

PART : MATHEMATICS

1. The number of ways to distribute 20 identical chocolates among three students such that each student will get at least one chocolate, is

(1) ${}^{22}C_2$

(2) ${}^{19}C_3$

(3) ${}^{19}C_2$

(4) ${}^{22}C_3$

Ans. (3)

Sol. $x_1 + x_2 + x_3 = 20$

let $x_1 = 1 + t_1, x_2 = 1 + t_2, x_3 = 1 + t_3$

when $t_1, t_2, t_3 \in \{0, 1, 2, \dots, 17\}$

$t_1 + t_2 + t_3 = 17 \dots (1)$

So number of such distribution is equal to number of non negative integral solutions of equation (1)

So required solution $= {}^{17+3-1}C_{3-1} = {}^{19}C_2$

2. Sum of first 20 terms of the series : $5 + 11 + 19 + 29 + 41 + \dots$ is

(1) 3250

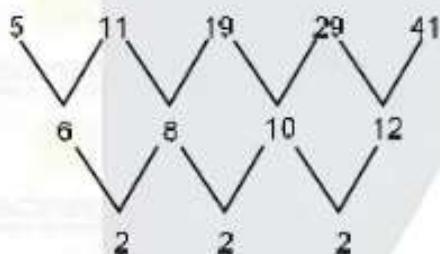
(2) 3520

(3) 3052

(4) 3205

Ans. (2)

Sol.



let $T_n = an^2 + bn + c$

$\therefore a + b + c = \dots \quad (1)$

$4a + 2b + c = 11 \dots (2)$

$9a + 3b + c = 19 \dots (3)$

$(2) - (1) \Rightarrow 3a + b = 6 \dots (4)$

$(3) - (2) \Rightarrow 5a + b = 8 \dots (5)$

$(5) - (4) \Rightarrow 2a = 2 \Rightarrow a = 1$

$\therefore b = 3$ and $c = 1$

$\therefore T_n = n^2 + 3n + 1$

$\therefore S_{20} = \sum_{n=1}^{20} T_n = \sum_{n=1}^{20} (n^2 + 3n + 1)$

$$= \frac{20 \times 21 \times 41}{6} + \frac{3 \times 20521}{2} + 20$$

$$= 70 \times 41 + 30 \times 21 + 20$$

$$= 10(7 \times 41 + 63 + 2)$$

$$= 10(287 + 65)$$

$$= 10(352)$$

$$= 3520$$

3. $|x^2 - 8x + 15| - 2x + 7 = 0$, then sum of roots of the equation is
 (1) $9 + \sqrt{3}$ (2) $6 + \sqrt{3}$ (3) $3 + \sqrt{3}$ (4) $9 - \sqrt{3}$

Ans. (1)

Sol.

case I

$$x < 3$$

$$x^2 - 8x + 15 - 2x + 7 = 0$$

$$x^2 - 10x + 22 = 0$$

$$(x - 5)^2 - 3 = 0$$

$$x - 5 = \pm\sqrt{3}$$

$$x = 5 + \sqrt{3}, 5 - \sqrt{3}$$

case II

case II

$$3 \leq x \leq 5$$

$$x^2 - 8x + 15 + 2x - 7 = 0$$

$$x^2 - 6x + 8 = 0$$

$$x = 2, 4$$

case III

$$x > 5$$

$$x - 5 = \pm\sqrt{3}$$

$$x = 5 + \sqrt{3}, 5 - \sqrt{3}$$

$$\text{sum} = 5 + \sqrt{3} + 4 = 9 + \sqrt{3}$$

4. If $\frac{\frac{2n}{n}C_3}{nC_3} = \frac{10}{1}$, then value of $\frac{n^2 + 3n}{n^2 - 3n + 4}$ is equal to
 (1) 1 (2) 2 (3) 4 (4) 5

Ans. (2)

Sol. $\frac{\frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)}}{1} = \frac{10}{1}$

$$\frac{4(2n-1)}{n-2} = \frac{10}{1}$$

$$8n - 4 = 10n - 20 \Rightarrow n = 8$$

$$\frac{n^2 + 3n}{n^2 - 3n + 4} = \frac{64 + 24}{64 - 24 + 4} = \frac{88}{44} = 2$$

