JEE MAIN (APRIL) 2023 (06-04-2023-FN)

Memory Based Question Paper

MATHEMATICS

MATHEMATICS

1. Find sum of all possible roots of $|x^2 - 8x + 15| - 2x + 7 = 0$

(1) $9 + \sqrt{3}$ (2) $5 + \sqrt{3}$

(3) $5 - \sqrt{3}$ (4) $4 + \sqrt{3}$

Ans. **(1)**

Sol. $|x^2 - 8x + 15| = 2x - 7$

Case-I : $x \geq 5$

$x^2 - 10x + 22 = 0$

$x = \dfrac{10 \pm \sqrt{12}}{2} = 5 \pm \sqrt{3}$

then $x = 5 + \sqrt{3}$

Case-II : $\dfrac{7}{2} \leq x \leq 5$

$x^2 - 8x + 15 = 7 - 2x$

$x^2 - 6x + 8 = 0$

$x = 4$

$\therefore$ Sum of roots $= 5 + \sqrt{3} + 4 = 9 + \sqrt{3}$

2. The coefficient of x^{18} in the expansion of $\left(x^4 - \dfrac{1}{x^3}\right)^{15}$ is

(1) ${}^{14}C_7$ (2) ${}^{15}C_8$

(3) ${}^{15}C_6$ (4) ${}^{14}C_8$

Ans. **(3)**

Sol. $T_{r+1} = {}^{15}C_r\,(x^4)^{15-r}\left(-\dfrac{1}{x^3}\right)^r$

$T_{r+1} = {}^{15}C_r\,(-1)^r\,x^{60-7r}$

$\therefore\ 60 - 7r = 18 \Rightarrow \quad r = 6$

$\therefore\ T_7 = {}^{15}C_6(-1)^6\,x^{18}$

$\Rightarrow$ Coefficient of x^{18} is ${}^{15}C_6$ or ${}^{15}C_9$

3. The sum of first 20 terms of series 5, 11, 19, 29, 41 …… is

(1) 3520 (2) 3510

(3) 3500 (4) 3505

Ans. **(1)**

Sol. 5 11 19 29 41….

6 8 10 12 $\rightarrow 1^{st}$ difference is AP

$\Rightarrow$ $T_n = an^2 + bn + c$

$T_1 = a + b + c = 5$

$T_2 = 4a + 2b + c = 11$

$T_3 = 9a + 3b + c = 19$

$3a + b = 6$

$5a + b = 8$

$a = 1, b = 3, c = 1$

$T_n = n^2 + 3n + 1$

$$S_{20} = \sum_{n=1}^{20} T_n = \sum_{n=1}^{20}\left(n^2 + 3n + 1\right)$$

$$S_{20} = \frac{20\times21\times41}{6} + 3\times\frac{20\times21}{2} + 20$$

$S_{20} = 2870 + 630 + 20$

$S_{20} = 3520$

4. The number of ways to distribute 20 chocolates among three students such that each student gets atleast one chocolate is

(1) ${}^{22}C_2$ (2) ${}^{19}C_2$ (3) ${}^{19}C_3$ (4) ${}^{22}C_3$

Ans. **(2)**

Sol. Let x, y, z are number of chocolates three students get

$x + y + z = 20$; $x, y, z \geq 1$

$\therefore$ no. of ways is ${}^{19}C_2$

5. In the expansion of $(2^{1/4} + 3^{-1/4})^n$, the ratio of 5^{th} term from start and 5^{th} term from end is $\sqrt{6} : 1$, then find 3^{rd} term

(1) $30\sqrt{3}$ (2) $60\sqrt{3}$ (3) 30 (4) $50\sqrt{3}$

Ans. **(2)**

Sol.
$$\frac{{}^nC_4 (2^{1/4})^{n-4}(3^{-1/4})^4}{{}^nC_4 (3^{-1/4})^{n-4}(2^{1/4})^4} = \sqrt{6}$$

$$\left(\frac{2^{1/4}}{3^{-1/4}}\right)^{n-8} = \sqrt{6}$$

$$(6)^{\frac{n-8}{4}} = \sqrt{6}$$

$n - 8 = 2$

$n = 10$

$T_3 = {}^{10}C_2\,(2^{1/4})^8(3^{-1/4})^2$

$$= {}^{10}C_2 \times (\sqrt{2})^4 \times \frac{1}{\sqrt{3}} = 60\sqrt{3}$$

6. Let $A = [a_{ij}]_{2\times2}$ be a matrix and $A^2 = I$ where $a_{ij} \neq 0$. If a is sum of diagonal elements and $b = \det(A)$, then $3a^2 + 4b^2$ is

