PART : MATHEMATICS

1. The number of ways to distribute 20 identical chocolates among three students such that each student will get at least one chocolate, is

(1) $^{22}C_2$ (2) $^{19}C_3$ (3) $^{19}C_2$ (4) $^{22}C_3$

Ans. **(3)**

Sol. $x_1 + x_2 + x_3 = 20$

let $x_1 = 1 + t_1$, $x_2 = 1 + t_2$, $x_3 = 1 + t_3$

when $t_1, t_2, t_3 \in \{0, 1, 2, ...17\}$

$t_1 + t_2 + t_3 = 17$---------(1)

So number of such distribution is equal to number of non negative integral solutions of equation (1)

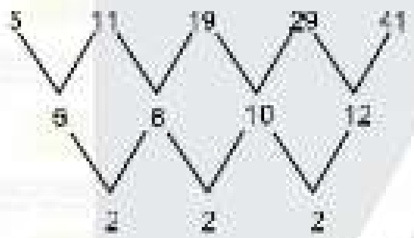
So required solution $= {}^{17+3-1}C_{3-1} = {}^{19}C_2$

2. Sum of first 20 terms of the series : 5 + 11 + 19 + 29 + 41+...........is

(1) 3250 (2) 3520 (3) 3052 (4) 3205

Ans. **(2)**

Sol.



let $Tn + an^2 + bn + c$

$\therefore$ $a + b + c =$ ---------- (1)

$4a + 2b + c = 11$------------(2)

$9a + 3n + v = 19$------------(3)

$(2) - (1) \Rightarrow$ $3a + b = 6$-------(4)

$(3) - (2) \Rightarrow$ $5a + b = 8$ -------(5)

$(5) - (4) \Rightarrow$ $2a = 2 \Rightarrow a = 1$

$\therefore$ $b = 3$ and $c = 1$

$\therefore$ $Tn = n^2 + 3n + 1$

$\therefore$ $S_{20} = \sum_{n=1}^{20} Tn = \sum_{n=1}^{20} (n^2 + 3n + 1)$

$= \frac{20 \times 21 \times 41}{6} + \frac{3 \times 20521}{2} + 20$

$= 70 \times 41 + 30 \times 21 + 20$

$= 10(7 \times 41 + 63 + 2)$

$= 10(287 + 65)$

$= 10(3521)$

$= 3520$

3. $|x^2-8x+15| - 2x + 7 = 0$, then sum of roots of the equation is

(1) $9+\sqrt{3}$ (2) $6+\sqrt{3}$ (3) $3+\sqrt{3}$ (4) $9-\sqrt{3}$

Ans. **(1)**

Sol.

case I
$x < 3$
$x^2 - 8x + 15 - 2x + 7 = 0$
$x^2 - 10x + 22 = 0$
$(x-5)^2 - 3 = 0$
$x - 5 = \pm\sqrt{3}$
$x = 5+\sqrt{3},\ 5-\sqrt{3}$

case II
$3 \le x \le 5$
$x^2 - 8x + 15 + 2x - 7 = 0$
$x^2 - 6x + 8 = 0$
$x = 2, 4$

case III
$5 < x$
$x - 5 = \pm\sqrt{3}$
$x = 5+\sqrt{3},\ 5-\sqrt{3}$
$\text{sum} = 5+\sqrt{3}+4 = 9+\sqrt{3}$

4. If $\frac{{}^{2n}C_3}{{}^{n}C_3} = \frac{10}{1}$, then value of $\frac{n^2+3n}{n^2-3n+4}$ is equal to

(1) 1 (2) 2 (3) 4 (4) 5

Ans. **(2)**

Sol. $\frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)} = \frac{10}{1}$

$\frac{4(2n-1)}{n-2} = \frac{10}{1}$

$8n - 4 = 10n - 20 \Rightarrow n = 8$

$\frac{n^2+3n}{n^2-3n+4} = \frac{64+24}{64-24+4} = \frac{88}{44} = 2$

5. $\int \frac{x^2(x\sec^2 x + \tan x)}{(x\tan x + 1)^2}dx$ is equal to

(1) $\frac{-x^2}{(x\tan x+1)} + 2\ln|x\sin x - \cos x| + c.$ (2) $\frac{x^2}{(x\tan x+1)} - 2\ln|x\sin x + \cos x| + c.$

