MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. If $f(x) + f(\pi - x) = \pi^2$ then $\int_0^{\pi} f(x)\sin x dx$

(1) π^2 (2) $\frac{\pi^2}{4}$

(3) $2\pi^2$ (4) $\frac{\pi^2}{8}$

Answer (1)

Sol. $I = \int_0^{\pi} f(x)\sin x\, dx$

$I = \int_0^{\pi} f(\pi - x)\sin x\, dx$

$2I = \int_0^{\pi} (\sin x)(f(x) + f(\pi - x))\, dx$

$2I = \pi^2 \int_0^{\pi} \sin x\, dx$

$2I = 2\pi^2 \int_0^{\frac{\pi}{2}} \sin x\, dx$

$I = \pi^2 (-\cos x)_0^{\frac{\pi}{2}}$

$= \pi^2$

2. The system of the equations

$x + y + z = 6$

$x + 2y + \alpha z = 5$ has

$x + 2y + 6z = \beta$

(1) Infinitely many solution for $\alpha = 6$, $\beta = 3$

(2) Infinitely many solution for $\alpha = 6$, $\beta = 5$

(3) Unique solution for $\alpha = 6$, $\beta = 5$

(4) No solution for $\alpha = 6$, $\beta = 5$

Answer (2)

Sol. Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \alpha \\ 1 & 2 & 6 \end{vmatrix} = 6 - \alpha$

$\therefore$ for $\alpha \neq 6$ system has unique solution

Now, when $\alpha = 6$

$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 5 & 2 & 6 \\ \beta & 2 & 6 \end{vmatrix} = 0 - (30 - 6\beta) + (10 - 2\beta)$

$= 4(\beta - 5)$

$\Delta_2 = \begin{vmatrix} 1 & 6 & 2 \\ 1 & 5 & 6 \\ 1 & \beta & 6 \end{vmatrix} = -4(\beta - 5)$

$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 5 \\ 1 & 2 & \beta \end{vmatrix} = \begin{vmatrix} 1 & 1 & 6 \\ 0 & 1 & -1 \\ 0 & 1 & \beta - 6 \end{vmatrix} = \beta - 5$

Clearly at $\beta = 5$, $\Delta_i = 0$ for $i = 1, 2, 3$

$\therefore$ at $\alpha = 6$, $\beta = 5$ system has infinite solutions.

3. Let statement 1 : $(2002)^{2023} - (1919)^{2002}$ is divisible by 8.

Statement 2 : $13.13^n - 12n - 13$ is divisible by 144 $\forall n \in N$, then

(1) Statement-1 and statement-2 both are true

(2) Only statement-1 is true

(3) Only statement-2 is true

(4) Neither statement-1 nor statement-2 are true

Answer (3)

Sol. $\because$ $(2002)^{2023} = 8m$

$\therefore$ $(2002)^{2023}$ is divisible by 8

and $(1919)^{2002}$ is not divisible by 8

$\therefore$ $(2002)^{2023} - (1919)^{2002}$ is not divisible by 8.

Now

$13.(13)^n - 12n - 13$

$= 13(1 + 12)^n - 12n - 13$

$= 13\left[1 + 12n + {}^nC_2 12^2 + --\right] - 12n - 13$

$= 144n + 144\,{}^nC_2 + --$

$= 144\left[n + {}^nC_2 + --\right]$

$= 144K$

$\therefore$ Statement-2 is correct

4. If the coefficient of x^7 in $\left(\alpha x^2 + \frac{1}{2\beta x}\right)^{11}$ and x^{-7} in $\left(x + \frac{1}{3\beta x^2}\right)^{11}$ are equal then

(1) $\alpha^6\beta = \frac{2^5}{3^6}$ (2) $\alpha^6\beta = \frac{2^6}{3^5}$

(3) $\alpha\beta^6 = \frac{2^5}{3^6}$ (4) $\alpha\beta^6 = \frac{2^6}{3^5}$

Answer (1)