(1) 10 (2) 12 (3) 4 (4) 8

Ans. **(3)**

Sol. $A^2 = I$

So $A^2 - 0A - I = O \Rightarrow \lambda^2 - 0\lambda - 1 = 0$

Here $a = \lambda_1 + \lambda_2 = 0$

$b = \lambda_1\lambda_2 = -1$

7. Let $I(x)=\int \frac{x^2(x\sec^2 x + \tan x)}{(x\tan x + 1)^2}dx$. If $I(0) = 1$ then $I\left(\frac{\pi}{4}\right)$ is equal to

(1) $-\frac{\pi^2}{4\pi+16} + 2\ell n\left(\frac{\pi+4}{4\sqrt{2}}\right) + 1$ (2) $\frac{\pi^2}{4\pi+16} - 2\ell n\left(\frac{\pi+4}{4\sqrt{2}}\right) + 1$

(3) $-\frac{\pi^2}{\pi+4} + 2\ell n\left(\frac{\pi+1}{\sqrt{2}}\right) + 1$ (4) $\frac{\pi^2}{\pi+16} + 2\ell n\left(\frac{\pi+1}{4\sqrt{2}}\right) + 1$

Ans. **(1)**

Sol. Using integration by parts

$$I(x) = x^2 . \frac{(-1)}{x\tan x + 1} - \int 2x . \frac{(-1)}{x\tan x + 1} dx$$

$$= -\frac{x^2}{x\tan x + 1} + 2\int \frac{x\cos x}{x\sin x + \cos x} dx$$

$$\Rightarrow I(x) = \frac{-x^2}{x\tan x + 1} + 2\ell n|x \sin x + \cos x| + c$$

put x = 0

c = 1

$$\therefore I\left(\frac{\pi}{4}\right) = \frac{\frac{-\pi^2}{16}}{\frac{\pi}{4}+1} + 2\ \ell n\left(\frac{\frac{\pi}{4}+1}{\sqrt{2}}\right) + 1$$

$$I\left(\frac{\pi}{4}\right) = -\frac{\pi^2}{4\pi+16} + 2\ell n\left(\frac{\pi+4}{4\sqrt{2}}\right) + 1$$

8. If $a_1, a_2, \ldots\ldots a_n$ are in arithmetic progression with common difference d > 0, then find

$$\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots+\frac{1}{\sqrt{a_n}+\sqrt{a_{n-1}}}\right)$$

Ans. **(1)**

Sol. $\frac{1}{d}\sqrt{\frac{d}{n}}\left[\left(\sqrt{a_2}-\sqrt{a_1}\right)+\left(\sqrt{a_3}-\sqrt{a_2}\right)+\ldots+\left(\sqrt{a_n}-\sqrt{a_{n-1}}\right)\right]$

$=\frac{1}{d}\sqrt{\frac{d}{n}}\left(\sqrt{a_n}-\sqrt{a_1}\right)$

$=\frac{1}{d}\sqrt{\frac{d}{n}}\left(a_1+nd\right)^{1/2}$

$=\frac{1}{d}\sqrt{\frac{d}{n}}\times\sqrt{n}\left(d+\frac{a_1}{n}\right)^{1/2}$

$=\frac{1}{d}\sqrt{d}\times\sqrt{d}\left(1+\frac{a_1}{nd}\right)^{1/2}$

$=1$

9. A pair of dice is rolled 5 times. Let getting a total of 5 in a single throw is considered as success. If probability of getting atleast four successes is $\frac{x}{3}$ then x is equal to

(1) $\frac{41}{9^5}$ (2) $\frac{41}{9^4}$ (3) $\frac{123}{9^5}$ (4) $\frac{123}{9^4}$