(3) $\frac{-x^2}{(x\tan x+1)} + 2\ln|x\sin x + \cos x| + c.$ (4) $\frac{x^2}{(x\tan x+1)} + 2\ln|x\sin x + \cos x| + c.$

Ans. **(3)**

Sol. Let $I = \int \frac{x^2.(x\sec^2 x + \tan x)}{(x\tan x + 1)^2}dx$

$\because \frac{d}{dx}(x\tan x + 1) = x\sec^2 x + \tan x$

$\therefore \int \frac{x\sec^2 x + \tan x}{(x\tan x + 1)^2}\,dx = -\frac{1}{x\tan x + 1}$

$\therefore I = x^2\left(\frac{-1}{x\tan x + 1}\right) - \int 2x\left(\frac{-1}{x\tan x + 1}\right)dx$

$= \frac{-x^2}{(x\tan x + 1)} + 2\int \frac{x\cos x}{x\sin x + \cos x}\,dx$

$= \frac{-x^2}{(x\tan x + 1)} + 2\int \frac{\frac{d}{dx}(x\sin x + \cos x)}{x\sin x + \cos x}\,dx$

$= \frac{-x^2}{(x\tan x + 1)} + 2\ln|x\sin x + \cos x| + c.$

6. The coefficient of x^{18}, in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$, is

(1) ${}^{14}C_7$ (2) ${}^{15}C_8$ (3) ${}^{15}C_6$ (4) ${}^{14}C_6$

Ans. (3)

Sol. $T_{r+1} = {}^{15}C_r (x^4)^{15-r}\left(-\frac{1}{x^3}\right)^r$

$60 - 4r - 3r = 18$

$7r = 42$

$r = 6$

Coff. $= {}^{15}C_6$

7. If $2x^y + 3y^x = 20$, then $\frac{dy}{dx}$ at point (2, 2) is equal to

(1) $\frac{(2-3\ln 2)}{(4-2\ln 2)}$ (2) $\frac{(2+3\ln 2)}{(3+4\ln 2)}$ (3) $\frac{-(2+3\ln 2)}{(3+4\ln 2)}$ (4) $\frac{-(2+3\ln 2)}{(3+2\ln 2)}$

Ans. (4)

Sol. Differentiating $2 \cdot x^y\left(\frac{dy}{dx}\ln x + y\frac{1}{x}\right) + 3 \cdot y^x\left(1.\ln y + x\frac{y'}{y}\right) = 0$

Put $x = 2$, $y = 2$

$2 \cdot 4\left(\frac{dy}{dx}\ln 2 + 1\right) + 3 \cdot 4\left(\ln 2 + \frac{dy}{dx}\right) = 0$

$(2\cdot\ln 2 + 3)\frac{dy}{dx} + (2 + 3\ln 2) = 0$

$\frac{dy}{dx} = -\frac{(2+3\ln 2)}{(3+2\ln 2)}$

8. Two groups G_1 and G_2 have 15 observations each, means of G_1 and G_2 are 12 and 14 respectively. If variance of G_1 is 14 and combined variance of G_1 and G_2 is 13, then the variance of G_2 is

(1) 12 (2) 10 (3) 11 (4) 13

Ans. (2)

Sol. We know that combined variance of two groups $= \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1+n_2} + \frac{n_1n_2(\bar{x}_1-\bar{x}_2)^2}{(n_1+n_2)^2}$

$\Rightarrow 13 = \frac{15(14)+15(\sigma_2^2)}{30} + \frac{225(4)}{30\times 30}$

$\Rightarrow 13 = \frac{14+\sigma_2^2}{2} + \frac{15\times 2}{30}$

$\Rightarrow \sigma_2^2 = 10$

9. In the expansion of $\left(2^{\frac{1}{4}} + \frac{1}{3^{\frac{1}{4}}}\right)^n$, if the ratio of 5^{th} term from the beginning and the end is $\sqrt{6} : 1$, then the 3^{rd} term from the beginning is :

