^2\left(x\sec^2 x+\tan x\right)}{\left(x\tan x+1\right)^2}dx$ is equal to

**Answer:** $\frac{-x^2}{x\tan x+1}+2\ln\left|x\sin x+\cos x\right|+c$

**Solution:**

$$\int \frac{x^2\left(x\sec^2 x+\tan x\right)}{\left(x\tan x+1\right)^2}dx$$

Integrating by parts

$$I=x^2\cdot\frac{-1}{x\tan x+1}-\int 2x\left(\frac{-1}{x\tan x+1}\right)dx$$

$$=\frac{-x^2}{x\tan x+1}+2\int\frac{x\cos x}{x\sin x+\cos x}dx$$

$$=\frac{-x^2}{x\tan x+1}+2\ln\left|x\sin x+\cos x\right|+c$$

**Question:** The coefficient of $x^{18}$ in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$ is

**Options:**

(a) ${}^{14}C_7$

(b) ${}^{15}C_8$

(c) ${}^{15}C_6$

(d) ${}^{14}C_8$

**Answer: (c)**

**Solution:**

$$\left(x^4-\frac{1}{x^3}\right)^{15}$$

$$T_{r+1}={}^{15}C_r\left(x^4\right)^{15-r}\left(-\frac{1}{x^3}\right)^r={}^{15}C_r\left(-1\right)^r x^{60-7r}$$

$$60-7r=18$$

$\Rightarrow 7r = 42$

$\Rightarrow r = 6$

Coefficient of $x^{18} = {}^{15}C_6$

**Question:** The number of ways of distributing 70 distinct oranges among three children such that each child gets atleast one orange is

**Answer:** $3^{70} - 3\left(2^{70} - 2\right) - 3$

**Solution:**

Number of ways $= 3^{70} - {}^{3}C_1 \cdot 2^{70} + {}^{3}C_2 \cdot 1^{70}$

$= 3^{70} - 3\left(2^{70} - 2\right) - 3$

**Question:** Sum of first 20 terms of the series $5, 11, 19, 29, 41, .....$ is

**Answer: 3250.00**

**Solution:**

Let $S_n = 5 + 11 + 19 + 29 + 41 + ...t_n$

$t_n = an^2 + bn + c$

$a + b + c = 5$

$4a + 2b + c = 11$

$\underline{9a + 3b + c = 19}$

$5a + b = 8$

$\underline{3a + b = 6}$

$2a = 2$

$a = 1$

$b = 3$

$c = 1$

$t_n = n^2 + 3n + 1$

$S_n = \sum t_n$

$S_n = \frac{n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + n$

$S_{20} = \frac{20 \cdot 21 \cdot 41}{6} + \frac{2 \cdot 20 \cdot 21}{2} + 20$

$= 2870 + 630 + 20$

$= 3520$

**Question:** Number of ways in which 20 chocolates can be given to 3 children such that each gets atleast one is ____.

**Answer:** ${}^{19}C_2$

**Solution:**

$x+y+z=20$

$x,y,z\geq 1$

$X+Y+Z=17$

$X,Y,Z\geq 0$

${}^{n+r-1}C_{r-1}$ i.e. ${}^{19}C_2$

**Question:** If 5 pairs of dice are thrown. Success is getting a sum 5. If the probability of getting atleast 4 success is $\frac{K}{3^{11}}$, then the value of $K$ is

**Answer: 123.00**

**Solution:**

$(1,4),(2,3),(3,2),(4,1)$

$p=\frac{4}{6}=\frac{1}{9},\ q=\frac{8}{9}$

$\frac{K}{3^{11}}=P(\text{atleast 4 success})={}^5C_4p^4q^1+{}^5C_5p^5$

$\frac{K}{3^{11}}=5\cdot\frac{1}{9^4}\cdot\frac{8}{9}+\frac{1}{9^5}$

$\Rightarrow\frac{K}{3^{11}}=\frac{40}{3^{10}}+\frac{1}{3^{10}}$

$\Rightarrow\frac{K}{3}=41$

$\Rightarrow K=123$

**Question:** If the ratio of the 5th term from the start to the 5th term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$. Find the 3rd term from start.