Ans. **(3)**

Sol. P(success) $=\frac{4}{36}=\frac{1}{9}$

P(atleast four success) $= {}^5C_4\left(\frac{1}{9}\right)^4\cdot\frac{8}{9}+\left(\frac{1}{9}\right)^5=\frac{x}{3}$

$\Rightarrow x=\frac{41\times 3}{9^5}=\frac{123}{9^5}$

10. Let f(x) satisfies $5f(x)+4f\left(\frac{1}{x}\right)=\frac{1}{x}+3$, then $18\int_1^2 f(x)\,dx$ is

(1) $10\ell n\,3-6$ (2) $5\ell n\,2-6$ (3) $10\ell n2-6$ (4) $5\ell n\,2-3$

Ans. **(3)**

Sol. $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3$ .....(i)

Replace $x \to \frac{1}{x}$

$5f\left(\frac{1}{x}\right) + 4f(x) = x + 3$ .....(ii)

By (i) & (ii)

$9f(x) = \frac{5}{x} - 4x + 3$

$18\int_1^2 f(x)\,dx = \int_1^2 \left(\frac{10}{x} - 8x + 6\right) dx = 10\ell n\, 2 - 6$

11. If image of point P(1, 2, 3) about the plane $2x - y + 3z = 2$ is point Q, then area of ΔPQR is where R is (4, 10, 12)

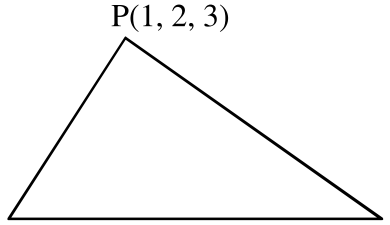
(1) $\frac{1}{2}\sqrt{1531}$ (2) $\sqrt{1531}$ (3) $\frac{1}{4}\sqrt{1531}$ (4) $\frac{1}{2}\sqrt{1351}$

Ans. **(1)**

Sol. Image formula w.r.t P

$$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{3} = -2\frac{(2\times1-2+3\times3-2)}{2^2+1^2+3^2}$$

$\Rightarrow$ Q(–1, 3, 0)



$$\text{Area} = \frac{1}{2}\left\|\begin{matrix}\hat{i} & \hat{j} & \hat{k}\\ 5 & 7 & 12\\ 2 & -1 & 3\end{matrix}\right\|$$

$= \frac{1}{2}\left|33\hat{i} + 9\hat{j} - 19\hat{k}\right|$

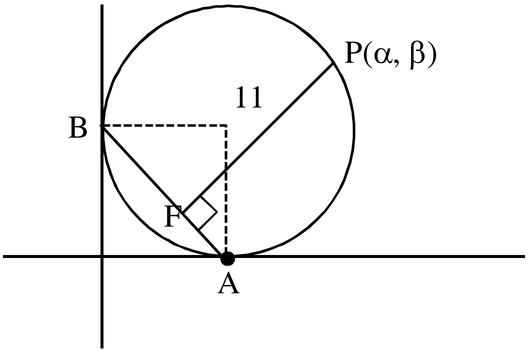
$\text{Area} = \frac{1}{2}\sqrt{(33)^2 + 9^2 + (19)^2}$

$= \frac{1}{2}\sqrt{1089+81+361} = \frac{1}{2}\sqrt{1531}$

12. Circle in Ist quadrant touches both the axes at A & B. If length of perpendicular from P(α, β) on circle to chord AB is equal to 11, Find α.β

Ans. **(121)**

Sol. $C : (x-r)^2 + (y-r)^2 = r^2;\ \alpha^2 + \beta^2 - 2r(\alpha + \beta) + r^2 = 0$