Sol. Coefficient of x^7 in $\left(\alpha x^2 + \frac{1}{2\beta x}\right)^{11}$

$$T_{r+1} = {}^{11}C_r \left(\alpha x^2\right)^{11-r} \left(\frac{1}{2\beta x}\right)^r$$

Now, $22 - 2r - r = 7$

$r = 5$

Coeff. = ${}^{11}C_5 \frac{\alpha^6}{(2\beta)^5}$

Coeff. of x^{-7} in $\left(x + \frac{1}{3\beta x^2}\right)^{11}$ will be, if $r = 6$ is $\frac{{}^{11}C_6}{3^6\beta^6}$

$${}^{11}C_5 \frac{\alpha^6}{2^5\beta^5} = \frac{{}^{11}C_5}{3^6\beta^6}$$

$$\alpha^6\beta = \frac{2^5}{3^6}$$

5. If $(21)^{18} + 20\cdot(21)^{17} + (20)^2 \cdot (21)^{16} + \ldots\ldots\ldots. (20)^{18} = k\,(21^{19} - 20^{19})$ then k =

(1) $\frac{21}{20}$ (2) 1

(3) $\frac{21}{20}$ (4) $\frac{20}{21}$

Answer (2)

Sol. $a = (21)^{18}, r = \frac{20}{21}, n = 19$

$$S = (21)^{18} \frac{\left(1 - \left(\frac{20}{21}\right)^{19}\right)}{1 - \frac{20}{21}}$$

$$\Rightarrow \frac{(21)^{19}}{(21)^{19}}\left((21)^{19} - (20)^{19}\right)$$

$= (21)^{19} - (20)^{19}$

6. If $1^2 - 2^2 + 3^2 - 4^2 + \ldots. - (2022)^2 + (2023)^2 = m^2 n$, where $m, n \in N$ and $m > 19$ then $n^2 - m$ is

(1) 615

(2) 562

(3) 812

(4) 264

Answer (1)

Sol. $1^2 - 2^2 + 3^2 - 4^2 + \ldots..(2021)^2 - (2022)^2 + (2023)^2$

$$= \underbrace{-3 - 7 - 11 \ldots\ldots..}_{1011 \text{ terms}} + (2023)^2$$

$$= -\frac{1011}{2}(6 + (1010)4) + (2023)^2$$

$$= -1011(3 + 2020) + (2023)^2$$

$$= (2023)(-1011) + (2023)^2$$

$$= (2023)(2023 - 1011)$$

$$= 2023(1012)$$

$$= (17)^2 \cdot 7\left(2^2 \cdot 253\right) = (34)^2(1771)$$

$= m = 34,\ n = 1771$

$\therefore$ n – m^2

= 1771 – 1156

= 615

7. If $a \neq b$ and are purely real, $z \in$ complex number, $\text{Re}(az^2 + bz) = a$ and $\text{Re}(bz^2 + az) = b$ then number of value of z possible is

(1) 0

(2) 1

(3) 2

(4) 3

Answer (1)

Sol. $a(x^2 - y^2) + bx = a$ ...(i)

$b(x^2 - y^2) + ax = b$ ...(ii)

(i) – (ii)

$(a - b)(x^2 - y^2) + (b - a)x = a - b$ $(a \neq b)$

$\Rightarrow x^2 - y^2 - x = 1$

(i) + (ii)

$(a + b)(x^2 - y^2) + x(a + b) = a + b$ $(a \neq -b)$

$\Rightarrow x^2 - y^2 + x = 1$

$\Rightarrow x = 0$

$\Rightarrow y^2 = -1$ (not possible $\because y \in R$)

$\therefore$ No complex number possible.

