MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. The number of ways to distribute 20 chocolates among three students such that each student will get atleast one chocolate is

(1) ${}^{22}C_2$ (2) ${}^{19}C_2$

(3) ${}^{19}C_3$ (4) ${}^{22}C_3$

Answer (2)

Sol. $x + y + z = 20$

$x \geq 1, y \geq 1, z \geq 1$

$x_1 + y_1 + z_1 = 17$ where $x_1 = x + 1$, $y_1 = y + 1$, $z_1 = z + 1$

Number of ways $= {}^{17+3-1}C_{3-1}$

$= {}^{19}C_2$

2. The coefficient of x^{18} in the expansion $\left(x^4 - \frac{1}{x^3}\right)^{15}$

(1) ${}^{14}C_7$ (2) ${}^{15}C_8$

(3) ${}^{15}C_6$ (4) ${}^{14}C_8$

Answer (3)

Sol. $T_{r+1} = {}^{15}C_r (x^4)^{15-r}\left(-\frac{1}{x^3}\right)^r$

$= {}^{15}C_r\, x^{60-4r-3r}(-1)^r$

Now, $60 - 7r = 18$

$r = 6$

$\therefore$ coefficient of x^{18} is ${}^{15}C_6$

3. Sum of first 20 turns of the series 5, 11, 19, 29, 41, …… is

(1) 3130 (2) 3520

(3) 2790 (4) 1880

Answer (2)

Sol.
$S = 5 + 11 + 19 + 29 + \ldots.. T_n$
$S = \quad 5 + 11 + 19 + \ldots\ldots T_{n-1} + T_n$

$0 = \underbrace{5 + 6 + 8 + 10 \ldots\ldots\ldots}_{n-1 \text{ turns}} - T_n$

$T_n = 5 + \frac{n-1}{2}\left(12 + (n-2)2\right)$

$T_n = 5 + 6(n-1) + (n-1)(n-2)$

$T_n = n^2 + 3n + 1$

$\sum T_n = \frac{(n)(n+1)(2n+1)}{6} + \frac{3(n)(n+1)}{2} + n$

$n = 20$

$S_{20} = \frac{20 \times 21 \times 41}{6} + \frac{3 \times 20 \times 21}{2} + 20$

$= 2870 + 630 + 20 = 3520$

4. Mean of first 15 numbers is 12 and variance is 14. Mean of next 15 numbers is 14 and variance is a. If variance of all 30 numbers is 13, then a is equal to

(1) 12 (2) 14

(3) 10 (4) 3

Answer (3)

Sol. $\sigma^2 = \frac{\Sigma x_i^2}{n} - (\bar{x})^2$

$13 = \frac{(14+144)\times 15 + (a+196)\times 15}{30} - (13)^2$

$\Rightarrow \quad a = 10$

5. If the image of point $P(1, 2, 3)$ about the plane $2x - y + 3y = 2$ is Q, then the area of triangle PQR, where coordinates of R is (4, 10, 12)

(1) $\frac{\sqrt{1531}}{2}$

(2) $\frac{\sqrt{1675}}{2}$

(3) $\frac{\sqrt{2443}}{2}$

(4) $\frac{\sqrt{1784}}{2}$

Answer (1)

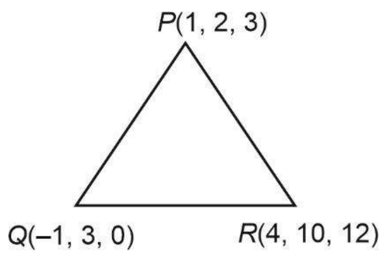
Sol. Image formula

$$\frac{x-1}{2}=\frac{y-2}{-1}=\frac{z-3}{3}=-2\left(\frac{2-2+9-2}{14}\right)$$

$\Rightarrow\ x = -1$

$y = 3$

$z = 0$



$\overrightarrow{PQ}=-2i+j-3k$

$\overrightarrow{PR}=3i+8j+9k$

Area $=\frac{1}{2}\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right|$

$=\frac{1}{2}\left|33i+9j-19k\right|$

$=\frac{\sqrt{1531}}{2}$

6. If $5f(x)+4f\left(\frac{1}{x}\right)=\frac{1}{x}+3$, then $18\int_1^2 f(x)dx$ is

(1) $10\log 2+6$ (2) $10\log 2-6$

(3) $5\log 2+6$ (4) $5\log 2-6$

Answer (2)