$\alpha^2 + \beta^2 - 2r\left(11\sqrt{2} + r\right) + r^2 = 0$

$\alpha^2 + \beta^2 - 22\sqrt{2}\ r - r^2 = 0$

$PF = \dfrac{\alpha + \beta - r}{\sqrt{2}} = 11$

$\alpha + \beta = 11\sqrt{2} + r$

$\alpha^2 + \beta^2 + 2\alpha\beta = 242 + r^2 + 22r\sqrt{2}$

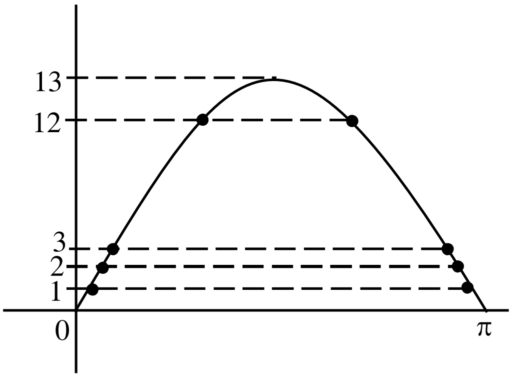
$\alpha\beta = 121$

13. If f(x) = [a + 13 sinx] & x ∈ (0, π), then number of non-differentiable points of f(x) are [where 'a' is integer]

Ans. **(25)**

Sol. Points where $\sin x = \dfrac{1}{13}, \dfrac{2}{13}, \ldots\ldots, \dfrac{12}{13}$ will be the points of non-derivability of f(x)

⇒ 24 points



And also where sin x = 1 ⇒ 1 points

∴ 25 points of non-derivability

14. If A(1, 1, 1), B(0, λ, 0), C(λ + 1, 0, 1), D(2, 2, –2) are coplanar then $\sum (\lambda_i + 2)^2$ is equal to

(1) $\frac{80}{3}$ (2) $\frac{320}{9}$ (3) $\frac{160}{9}$ (4) $\frac{160}{3}$

Ans. **(3)**

Sol. $[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 0$

$\Rightarrow \lambda = 2, -\frac{2}{3}$

$\sum(\lambda_i + 2)^2 = 16 + \frac{16}{9} = \frac{160}{9}$

DATA FICTITIOUS

15. If [x + 6] + [x + 3] ≤ 7 and let call its solution as set A and set B is the solution of inequality $3^{5x-8} < 3^{-3x}$

(1) $B \subset A, A \neq B$ (2) $A \subset B, A \neq B$ (3) $A \cap B = \phi$ (4) $A \cup B = R$

Ans. **(2)**

Sol. $2[x] \leq -2 \Rightarrow [x] \leq -1 \Rightarrow x < 0$

A is $(-\infty, 0)$

$5x - 8 < -3x \Rightarrow x < 1 \Rightarrow$ B is $(-\infty, 1)$

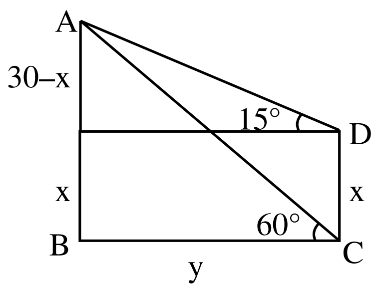
Hence $A \subset B, A \neq B$

16. Height of tower AB is 30 m where B is foot of tower. Angle of elevation from a point C on level ground to top of tower is 60° and angle of elevation of A from a point D x m above C is 15° then find area of quadrilateral ABCD.

(1) $300(\sqrt{3}-1)$ (2) $600(\sqrt{3}-1)$ (3) $150(\sqrt{3}-1)$ (4) $100(\sqrt{3}-1)$

Ans. **(2)**

Sol.



$\tan 60° = \frac{30}{y} = \sqrt{3}$

$\Rightarrow y = 10\sqrt{3}$

$\tan 15° = \dfrac{30 - x}{y}$

$(2 - \sqrt{3})10\sqrt{3} = 30 - x$

$x = 30 - 20\sqrt{3} + 30$

$x = 60 - 20\sqrt{3}$

Area of ABCD = $xy = (60 - 2\sqrt{3}).\ 10\sqrt{3}$

$= 600(\sqrt{3} - 1)$

17. Equivalent statement to $(p \to q) \vee (r \to q)$ will be

(1) $(p \wedge r) \to q$ (2) $(p \vee r) \to q$

(3) $(q \to r) \vee (p \vee r)$ (4) $(r \to p) \wedge (q \to r)$

Ans. **(1)**

Sol.

p	q	r	$p \to q$	$r \to q$	$(p\to q) \vee (r \to q)$	$(p \wedge r)$	$(p \wedge r) \to q$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	F	F	F	T	F
T	F	F	F	T	T	F	T
F	T	T	T	T	T	F	T
F	T	F	T	T	T	F	T
F	F	T	T	F	T	F	T
F	F	F	T	T	T	F	T

18. For two groups of 15 sizes each, mean and variance of first group is 12, 14 respectively, and second group has mean 14 and variance of σ^2. If combined variance is 13 then find variance of second group ?