(1) $\frac{60}{\sqrt{3}}$ (2) $30\sqrt{3}$ (3) $90\sqrt{3}$ (4) $60\sqrt{3}$

Ans. (4)

Sol. given $\frac{{}^nC_4\left(2^{\frac{1}{4}}\right)^{n-4}\cdot\left(3^{\frac{-1}{4}}\right)^4}{{}^nC_4\left(3^{\frac{-1}{4}}\right)^{n-4}\cdot\left(2^{\frac{1}{4}}\right)^4} = \sqrt{6}$

$\Rightarrow 2^{\frac{n-8}{4}}\cdot 3^{\frac{n-8}{4}} = \sqrt{6}$

$\Rightarrow 6^{n-8} = 6^2$

$\Rightarrow n = 10$

So $T_3 = {}^{10}C_2\left(2^{\frac{1}{4}}\right)^8\cdot\left(3^{\frac{-1}{4}}\right)^2$

$= \frac{15\times 3\times 4}{\sqrt{3}} = 60\sqrt{3}$

10. If the image of point P(1,2,3) in the plane $2x-y+3z = 2$ is Q, then the area of triangle PQR is, (where point R is (4, 10, 12))

(1) $\frac{\sqrt{1543}}{4}$ (2) $\frac{\sqrt{2131}}{2}$ (3) $\frac{\sqrt{1531}}{2}$ (4) $\frac{\sqrt{3215}}{4}$

Ans. (3)

Sol. Image of P in plane is Q

$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{3} = -2\frac{(2-2+9-2)}{4+1+9}$

Q = (−1, 3, 0)

$\vec{PQ} = -2\hat{i} + \hat{j} - 3\hat{k}$, $\vec{PR} = 3\hat{i} + 8\hat{j} + 9\hat{k}$

Area of $\Delta PQR = \frac{1}{2}|\vec{PQ} \times \vec{PR}| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -3 \\ 3 & 8 & 9 \end{vmatrix}$

$= \frac{1}{2}|\hat{i}(9+24) - \hat{j}(-18+9) + \hat{k}(-16-3)|$

$= \frac{1}{2}\sqrt{33^2 + 9^2 + 19^2} = \frac{1}{2}\sqrt{1531}$

11. If $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3$ $(x > 0)$, then $18\int_1^2 f(x)dx$ is

(1) $10 \ln 2 + 6$ (2) $10 \ln 2 - 6$ (3) $5 \ln 2 + 16$ (4) $10 \ln 2 - 16$

Ans. (2)

Sol. $\because 5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3$

$5f\left(\frac{1}{x}\right) + 4f(x) = x + 3$

$9f(x) = \frac{5}{x} + 15 - 4x - 12$

$18f(x) = \frac{10}{x} - 8x + 6$

$\int_1^2 18f(x)dx = \left(10\ln x - 4x^2 + 6x\right)_1^2$

$= 10 \ln 2 - 16 + 12 + 4 - 6$

$= 10 \ln 2 - 6$

12. $(P \Rightarrow Q) \wedge (R \Rightarrow Q)$ is equivalent to :

(1) $(P \wedge R) \Rightarrow Q$ (2) $(P \vee R) \Rightarrow Q$

(3) $(P \vee Q) \Rightarrow R$ (4) $(P \wedge Q) \Rightarrow R$

Ans. (2)

Sol. $(\sim P \vee Q) \wedge (\sim R \vee Q)$

$= (\sim P \wedge \sim R) \vee Q$

$= \sim (P \vee R) \vee Q$

$= (P \vee R) \Rightarrow Q$

13. If $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}$, $\vec{c} = -\hat{i} + 4\hat{j} + 3\hat{k}$ and $\vec{d}$ is a vector perpendicular to both $\vec{b}$ and $\vec{c}$ such that $\vec{a}.\vec{d} = 18$, then $\left|\vec{a} \times \vec{d}\right|^2 =$