$$5\left(5f(x)+4f\left(\frac{1}{x}\right)=\frac{1}{x}+3\right)\quad ...(i)$$

Sol. $x\to\frac{1}{x}$

$$4\left(5f\left(\frac{1}{x}\right)+4f(x)=x+3\right)\quad ...(ii)$$

$$9f(x)=\frac{5}{x}+15-4x-12$$

$$9f(x)=\frac{5}{x}-4x+3$$

$$\int_1^2 f(x)dx=\frac{1}{9}\int_1^2\left(\frac{5}{x}-4x+3\right)dx$$

$$=\frac{1}{9}\left[5\log x-2x^2+3x\right]_1^2$$

$$=\frac{1}{9}\left[5\log 2-6+3\right]$$

$$=\frac{1}{9}\left[5(\log 2)-3\right]$$

$$18\int_1^2 f(x)dx=10\log 2-6$$

7. The sum of roots of $\left|x^2-8x+15\right|-2x+7=0$ is

(1) $11+\sqrt{3}$

(2) $11-\sqrt{3}$

(3) $9+\sqrt{3}$

(4) $9-\sqrt{3}$

Answer (3)

Sol. $\left|x^2-8x+15\right|-2x+7=0$

$\Rightarrow\ \left|(x-3)(x-5)\right|-2x+7=0$

Let $x\le 3$ or $x\ge 5$, then

$x^2 - 8x + 15 - 2x + 7 = 0$

$x^2 - 10x + 22 = 0$

$x=\frac{5\pm\sqrt{3}}{3}$ but $x\in(-\infty, 3]\cup[5,\infty)$

$\therefore\ x=5+\sqrt{3}$

Now,

$x\in(3, 5)$, then

$-x^2 + 8x - 15 - 2x + 7 = 0$

$x^2 - 6x + 8 = 0$

$x = 2, 4$

$\therefore\ x = 4$

$\therefore$ Sum of roots $=9+\sqrt{3}$

8. $(P\Rightarrow Q)\vee(R\Rightarrow Q)$ is equivalent to

(1) $(P\wedge R)\Rightarrow Q$ (2) $(P\vee R)\Rightarrow Q$

(3) $(Q\Rightarrow R)\vee(P\Rightarrow R)$ (4) $(R\Rightarrow P)\wedge(Q\Rightarrow R)$

Answer (1)

Sol. $(P\Rightarrow Q)\vee(R\Rightarrow Q)$

$=(\sim P\vee Q)\vee(\sim R\vee Q)$

$=(\sim P\vee\sim R)\vee Q$

$=\sim(P\wedge R)\vee Q$

$=(P\wedge R)\Rightarrow Q$

9. Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}, \vec{c} = -\hat{i} + 4\hat{j} + 3\hat{k}$ and $\vec{d}$ is a vector perpendicular to both $\vec{b}$ and $\vec{c}$ and $\vec{a} \cdot \vec{d} = 18$, then $|\vec{a} \times \vec{d}|^2$ is

(1) 720 (2) 640

(3) 680 (4) 760

Answer (1)

Sol. $\vec{b} \times \vec{c} = 2\hat{i} - \hat{j} + 2\hat{k}$

$\therefore \quad \vec{d} = \lambda(2\hat{i} - \hat{j} + 2\hat{k})$

$\vec{a} \cdot \vec{d} = 18$

$\Rightarrow \quad \lambda = 2$

$\therefore \quad |\vec{a} \times \vec{d}|^2 = |\vec{a}|^2 \cdot |\vec{d}|^2 - (\vec{a} \cdot \vec{d})^2$

$= 720$

10. The integration $\int \frac{x^2(x\sec^2 x + \tan x)}{(x\tan x + 1)^2} dx$ is

(1) $\frac{x}{x\tan x + 1} + \log|x\sin x + \cos x| + c$

(2) $\frac{x}{x\tan x + 1} - \log|x\sin x + \cos x| + c$

(3) $\frac{-x^2}{x\tan x + 1} + 2\log|x\sin x + \cos x| + c$

(4) $\frac{x^2}{x\tan x + 1} + 2\log|x\sin x + \cos x| + c$

Answer (3)

Sol. $\int \underset{\text{I}}{x^2} \cdot \underset{\text{II}}{\frac{(x\sec^2 x + \tan x)}{(x\tan x + 1)^2}} dx$

$= \frac{-x^2}{(x\tan x + 1)} + \underset{\text{I}}{\int \frac{2x}{x\tan x + 1} dx}$