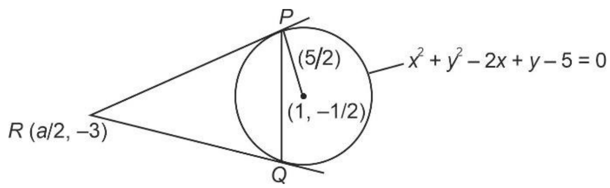
8. Two tangents are drawn from point $R\left(\frac{a}{2}, -3\right)$ intersect the circle $x^2 + y^2 - 2x + y = 5$ at P and Q then the area of ΔPQR is

(1) $\frac{1710}{290}$ (2) $\frac{1715}{296}$

(3) $\frac{296}{1715}$ (4) $\frac{290}{1710}$

Answer (2)

Sol.



$$\text{Area} = \frac{RL^3}{R^2 + L^2}\,r$$

$$L = \frac{7}{2}$$

$$\text{Area} = \frac{\frac{5}{2}\times\left(\frac{7}{2}\right)^3}{\left(\frac{5}{2}\right)^2 + \left(\frac{7}{2}\right)^2}$$

$$= \frac{\frac{1715}{16}}{\frac{25+49}{4}} = \frac{1715\times 4}{16\times 74} = \frac{1715}{296}$$

9. Equation of plane passing through intersection of $P_1 : \vec{r}\,(\hat{i} + \hat{j} + \hat{k}) = 6$ and $P_2 : \vec{r}\cdot(2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$ and (0, 2, –2) is P. Then square of distance of (12, 12, 18) from P is

(1) 310 (2) 1240

(3) 155 (4) 620

Answer (4)

Sol. $P_1 + \lambda P_2 = 0$

$(x + y + z - 6) + \lambda\,(2x + 3y + 4z + 5) = 0$

$(1 + 2\lambda)\,x + (1 + 3\lambda)y + (1 + 4\lambda)z + 5\lambda - 6 = 0$

Passing through (0, 2, –2)

$\Rightarrow\ \lambda = 2$

$\therefore$ Plane : $5x + 7y + 9z = -4$

$$\text{Distance} = \left|\frac{5(12) + 7(12) + 9(18) + 4}{\sqrt{5^2 + 7^2 + 9^2}}\right|^2$$

$= 620$

10. If V is volume of parallelopiped whose three coterminous edges are $\vec{a}, \vec{b}, \vec{c}$, then volume of a parallelopiped whose coterminous edges are $\vec{a}, \vec{b} + \vec{c}, \vec{a} + 2\vec{b} + 3\vec{c}$ is

(1) $6V$ (2) V

(3) $2V$ (4) $3V$

Answer (2)

Sol. $\left[\vec{a}\quad \vec{b}+\vec{c}\quad \vec{a}+2\vec{b}+3\vec{c}\right]$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]$$

$$= \left[\vec{a}\quad \vec{b}\quad \vec{c}\right]$$

$= V$

11. $S_1 : (p \Rightarrow q) \vee (\sim p \wedge q)$ is a tautology

$S_2 : (q \Rightarrow p) \Rightarrow (\sim p \wedge q)$ is a contradiction

(1) Both S_1 and S_2 are true

(2) Neither S_1 Nor S_2 are true

(3) Only S_1 are true

(4) Only S_2 are true

Answer (2)

Sol. $(p \Rightarrow q) \vee (\sim p \wedge q)$

$\equiv (\sim p \vee q) \vee \sim p \wedge q$

$\equiv \sim p \vee q \vee \sim pq$

$\equiv \sim p \vee q$

Which is not a tautology

$(q \Rightarrow p) \Rightarrow (\sim p \wedge q)$

$\equiv (\sim q \vee p) \Rightarrow (\sim p \wedge q)$

$\equiv \sim(\sim q \vee p) \vee (\sim p \wedge q)$

$\equiv (q \wedge (\sim p) \vee (\sim p \wedge q)$

$\equiv \sim p \wedge q$

Which is not a contradiction.