(1) 770 (2) 820 (3) 720 (4) 860

Ans. (3)

Sol. As $\vec{d}$ is perpendicular to both $\vec{b}$ and $\vec{c}$ $\Rightarrow \vec{d} = \lambda\left(\vec{b} \times \vec{c}\right)$, $\lambda \in R$

$$\vec{d} = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -2 \\ -1 & 4 & 3 \end{vmatrix} = \lambda\left(\hat{i}(2) - \hat{j}(1) + \hat{k}(2)\right)$$

$$= \lambda\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$$

$\vec{a}.\vec{d} = 18 \Rightarrow |\lambda| = 2$

$$\left(\vec{a} \times \vec{d}\right)^2 = \lambda^2 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 2 & -1 & 2 \end{vmatrix} = 4\left(10\hat{i} + 4\hat{j} - 8\hat{k}\right)^2$$

$= 4(100 + 16 + 64) = 4(180) = 720$

14. Given that a_1, a_2, a_3 .... a_n are in A.P with common difference. Then value of

$$\underset{n \to \infty}{\text{Lt}} \sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \cdots\cdots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}}\right) \text{ is}$$

Ans. (1)

Sol. $$\left(\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \cdots\cdots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}}\right)$$

$$= \left(\frac{\sqrt{a_2} - \sqrt{a_1}}{a_2 - a_1} + \frac{\sqrt{a_3} - \sqrt{a_2}}{a_3 - a_2} + \frac{\sqrt{a_4} - \sqrt{a_3}}{a_4 - a_3} + \cdots\cdots + \frac{\sqrt{a_n} - \sqrt{a_{n-1}}}{a_n - a_{n-1}}\right)$$

$$= \frac{1}{d}\left(\sqrt{a_n} - \sqrt{a_1}\right)$$

$$\therefore \underset{n \to \infty}{\text{Lt}} \sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \cdots\cdots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}}\right)$$

$$= \underset{n \to \infty}{\text{Lt}} \sqrt{\frac{d}{n}}\left(\frac{\sqrt{a_n} - \sqrt{a_1}}{d}\right)$$

$$= \underset{n \to \infty}{\text{Lt}} \left(\frac{\sqrt{a_1 + (n-1)d} - \sqrt{a_1}}{\sqrt{n}\sqrt{d}}\right) = \underset{n \to \infty}{\text{Lt}} \left(\sqrt{\frac{a_1}{nd} + \left(1 - \frac{1}{n}\right)} - \sqrt{\frac{a_1}{nd}}\right)$$

$= 1$

15. $I = \int_0^{\frac{\pi}{2}} \cos^5 x(\sin^7 x)dx$ is equal to

(1) $\frac{1}{120}$ (2) $\frac{1}{60}$ (3) $\frac{1}{240}$ (4) $\frac{\pi}{120}$

Sol. By wall's formulas

$I = \frac{(4\times2)(6\times4\times2)}{12\times10\times8\times6\times4\times2} = \frac{1}{120}$

16. Let A be a 2×2 matrix with $A^2 = I$ and all entries of A are non-zero. If sum of leading diagonal elements is 'a' and det(A) = b, then the value of $(3a^2 + b^2)$ is

Ans. **1**

Sol. $|A|^2 = 1 \Rightarrow b^2 = 1$

Let $A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$

$A^2 = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} = \begin{bmatrix} a_1^2 + b_1a_2 & a_1b_1 + b_1b_2 \\ a_1a_2 + b_2a_2 & a_2b_1 + b_2^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \quad a_2(a_1+b_2) = 0$ and $b_1(a_1 + b_2) = 0$

$\Rightarrow \quad a_1 + b_2 = 0$

$\Rightarrow \quad Tr(A) = 0$

$\Rightarrow \quad a = 0$

So $(3a^2 + b^2) = 0 + 1 = 1$

17. A pair of dice is rolled 5 times. If getting a sum 5 is considered as a success, then the probability of four successes is