**Answer:** $60\sqrt{3}$

**Solution:**

$\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$

$$\frac{{}^{n}C_4\left(\sqrt[4]{2}\right)^{n-4}\left(\frac{1}{\sqrt[4]{3}}\right)^{4}}{{}^{n}C_4\left(\sqrt[4]{2}\right)^{4}\left(\frac{1}{\sqrt[4]{3}}\right)^{n4}}=\frac{\sqrt{6}}{1}$$

$$\frac{\left(\sqrt[4]{2}\right)^{n-8}}{\left(\frac{1}{\sqrt[4]{3}}\right)^{n-8}}=\sqrt{6}$$

$$\left(\sqrt[4]{6}\right)^{n-8}=\sqrt{6}$$

$$\left(\sqrt{6}\right)^{\frac{n-8}{2}}=\sqrt{6}$$

$$n=10$$

$${}^{10}C_2\left(2^{\frac{1}{4}}\right)^{8}\times\left(\frac{1}{3^{\frac{1}{4}}}\right)^{2}$$

$$=45\times4\times\frac{1}{\sqrt{3}}$$

$$=60\sqrt{3}$$

**Question:** Mean of 15 observations is 12 and its variance is 14. Mean of another 15 observations is 14 and variance $\sigma^2$. Combined variance is 13. Find $\sigma^2$.
**Answer: 10.00**
**Solution:**

Subtract each entry by 13

So $14=\frac{\sum x_i^{\,2}}{15}-(-1)^2\Rightarrow\sum x_i^{\,2}=2.25$

& $\sigma^2=\frac{\sum y_i^{\,2}}{15}-1^2\Rightarrow\sum y_i^{\,2}=15\left(\sigma^2\right)+1$

$$13=\frac{225+15\sigma^2+15}{30}$$

$$\Rightarrow 8+\frac{\sigma^2}{2}=0$$

$$\sigma^2=10$$

**Question:** A $2\times2$ matrix $A$ is such that none of its elements is 0, and $A^2 = I$. '$a$' is the sum of diagonal elements and '$b$' is $|A|$. Find $3a^2+4b^2$.

**Answer: 4.00**
**Solution:**

$$A^2 = \begin{bmatrix} p & q \\ r & s \end{bmatrix}^2 = \begin{bmatrix} p^2+qr & pq+qs \\ pr+rs & qr+s^2 \end{bmatrix}$$

$$(p+s)q = r(p+s) = 0$$

$$p+s=0 \qquad p=-s$$

$$p^2+qr=1$$

So, $a=0, \quad b=ps-qr$

$$=-\left(p^2+qr\right)=-1$$

So $3a^2+4b^2=4$

**Question:** If $2x^y+3y^x=20$. Find $\frac{dy}{dx}$ at (2, 2).

**Answer:**
**Solution:**

$$2x^y+3y^x=20$$

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$$

$$=-\frac{2y\,x^{y-1}+3y^x\ln y}{2x^y\cdot\ln x+3xyx-1}$$

$$=-\frac{2+3\ln 2}{1\ln 2+3}$$

$$=-\frac{2+\ln 8}{\ln 4+3}$$

**Question:** If $a_1, a_2, a_3, ...., a_n$ are in A.P.

$$\lim_{n\to\infty}\frac{\sqrt{d}}{\sqrt{n}}\left(\frac{1}{\sqrt{a_2}+\sqrt{a_1}}+\frac{1}{\sqrt{a3}+\sqrt{a_2}}....\right)$$

**Answer: 1.00**
**Solution:**

$a_1, a_2, a_3$ are in A.P.