$I = 2\int \frac{x}{x\tan x + 1} dx$

$= 2\int \frac{x\cos x}{x\sin x + \cos x} dx$

Let $x\sin x + \cos x = t$

$(x\cos x + \sin x - \sin x)\, dx = dt$

$= 2\int \frac{dt}{t} = 2\log t + c'$

$= 2\log|x\sin x + \cos x| + c'$

$\therefore \quad \int \frac{x^2(x\sec^2 x + \tan x)}{(x\tan x + 1)^2} dx$

$= \frac{-x^2}{x\tan x + 1} + 2\log|x\sin x + \cos x| + c$

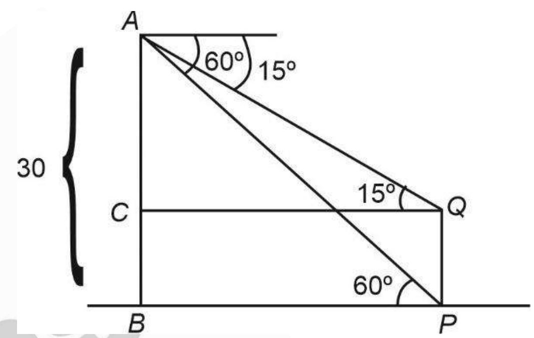
11. From the top of 30 m tower AB the angle of depression to another tower's QP base and top is 60° and 30° respectively. Another point C lies on tower AB such that CQ is parallel to BP (where B and P are the base of towers). Then the area of $BCQP$ is

(1) $600(\sqrt{3} - 1)$ (2) $600(\sqrt{3} + 1)$

(3) 600 (4) $300(\sqrt{3} - 1)$

Answer (1)

Sol.



$\tan 60° = \frac{30}{BP}$

$BP = 10\sqrt{3}$

$\tan 15° = \frac{AC}{10\sqrt{3}}$

$\therefore AC = 10\sqrt{3}(2 - \sqrt{3})$

$\therefore QP = 30 - 10\sqrt{3}(2 - \sqrt{3})$

$= 60 - 20\sqrt{3}$

Now area = $BP \times QP$

$= 10\sqrt{3}(60 - 20\sqrt{3})$

$= 600\sqrt{3} - 600$

$= 600(\sqrt{3} - 1)$

12. If $2y^x + 3x^y = 20$, then $\left(\frac{dy}{dx}\right)_{at(2,2)}$ is equal to

(1) $\frac{-(8 + 12\ln 2)}{(12 + 8\ln 2)}$ (2) $\frac{-(8\ln 2 + 12)}{(8 + 12\ln 2)}$

(3) 8ln + 12 (4) 8 + 12ln2

Answer (2)

Sol. $2y^x + 3x^y = 20$ ...(i)

Let $u = y^x$

$\ln u = x \ln y$

$$\frac{1}{u}\frac{dy}{dx} = \frac{x}{y}\frac{dy}{dx} + \ln y$$

Let $v = x^y$

$\ln v = y \ln x$

$$\frac{1}{v}\frac{dy}{dx} = \frac{y}{x} + \frac{dy}{dx}\ln x$$

Now (i) differentiate w.r.t. x

$$2\frac{dy}{dx} + 3\frac{dv}{dx} = 0$$

$$2y^x\left[\frac{x}{y}\frac{dy}{dx} + \ln y\right] + 3\left[x^y\left[\frac{y}{x} + \frac{dy}{dx}\ln x\right]\right] = 0$$

Put $x = 2$, $y = 2$

$$8\left[\frac{dy}{dx} + \ln 2\right] + 12\left[1 + \frac{dy}{dx}\ln 2\right] = 0$$

13. Number of words with (or) without meaning using all the letters of the word ASSASSINATION such that all the vowels come together is

(1) 38004 (2) 38042

(3) 50400 (4) 60200

Answer (3)

Sol. $\underbrace{\text{SSSSNTN}}_{7}$ $\boxed{\text{AAIAIO}}$

Number of required words

$$\Rightarrow \frac{8!}{4!\,2!} \times \frac{6!}{3!\,2!} = 50400$$

14. If a cuboid has its sides along axes with lengths 3, 4 and 5, find the shortest distance between body diagonal and the side not containing the vertices of body diagonal.

(1) $\frac{20}{\sqrt{41}}$

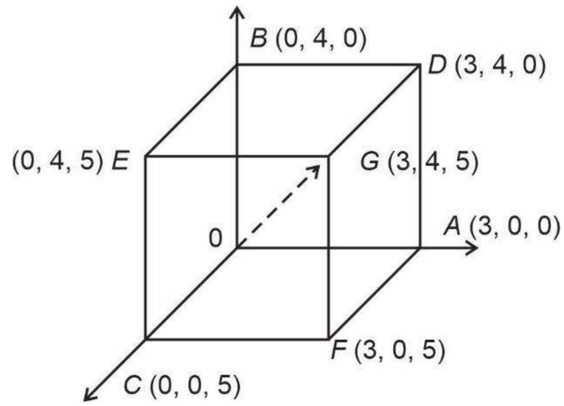
(2) $\frac{12}{5}$

(3) $\frac{15}{\sqrt{34}}$

(4) $\frac{18}{5}$

Answer (2)

Sol.