12. $(1 + \ln x)\frac{dx}{dy} + x\ln x = e^y$

Solution of this differential equation satisfies (1, 90) and (α, 92) then α^α is

(1) $\frac{e^{90}}{90}$ (2) $\frac{e^{92} - e^{90}}{45}$

(3) $e^{\left(\frac{e^{92} - e^{90}}{90}\right)}$ (4) $e^{92} - e^{90}$

Answer (3)

Sol. $(1+\ln x)dx + x\ln x\,dy = e^y dy$

$d(y.x\ln x) = d(e^y)$

$\Rightarrow \boxed{xy\ln x = e^y + C}$

Through (1, 90) $\Rightarrow C = -e^{90}$

$xy\ln x = e^y - e^{90}$

$\therefore \quad \alpha.92\ln\alpha = e^{92} - e^{90}$

$\ln\alpha^\alpha = \dfrac{e^{92}-e^{90}}{92}$

$\alpha^\alpha = e^{\left(\frac{e^{92}-e^{90}}{92}\right)}$

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. The rank of the word "PUBLIC" is

Answer (198)

Sol. $\therefore$

5	6	1	4	3	2
P	U	B	L	I	C
4	4	0	2	1	0
4!	4!	3!	2!	1!	0!

$\therefore$ Rank = (1 × 1! + 2 × 2! + 4 × 4! + 4 × 4!) + 1

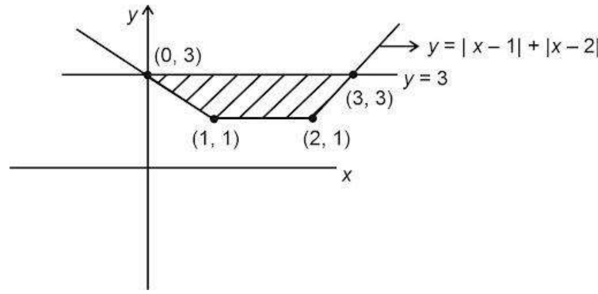
= (1 + 4 + 96 + 96) + 1

= 198

22. The area enclosed by $y = |x-1| + |x-2|$ and $y = 3$ is

Answer (04)

Sol.



$= \dfrac{1}{2}[1+3]\times 2$

= 4

23. If the number of all 4 letter words with 2 vowels and 2 consonants from the word UNIVERSE is n, then $n - 500$ is

Answer (4)

Sol. Vowels $\rightarrow$ I, U, E, E

Consonants $\rightarrow$ N, V, R, S

(I) 2 Vowels some (II) 2 Vowels different

$4C_2 \times \dfrac{4!}{2!} = 72$ $\qquad 3C_2 \times 4C_2 \times 4! = 432$

72 + 432 = 504

24. Three dice are thrown. Then the probability that no outcomes is similar is $\dfrac{p}{q}$ then $q - p$ is (where p and q are co-prime)

Answer (04)

Sol. $P(E) = \dfrac{6\times5\times4}{6\times6\times6}$

$= \dfrac{20}{36} = \dfrac{5}{9} = \dfrac{p}{q}$

$q - p = 9 - 5 = 4$

25. $P^2 = I - P$

$P^\alpha + P^\beta = \gamma I - 2qp$

$P^\alpha - P^\beta = \delta I - 13P$

Then find the value of $\alpha + \beta + \gamma - \delta$

Answer (24)

Sol. $P^3 = P - P^2$

$= P - (I - P) = 2P - I$

$P^4 = 2P^2 - P$

$= 2(I - P) - P = 2I - 3P$

Similarly

$P^6 = 5I - 8P$

and $P^8 = 13I - 21P$

$P^8 + P^6 = 18I - 29P$

$P^8 - P^6 = 8I - 13P$

$\alpha = 8, \beta = 6, \gamma = 18, \delta = 8$

$\alpha + \beta + \gamma - \delta = 8 + 6 + 18 - 8 = 24$

26.

27.

28.

29.

30.

❑ ❑ ❑