$$\lim_{n\to\infty}\frac{\sqrt{d}}{\sqrt{n}}\left(\frac{1}{\sqrt{a_2}+\sqrt{a_1}}+\frac{1}{\sqrt{a_3}+\sqrt{a_2}}....\right)$$

$$=\lim_{n\to\infty}\frac{\sqrt{d}}{\sqrt{n}}\left[\frac{\sqrt{a_n}-\sqrt{a_1}}{d}\right]$$

$$=\lim_{n\to\infty}\left[\frac{\sqrt{a_1+(n-1)d}}{\sqrt{n}\cdot\sqrt{d}}-\frac{\sqrt{d}\sqrt{a_1}}{\sqrt{n}d}\right]$$

$=1$

**Question:** $f=[9+13\sin x]$ when $x\in[0,\pi]$. Find the number of points where $f$ is not differentiable.

**Answer: 25.00**

**Solution:**

$[a+b\sin x];\ x\in[0,\pi]$

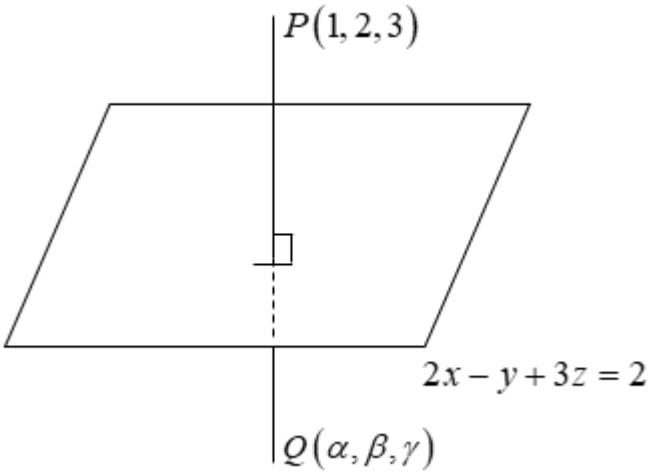
Non-differentiable at $2b-1$ points

$2b-1=2\times13-1=25$

**Question:** Image of point $P(1,2,3)$ about the plane $2x-y+3z=2$ is $Q$, then the area of $\Delta PQR=?$ where $R=(4,10,12)$