Equation of $\overrightarrow{OG}$:

$$\vec{r} = \lambda\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$$

Equation of BE :

$$\vec{r} = 4\hat{j} + \mu\left(5\hat{k}\right)$$

$$SD = \frac{\begin{vmatrix} 0 & 4 & 0 \\ 3 & 4 & 5 \\ 0 & 0 & 5 \end{vmatrix}}{\left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 5 \\ 0 & 0 & 5 \end{matrix}\right\|} = \frac{60}{\left|20\hat{i} - 15\hat{j}\right|} = \frac{12}{5}$$

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Let $a_1, a_2, a_3, \ldots a_n$ are in arithmetic progression having common difference as 'd'. The value of

$$\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$$

is ________

Answer (1)

Sol. $\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$

$=\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{\sqrt{a_2}-\sqrt{a_1}}{a_2-a_1}+\frac{\sqrt{a_3}-\sqrt{a_2}}{a_3-a_2}+\ldots\frac{\sqrt{a_n}-\sqrt{a_{n-1}}}{a_n-a_{n-1}}\right)$

$=\lim_{n\to\infty}\sqrt{\frac{d}{n}}-\frac{1}{d}\left(\sqrt{a_n}-\sqrt{a_1}\right)$

$=\lim_{n\to\infty}\frac{1}{\sqrt{d}}\left(\frac{\sqrt{a_1+(n-1)d}-\sqrt{a_1}}{\sqrt{n}}\right)$

$=\lim_{n\to\infty}\left[\frac{1}{\sqrt{d}}\left(\sqrt{\frac{a_1}{n}+d-\frac{d}{n}}-\sqrt{\frac{a_1}{d}}\right)\right]$

= 1

22. Matrix A is 2 × 2 matrix and $A^2 = I$, no elements of the matrix is zero, let sum of diagonal elements is a and det(A) = b, then the value of $3a^2 + b^2$ is

Answer (1)

Sol. $\begin{bmatrix} u & v \\ w & x \end{bmatrix}\begin{bmatrix} u & v \\ w & x \end{bmatrix}$

$u^2 + vw = 1$

$uv + vx = 0 \Rightarrow u = -v$

$wu + wx = 0 \Rightarrow u = -x \Rightarrow u + x = 0 = a$

$wv + x^2 = 1$

$|A^2| = |I|$

det(A) = ±1

$\Rightarrow b = \pm1$

$\Rightarrow 3a^2 + b^2 = 3 \times 0 + 1 = 1$

23. Ratio of terms of 5th term from beginning and 5th term from end is $\sqrt{6}:1$ in $\left(2^{\frac{1}{4}}+\frac{1}{3^{\frac{1}{4}}}\right)^n$. The value of n is ________.

Answer (10)

Sol. $\frac{T_5}{T_{5'}}=\frac{{}^nC_4\left(2^{\frac{1}{4}}\right)^{n-4}\left(3^{-\frac{1}{4}}\right)^4}{{}^nC_4\left(3^{-\frac{1}{4}}\right)^{n-4}\left(2^{\frac{1}{4}}\right)^4}=\sqrt{6}$

$=\left(2^{\frac{1}{4}}\right)^{n-4-4}.\left(3^{\frac{1}{4}}\right)^{n-4-4}=(6)^{\frac{1}{2}}$

$\Rightarrow (6)^{\frac{n-8}{4}}=(6)^{\frac{1}{2}}$

$\Rightarrow \frac{n-8}{4}=\frac{1}{2}$

$\Rightarrow n - 8 = 2$

$\Rightarrow n = 10$

24. If $2n_{C_3} : n_{C_3} = 10$, then $\frac{n^2+3n}{n^2-3n+4}$ is equal to

Answer (02)

Sol. $\frac{2n!}{3!(2n-3)!}\times\frac{3!(n-3)!}{n!}=10$

$\Rightarrow \frac{(2n)(2n-1)(2n-2)}{n(n-1)(n-2)}=10$

$\Rightarrow 4(2n-1)=10n-20$

$\Rightarrow 2n=16 \Rightarrow \boxed{n=8}$

$\therefore \frac{n^2+3n}{n^2-3n+4}=\frac{64+24}{64-24+4}=\frac{88}{44}=02$

25. The number of points of non-differentiability of the function $f(x) = [4 + 13\sin x]$ in $(0, 2\pi)$ is _____.

Answer (50)

Sol. Number of points of non-differentiability for $4 + [13 \sin x]$ is 4 × 12 + 2 = 50 (by graph)

26.

27.

28.

29.

30.

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