(1) 9 (2) 11 (3) 10 (4) 12

Ans. **(3)**

Sol. $\bar{x} = 12,\ \sigma_1^2 = 14,\ \bar{y} = 14,\ \sigma_2^2 = \sigma^2,\ n_1 = n_2 = 15$

$\sigma_1^2 = 14 = \dfrac{\sum x_i^2}{15} - (12)^2 \Rightarrow \sum x_i^2 = 2370,\ \sum xi = 180$

$\sigma_2^2 = \dfrac{\sum y_i^2}{15} - (14)^2,\ \sum y_i = 210$

$13 = \dfrac{\sum x_i^2 + \sum y_i^2}{30} - \left(\dfrac{15\bar{x} + 15\bar{y}}{30}\right)^2$

$13 = \dfrac{2370 + \sum y_i^2}{30} - (13)^2$

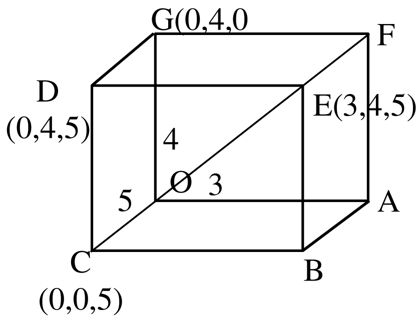
$\sum y_i^2 = 3090 \Rightarrow \sigma_2^2 = \dfrac{3090}{15} - (14)^2 = 10$

19. A rectangular parallelepiped with edges along x, y, z axis has length of 3, 4, 5 respectively. Find the shortest distance of the body diagonal from one of the edges parallel to z-axis which is skew to the diagonal

(1) $\frac{16}{5}$ (2) $\frac{15}{\sqrt{34}}$ (3) $\frac{12}{5}$ (4) $\frac{9}{5}$

Ans. **(3)**

Sol. Equation of diagonal OE $\vec{r} = 0 + \lambda\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$



Equation of edge GD

$\vec{r} = 4\hat{j} + \mu\hat{k}$

Shortest distance = $\left|\text{projection of } 4\hat{j} \text{ on} \left(3\hat{j} - 4\hat{i}\right)\right|$

$= \frac{12}{\sqrt{9+16}} = \frac{12}{5}$

20. If ${}^{2n}C_3 : {}^{n}C_3 = 10$, then $\frac{n^2+3n}{n^2-3n+4}$ is equal to

Ans. **(2)**

Sol. $\frac{{}^{2n}C_3}{{}^{n}C_3} = 10 \Rightarrow \frac{2n\,.\,(2n-1)\,.\,(2n-2)}{n\,.\,(n-1)(n-2)} = 10$

$\Rightarrow \frac{(2n-1)\,.\,2}{n-2} = 5$

$\Rightarrow n = 8$

$\therefore \frac{n^2+3n}{n^2-3n+4} = \frac{88}{44} = 2$

21. Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}$, $\vec{c} = -\hat{i} + 4\hat{j} + 3\hat{k}$ and $\vec{d}$ is a vector perpendicular to $\vec{b}$ and $\vec{c}$, $\vec{a}.\vec{d} = 18$ then find $|\vec{a} \times \vec{d}|^2$

(1) 720 (2) 700 (3) 360 (4) 300

Ans. **(1)**

Sol. $\vec{d} = \lambda(\vec{b} \times \vec{c}) = \lambda(2\hat{i} - \hat{j} + 2\hat{k})$

$\vec{a}.\vec{d} = 18$

$\Rightarrow \lambda = 2$

$\therefore |\vec{a} \times \vec{d}|^2 = \vec{a}^2 \vec{d}^2 - (\vec{a}.\vec{d})^2$

$\Rightarrow |\vec{a} \times \vec{d}|^2 = 29 \times 36 - 324 = 1044 - 324 = 720$