5. $\int \frac{x^2(x \sec^2 x + \tan x)}{(x \tan x + 1)^2} dx$ is equal to

- (1) $\frac{-x^2}{(x \tan x + 1)} + 2 \ln |x \sin x - \cos x| + C$. (2) $\frac{-x^2}{(x \tan x + 1)} - 2 \ln |x \sin x + \cos x| + C$.
 (3) $\frac{-x^2}{(x \tan x + 1)} + 2 \ln |x \sin x + \cos x| + C$. (4) $\frac{x^2}{(x \tan x + 1)} + 2 \ln |x \sin x + \cos x| + C$.

Ans. (3)

Sol. Let $I = \int \frac{x^2(x \sec^2 x + \tan x)}{(x \tan x + 1)^2} dx$

$$\because \frac{d}{dx}(x \tan x + 1) = x \sec^2 x + \tan x$$

$$\begin{aligned} \therefore \int \frac{x \sec^2 x + \tan x}{(x \tan x + 1)^2} dx &= -\frac{1}{x \tan x + 1} \\ \therefore I &= x^2 \left(\frac{-1}{x \tan x + 1} \right) - \int 2x \left(\frac{1}{x \tan x + 1} \right) dx \\ &= \frac{-x^2}{(x \tan x + 1)} + 2 \int \frac{x \cos x}{x \sin x + \cos x} dx \\ &= \frac{-x^2}{(x \tan x + 1)} + 2 \int \frac{\frac{d}{dx}(x \sin x + \cos x)}{x \sin x + \cos x} dx \\ &= \frac{-x^2}{(x \tan x + 1)} + 2 \ln |x \sin x + \cos x| + C. \end{aligned}$$

6. The coefficient of x^{18} , in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$, is

(1) ${}^{14}C_7$

(2) ${}^{15}C_6$

(3) ${}^{15}C_6$

(4) ${}^{14}C_9$

Ans.

(3)

Sol. $T_{r+1} = {}^{15}C_r (x^4)^{15-r} \left(-\frac{1}{x^3}\right)^r$

$60 - 4r - 3r = 18$

$7r = 42$

$r = 6$

Coff. = ${}^{15}C_6$

7. If $2x^y + 3y^x = 20$, then $\frac{dy}{dx}$ at point $(2, 2)$ is equal to

(1) $\frac{(2-3\ln 2)}{(4-2\ln 2)}$

(2) $\frac{(2+3\ln 2)}{(3+4\ln 2)}$

(3) $\frac{-(2+3\ln 2)}{(3+4\ln 2)}$

(4) $\frac{-(2+3\ln 2)}{(3+2\ln 2)}$

Ans. (4)

Sol. Differentiating $2 \cdot x^y \left(\frac{dy}{dx} \ln x + y \frac{1}{x} \right) + 3 \cdot y^x \left(1 \cdot \ln y + x \frac{y'}{y} \right) = 0$

Put $x = 2, y = 2$

$2 \cdot 4 \left(\frac{dy}{dx} \ln 2 + 1 \right) + 3 \cdot 4 \left(\ln 2 + \frac{dy}{dx} \right) = 0$

$(2 \cdot \ln 2 + 3) \frac{dy}{dx} + (2 + 3 \ln 2) = 0$

$\frac{dy}{dx} = -\frac{(2+3\ln 2)}{(3+2\ln 2)}$

8. Two groups G_1 and G_2 have 15 observations each, means of G_1 and G_2 are 12 and 14 respectively. If variance of G_1 is 14 and combined variance of G_1 and G_2 is 13, then the variance of G_2 is
 (1) 12 (2) 10 (3) 11 (4) 13

Ans. (2)

Sol. We know that combined variance of two groups = $\frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1n_2(\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}$

$$\Rightarrow 13 = \frac{15(14) + 15(\sigma_2^2)}{30} + \frac{225(4)}{30 \times 30}$$

$$\Rightarrow 13 = \frac{14 + \sigma_2^2}{2} + \frac{15 \times 2}{30}$$

$$\Rightarrow \sigma_2^2 = 10$$

9. In the expansion of $\left(2^{\frac{1}{4}} + \frac{1}{3^{\frac{1}{4}}}\right)^n$, if the ratio of 5th term from the beginning and the end is $\sqrt{6} : 1$, then the 3rd term from the beginning is :

$$(1) \frac{60}{\sqrt{3}}$$

$$(2) 30\sqrt{3}$$

$$(3) 90\sqrt{3}$$

$$(4) 60\sqrt{3}$$

Ans. (4)