(1) $\frac{4}{9^5}$ (2) $\frac{20}{9^4}$ (3) $\frac{5}{9^4}$ (4) $\frac{40}{9^5}$

Ans. **(4)**

Sol. S = {(1, 1), (1, 2)………………(1, 6)

(2, 1), (2, 2)……………………(2, 6)

(6, 1), (6, 2)……………………(6, 6)}

Event of success = {(1, 4), (4, 1), (2, 3), (3, 2)}

$p = \frac{4}{36}$

Probability of exactly four successes is = ${}^5C_4\left(\frac{8}{9}\right)^1\left(\frac{1}{9}\right)^4$

18. If f(x) = [a + 13 sinx], where x ∈ (0, π) and [•] denotes G.I.F. and a ∈ I, then the number of points of non-differentiability is

(1) 24 (2) 26 (3) 25 (4) 23

Ans. **(3)**

Sol. f(x) = a + [13 sinx] ∵ a ∈ I and x ∈ (0, π)

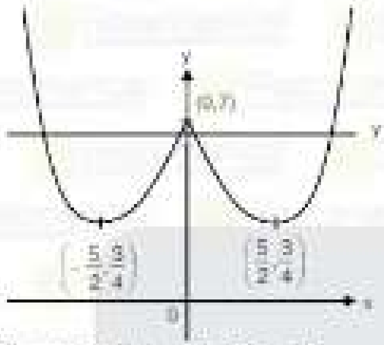
∵ total number of points of non-differentiability of [p sinx] = 2p − 1 here p = 13

∴ total number of points of non-differentiability of [13 sinx] = 25

19. If the equation $|x^2 - 5|x| + 7| = P$, $P \in I$, has 4 solutions, then the number of possible values of P is.

Ans. (6)

Sol. Let $y = ||x|^2 - 5|x| + 7|$ $\quad \because x^2 = |x|^2$ as $x \in R$



Suppose $f(x) = x^2 - 5x + 7$

So $y = |f(|x|)|$

for 4 solutions

$\frac{3}{4} < P < 7$

But $P \in I$ so $P = 1, 2, 3, 4, 5, 6$

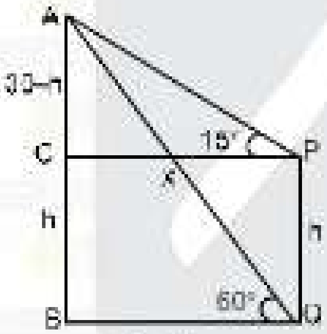
6 values

20. AB is a building of height 30 m with base B. PQ is a pole with base Q. Angle of depression from point A to P and Q are 15° and 60° respectively. A point C is on the same level of point P, on the building. Then area of rectangle PCBQ is

(1) $60(\sqrt{3}-1)$ (2) $600(\sqrt{3}+1)$ (3) $600(\sqrt{3}-1)$ (4) $60(\sqrt{3}+1)$

Ans. (3)

Sol.



$\tan 15° = \frac{30-h}{x}$

$\tan 60° = \frac{30}{x} \Rightarrow x = \frac{30}{\sqrt{3}}$

$\frac{2-\sqrt{3}}{\sqrt{3}} = \frac{30-h}{30} \Rightarrow 30\left(\frac{2-\sqrt{3}}{\sqrt{3}}\right) = 30 - h$

$h = 30 - 30 \times \frac{2}{\sqrt{3}} + 30 = 60 - \frac{60}{\sqrt{3}}$

Area $= \frac{30(60)(\sqrt{3}-1)}{3} = 600(\sqrt{3}-1)$

21. Number of words with or without meaning using all the letters of the word ASSASSINATION such that all the vowels come together is :

(1) 38004 (2) 38042 (3) 50400 (4) 60200

Ans. (3)

Sol. We have to arrange bundle of AAAIIO with seven consonants SSSSNTN :

So, no of words = $\frac{8!}{4!2!} \times \frac{6!}{3!2!1!}$

$= 50400$