**Answer:** $\frac{\sqrt{1531}}{2}$

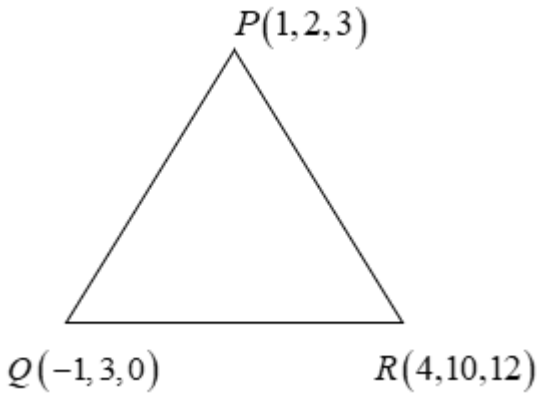
**Solution:**



$$\frac{\alpha-1}{2}=\frac{\beta-2}{-1}=\frac{\alpha-3}{3}=\frac{-2(7)}{14}$$

$$\frac{\alpha-1}{2}=\frac{\beta-2}{-1}=\frac{\alpha-3}{3}=-1$$

$$Q(-1,3,0)$$




$$\text{Area}=\frac{1}{2}\times\left|\overline{PQ}\times\overline{PR}\right|$$

$$=\frac{\sqrt{1531}}{2}$$

**Question:** If $5f(x)+4f\left(\frac{1}{x}\right)=\frac{1}{x}+3$, then $18\int_1^2 f(x)dx$ is:

**Answer:** $10\log_e 2-6$

**Solution:**

$$5f(x)+4f\left(\frac{1}{x}\right)=\frac{1}{x}+3$$

$$5f(x)+4f\left(\frac{1}{x}\right) \quad ....(1)$$

Take $x=\frac{1}{x}$

$$5f\left(\frac{1}{x}\right)+4f(x)=x+3 \quad ....(2)$$

$$(1)\times 5-4\times(2)$$

$$9f(x)=\frac{5}{x}+15-4x-12$$

$$9f(x)=\frac{5}{x}-4x+3$$

By integrating

$$9\int_1^2 f(x)\,dx=\int_1^2 \frac{5}{x}-4x+3\,dx$$

$$2\times 9\int_1^2 f(x)=\int_1^2 \frac{10}{x}-8x+6$$

$$=10\ln|x|-\frac{8x^2}{2}+6x\Big|_1^2$$

$$=(10\ln 2-16+12)-(0-4+6)$$

$$=10\log_e 2-6$$

**Question:** The sum of roots of $|x^2-8x+15|-2x+7=0$ is

**Answer:** $9+\sqrt{3}$

**Solution:**

$$|x^2-8x+15|-2x+7=0$$

$$|(x-3)(x-5)|-2x+7=0$$

$x\le 3$ or $x\ge 5$

$$x^2-8x+15-2x+7=0$$

$$x^2-10x+22=0$$

$$x=\frac{10\pm\sqrt{100-88}}{2}$$

$$=5\pm\sqrt{3}$$

Take intersection

$$x=5+\sqrt{3}$$

$$3<x<5$$

$$-x^2+8x-15-2x+7=0$$

$$x^2-6x+8=0$$

$$(x-2)(x-4)=0$$

$x = 2, 4$

$x = 4$

So, sum of roots is $9 + \sqrt{3}$

**Question:** $(P \Rightarrow Q) \vee (R \Rightarrow Q)$ is equivalent to:

**Options:**

(a) $(P \wedge R) \Rightarrow Q$

(b) $(P \vee R) \Rightarrow Q$

(c) $(Q \Rightarrow R) \vee (P \Rightarrow R)$

(d) $(R \Rightarrow P) \vee (Q \Rightarrow R)$

**Answer: (a)**

**Solution:**

$(P \Rightarrow Q) \vee (R \Rightarrow Q)$

$\equiv (\sim P \vee Q) \vee (\sim R \vee Q)$

$\equiv (\sim P \vee \sim R) \vee Q$

$\equiv \sim (P \wedge R) \vee Q$

$\equiv (P \wedge R) \Rightarrow Q$

**Question:** Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}, \vec{c} = -\hat{i} + 4\hat{j} + 3\hat{k}$ and $\vec{d}$ is a vector perpendicular to both $\vec{b}$ and $\vec{c}$ and $\vec{a} \cdot \vec{d} = 18$, then $\left|\vec{a} \times \vec{d}\right|^2$ is

**Answer: 720.00**

**Solution:**

$\vec{d} = \lambda\left(\vec{b} \times \vec{c}\right)$

$$= \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -2 \\ -1 & 4 & 3 \end{vmatrix}$$

$= \lambda\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$

$\vec{a} \cdot \vec{d} = 18$

$\Rightarrow \lambda(4-3+8)=18$

$\Rightarrow \lambda=2$

$$\left|\vec{a}\times\vec{d}\right|^2 = a^2d^2-\left(\vec{a}\cdot\vec{d}\right)^2$$

$$=29\times36-18^2$$

$$=18(58-18)$$

$$=18\times40$$

$$=720$$

**Question:** If ${}^{2n}C_3 : {}^{n}C_3 = 10$, then $\dfrac{n^2+3n}{n^2-3n+4}$ is equal to

**Answer: 2.00**

**Solution:**

${}^{2n}C_3 : {}^{n}C_3 = 10$

$$\frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)}=10$$

$$\Rightarrow \frac{2(2n-1)\cdot 2(n-1)}{(n-1)(n-2)}=10$$

$4n-2=5n-10$

$\Rightarrow n=8$

$$\frac{n^2+3n}{n^2-3n+4}=\frac{64+24}{64-24+4}=\frac{88}{44}=2$$