Sol. given $\frac{{}^n C_4 \left(\frac{1}{2^{\frac{1}{4}}}\right)^{n-4} \cdot \left(\frac{1}{3^{\frac{1}{4}}}\right)^4}{{}^n C_4 \left(\frac{1}{3^{\frac{1}{4}}}\right)^{n-4} \cdot \left(\frac{1}{2^{\frac{1}{4}}}\right)^4} = \sqrt{6}$

$$\Rightarrow 2^{\frac{n-8}{4}} \cdot 3^{\frac{n-8}{4}} = \sqrt{6}$$

$$\Rightarrow 6^{n-8} = 6^2$$

$$\Rightarrow n = 10$$

$$\text{So } T_3 = {}^{10} C_2 \left(\frac{1}{2^{\frac{1}{4}}}\right)^8 \cdot \left(\frac{1}{3^{\frac{1}{4}}}\right)^2$$

$$= \frac{15 \times 3 \times 4}{\sqrt{3}} = 60\sqrt{3}$$

10. If the image of point P(1,2,3) in the plane $2x-y+3z=2$ is Q, then the area of triangle PQR is, (where point R is (4, 10, 12))

$$(1) \frac{\sqrt{1543}}{4}$$

$$(2) \frac{\sqrt{2131}}{2}$$

$$(3) \frac{\sqrt{1531}}{2}$$

$$(4) \frac{\sqrt{3215}}{4}$$

Ans. (3)

Sol. Image of P in plane is Q

$$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{3} = -2 \frac{(2-2+9-2)}{4+1+9}$$

$$\vec{PQ} = -2\hat{i} + \hat{j} - 3\hat{k}, \vec{PR} = 3\hat{i} + 8\hat{j} + 9\hat{k}$$

$$\begin{aligned}\text{Area of } \triangle PQR &= \frac{1}{2} |\vec{PQ} \times \vec{PR}| = \frac{1}{2} \left| \begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 8 & 9 \end{array} \right| \\ &= \frac{1}{2} \left| \hat{i}(9+24) - \hat{j}(-18+9) + \hat{k}(-16-3) \right| \\ &= \frac{1}{2} \sqrt{33^2 + 9^2 + 19^2} = \frac{1}{2} \sqrt{1531}\end{aligned}$$

11. If $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3$ ($x > 0$), then $18 \int_1^2 f(x)dx$ is
 (1) $10 \ln 2 + 6$ (2) $10 \ln 2 - 6$ (3) $5 \ln 2 + 16$ (4) $10 \ln 2 - 16$

Ans. (2)

$$\text{Sol. } \because 5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3$$

$$5f\left(\frac{1}{x}\right) + 4f(x) = x + 3$$

$$9f(x) = \frac{5}{x} + 15 - 4x - 12$$

$$18f(x) = \frac{10}{x} - 8x + 6$$

$$\int_1^2 18f(x)dx = \left(10 \ln x - 4x^2 + 6x\right)_1^2$$

$$= 10 \ln 2 - 16 + 12 + 4 - 6$$

$$= 10 \ln 2 - 6$$

12. $(P \Rightarrow Q) \wedge (R \Rightarrow Q)$ is equivalent to :

- | | |
|----------------------------------|----------------------------------|
| (1) $(P \wedge R) \Rightarrow Q$ | (2) $(P \vee R) \Rightarrow Q$ |
| (3) $(P \vee Q) \Rightarrow R$ | (4) $(P \wedge Q) \Rightarrow R$ |

Ans. (2)

$$\text{Sol. } (\neg P \vee Q) \wedge (\neg R \vee Q)$$

$$\equiv (\neg P \wedge \neg R) \vee Q$$

$$\equiv \neg (P \vee R) \vee Q$$

$$\equiv (P \vee R) \Rightarrow Q$$

13. If $\vec{a} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$, $\vec{b} = \mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$, $\vec{c} = -\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ and \vec{d} is a vector perpendicular to both \vec{b} and \vec{c} such that $|\vec{a} \cdot \vec{d}| = 18$, then $|\vec{a} \times \vec{d}|^2 =$

Ans. (3)

Sol. As \vec{d} is perpendicular to both \vec{b} and $\vec{c} \Rightarrow \vec{d} = \lambda (\vec{b} \times \vec{c})$, $\lambda \in \mathbb{R}$

$$\vec{d} = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -2 \\ -1 & 4 & 3 \end{vmatrix} = \lambda (\hat{i}(2) - \hat{j}(1) + \hat{k}(2))$$

$$= \lambda (2\hat{i} - \hat{j} + 2\hat{k})$$

$$\bar{a}\bar{d} = 18 \Rightarrow |\lambda| = 2$$

$$(\vec{a} \times \vec{d})^2 = \lambda^2 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 2 & -1 & 2 \end{vmatrix} = 4 (10\hat{i} + 4\hat{j} - 8\hat{k})^2$$

$$= 4(100 + 16 + 64) = 4(180) = 720$$

14. Given that $a_1, a_2, a_3, \dots, a_n$ are in A.P with common difference. Then value of

$$\lim_{n \rightarrow \infty} \sqrt{\frac{d}{n}} \left(\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} \right) \text{ is}$$

Ans. (1)

$$\begin{aligned}
 \text{Sol.} & \left(\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} \right) \\
 & = \left(\frac{\sqrt{a_2} - \sqrt{a_1}}{a_2 - a_1} + \frac{\sqrt{a_3} - \sqrt{a_2}}{a_3 - a_2} + \frac{\sqrt{a_4} - \sqrt{a_3}}{a_4 - a_3} + \dots + \frac{\sqrt{a_n} - \sqrt{a_{n-1}}}{a_n - a_{n-1}} \right) \\
 & = \frac{1}{d} (\sqrt{a_n} - \sqrt{a_1})
 \end{aligned}$$

$$\therefore \lim_{n \rightarrow \infty} \sqrt{\frac{d}{n}} \left(\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} \right)$$

$$= \lim_{n \rightarrow \infty} \sqrt{\frac{d}{n}} \left(\frac{\sqrt{a_n} - \sqrt{a_1}}{d} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{\sqrt{a_1 + (n-1)d} - \sqrt{a_1}}{\sqrt{n}\sqrt{d}} \right) = \lim_{n \rightarrow \infty} \left(\sqrt{\frac{a_1}{nd} + \left(1 - \frac{1}{n}\right)} - \sqrt{\frac{a_1}{nd}} \right)$$

1

15. $I = \int_0^{\frac{\pi}{2}} \cos^5 x (\sin^7 x) dx$ is equal to

(1) $\frac{1}{120}$

(2) $\frac{1}{60}$

(3) $\frac{1}{240}$

(4) $\frac{\pi}{120}$

Sol. By wall's formulas

$$I = \frac{(4 \times 2)(6 \times 4 \times 2)}{12 \times 10 \times 8 \times 6 \times 4 \times 2} = \frac{1}{120}$$

16. Let A be a 2×2 matrix with $A^2 = I$ and all entries of A are non-zero. If sum of leading diagonal elements is 'a' and $\det(A) = b$, then the value of $(3a^2 + b^2)$ is

Ans. 1

Sol. $|A|^2 = 1 \Rightarrow b^2 = 1$

$$\text{Let } A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} = \begin{bmatrix} a_1^2 + b_1 a_2 & a_1 b_1 + b_1 b_2 \\ a_1 a_2 + b_2 a_2 & a_2 b_1 + b_2^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow a_2(a_1 + b_2) = 0 \text{ and } b_1(a_1 + b_2) = 0$$

$$\Rightarrow a_1 + b_2 = 0$$

$$\Rightarrow \text{Tr}(A) = 0$$

$$\Rightarrow a = 0$$

$$\text{So } (3a^2 + b^2) = 0 + 1 = 1$$

17. A pair of dice is rolled 5 times. If getting a sum 5 is considered as a success, then the probability of four successes is

(1) $\frac{4}{9^5}$

(2) $\frac{20}{9^4}$

(3) $\frac{5}{9^4}$

(4) $\frac{40}{9^5}$

Ans. (4)

Sol. $S = \{(1, 1), (1, 2), \dots, (1, 6)$

$(2, 1), (2, 2), \dots, (2, 6)$

$(6, 1), (6, 2), \dots, (6, 6)\}$

Event of success = $\{(1, 4), (4, 1), (2, 3), (3, 2)\}$

$$p = \frac{4}{36}$$

$$\text{Probability of exactly four successes is} = {}^5C_4 \left(\frac{8}{9}\right)^1 \left(\frac{1}{9}\right)^7$$

18. If $f(x) = [a + 13 \sin x]$, where $x \in (0, \pi)$ and $[\cdot]$ denotes G.I.F. and $a \in I$, then the number of points of non-differentiability is

(1) 24

(2) 26

(3) 25

(4) 23

Ans. (3)

Sol. $f(x) = a + [13 \sin x] \quad \because a \in I \text{ and } x \in (0, \pi)$

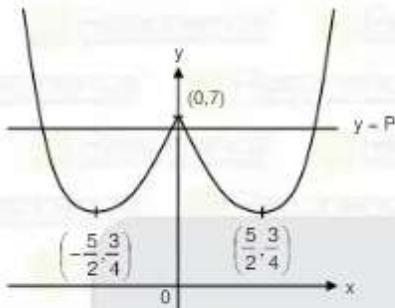
\therefore total number of points of non-differentiability of $[p \sin x] = 2p - 1$ here $p = 13$

\therefore total number of points of non-differentiability of $[13 \sin x] = 25$

19. If the equation $|x^2 - 5|x| + 7| = P$, $P \in I$, has 4 solutions, then the number of possible values of P is.

Ans. (6)

Sol. Let $y = ||x|^2 - 5|x| + 7|$ $\because |x|^2 = x^2$ as $x \in R$



Suppose $f(x) = x^2 - 5x + 7$

So $y = |f(|x|)|$

for 4 solutions

$$\frac{3}{4} < P < 7$$

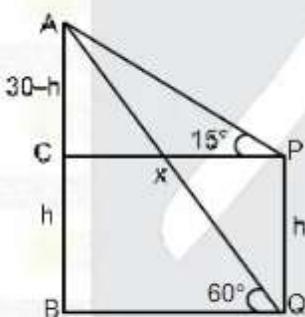
But $P \in I$ so $P = 1, 2, 3, 4, 5, 6$
6 values

20. AB is a building of height 30 m with base B. PQ is a pole with base Q. Angle of depression from point A to P and Q are 15° and 60° respectively. A point C is on the same level of point P, on the building. Then area of rectangle PCBQ is

- (1) $60(\sqrt{3}-1)$ (2) $600(\sqrt{3}+1)$ (3) $600(\sqrt{3}-1)$ (4) $60(\sqrt{3}+1)$

Ans. (3)

Sol.



$$\tan 15^\circ = \frac{30-h}{x}$$

$$\tan 60^\circ = \frac{30}{x} \Rightarrow x = \frac{30}{\sqrt{3}}$$

$$\frac{2-\sqrt{3}}{\sqrt{3}} = \frac{30-h}{30} \Rightarrow 30\left(\frac{2-\sqrt{3}}{\sqrt{3}}\right) = 30-h$$

$$h = 30 - 30 \times \frac{2}{\sqrt{3}} + 30 = 60 - \frac{60}{\sqrt{3}}$$

$$\text{Area} = \frac{30(60)(\sqrt{3}-1)}{3} = 600(\sqrt{3}-1)$$

21. Number of words with or without meaning using all the letters of the word ASSASSINATION such that all the vowels come together is :
(1) 38004 (2) 38042 (3) 50400 (4) 60200

Ans. (3)

Sol. We have to arrange bundle of AAAIO with seven consonants SSSSNTN :

$$\text{So, no of words} = \frac{8!}{4!2!} \times \frac{6!}{3!2!1!} = 50400$$