

JEE-Mains-06-04-2023 [Memory Based]

[Morning Shift]

Mathematics

Question: $\int \frac{x^2(x \sec^2 x + \tan x)}{(x \tan x + 1)^2} dx$ is equal to

Answer: $\frac{-x^2}{x \tan x + 1} + 2 \ln|x \sin x + \cos x| + c$

Solution:

$$\int \frac{x^2(x \sec^2 x + \tan x)}{(x \tan x + 1)^2} dx$$

Integrating by parts

$$\begin{aligned} I &= x^2 \cdot \frac{-1}{x \tan x + 1} - \int 2x \left(\frac{-1}{x \tan x + 1} \right) dx \\ &= \frac{-x^2}{x \tan x + 1} + 2 \int \frac{x \cos x}{x \sin x + \cos x} dx \\ &= \frac{-x^2}{x \tan x + 1} + 2 \ln|x \sin x + \cos x| + c \end{aligned}$$

Question: The coefficient of x^{18} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is

Options:

- (a) ${}^{14}C_7$
- (b) ${}^{15}C_8$
- (c) ${}^{15}C_6$
- (d) ${}^{14}C_8$

Answer: (c)

Solution:

$$\left(x^4 - \frac{1}{x^3}\right)^{15}$$

$$T_{r+1} = {}^{15}C_r \left(x^4\right)^{15-r} \left(-\frac{1}{x^3}\right)^r = {}^{15}C_r (-1)^r x^{60-7r}$$

$$60 - 7r = 18$$

$$\Rightarrow 7r = 42$$

$$\Rightarrow r = 6$$

$$\text{Coefficient of } x^{18} = {}^{15}C_6$$

Question: The number of ways of distributing 70 distinct oranges among three children such that each child gets atleast one orange is

$$\text{Answer: } 3^{70} - 3(2^{70} - 2) - 3$$

Solution:

$$\begin{aligned}\text{Number of ways} &= 3^{70} - {}^3C_1 \cdot 2^{70} + {}^3C_2 \cdot 1^{70} \\ &= 3^{70} - 3(2^{70} - 2) - 3\end{aligned}$$

Question: Sum of first 20 terms of the series 5, 11, 19, 29, 41,..... is

Answer: 3250.00

Solution:

$$\text{Let } S_n = 5 + 11 + 19 + 29 + 41 + \dots t_n$$

$$t_n = an^2 + bn + c$$

$$a + b + c = 5$$

$$4a + 2b + c = 11$$

$$\underline{9a + 3b + c = 19}$$

$$5a + b = 8$$

$$\underline{3a + b = 6}$$

$$2a = 2$$

$$a = 1$$

$$b = 3$$

$$c = 1$$

$$t_n = n^2 + 3n + 1$$

$$S_n = \sum t_n$$

$$S_n = \frac{n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + n$$

$$S_{20} = \frac{20 \cdot 21 \cdot 41}{6} + \frac{2 \cdot 20 \cdot 21}{2} + 20$$

$$= 2870 + 630 + 20$$

$$= 3520$$

Question: Number of ways in which 20 chocolates can be given to 3 children such that each gets atleast one is ____.

Answer: ${}^{19}C_2$

Solution:

$$x + y + z = 20$$

$$x, y, z \geq 1$$

$$X + Y + Z = 17$$

$$X, Y, Z \geq 0$$

$${}^{n+r-1}C_{r-1} \text{ i.e. } {}^{19}C_2$$

Question: If 5 pairs of dice are thrown. Success is getting a sum 5. If the probability of getting atleast 4 success is $\frac{K}{3^{11}}$, then the value of K is

Answer: 123.00

Solution:

$$(1,4), (2,3), (3,2), (4,1)$$

$$p = \frac{4}{6} = \frac{1}{9}, q = \frac{8}{9}$$

$$\frac{K}{3^{11}} = P(\text{atleast 4 success}) = {}^5C_4 p^4 q^1 + {}^5C_5 p^5$$

$$\frac{K}{3^{11}} = 5 \cdot \frac{1}{9^4} \cdot \frac{8}{9} + \frac{1}{9^5}$$

$$\Rightarrow \frac{K}{3^{11}} = \frac{40}{3^{10}} + \frac{1}{3^{10}}$$

$$\Rightarrow \frac{K}{3} = 41$$

$$\Rightarrow K = 123$$

Question: If the ratio of the 5th term from the start to the 5th term from the end in the expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$. Find the 3rd term from start.

Answer: $60\sqrt{3}$

Solution:

$$\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$$

$$\frac{{}^n C_4 \left(\sqrt[4]{2}\right)^{n-4} \left(\frac{1}{\sqrt[4]{3}}\right)^4}{{}^n C_4 \left(\sqrt[4]{2}\right)^4 \left(\frac{1}{\sqrt[4]{3}}\right)^{n-4}} = \frac{\sqrt{6}}{1}$$

$$\frac{\left(\sqrt[4]{2}\right)^{n-8}}{\left(\frac{1}{\sqrt[4]{3}}\right)^{n-8}} = \sqrt{6}$$

$$\left(\sqrt[4]{6}\right)^{n-8} = \sqrt{6}$$

$$\left(\sqrt{6}\right)^{\frac{n-8}{2}} = \sqrt{6}$$

$$n=10$$

$${}^{10} C_2 \left(2^{\frac{1}{4}}\right)^8 \times \left(\frac{1}{3^{\frac{1}{4}}}\right)^2$$

$$= 45 \times 4 \times \frac{1}{\sqrt{3}}$$

$$= 60\sqrt{3}$$

Question: Mean of 15 observations is 12 and its variance is 14. Mean of another 15 observations is 14 and variance σ^2 . Combined variance is 13. Find σ^2 .

Answer: 10.00

Solution:

Subtract each entry by 13

$$\text{So } 14 = \frac{\sum x_i^2}{15} - (-1)^2 \Rightarrow \sum x_i^2 = 2.25$$

$$\& \sigma^2 = \frac{\sum y_i^2}{15} - 1^2 \Rightarrow \sum y_i^2 = 15(\sigma^2) + 1$$

$$13 = \frac{225 + 15\sigma^2 + 15}{30}$$

$$\Rightarrow 8 + \frac{\sigma^2}{2} = 0$$

$$\sigma^2 = 10$$

Question: A 2×2 matrix A is such that none of its elements is 0, and $A^2 = I$. ‘ a ’ is the sum of diagonal elements and ‘ b ’ is $|A|$. Find $3a^2 + 4b^2$.

Answer: 4.00

Solution:

$$A^2 = \begin{bmatrix} p & q \\ r & s \end{bmatrix}^2 = \begin{bmatrix} p^2 + qr & pq + qs \\ pr + rs & qr + s^2 \end{bmatrix}$$

$$(p+s)q = r(p+s) = 0$$

$$p+s=0 \quad p=-s$$

$$p^2 + qr = 1$$

$$\text{So, } a=0, \quad b=ps-qr$$

$$= -(p^2 + qr) = -1$$

$$\text{So } 3a^2 + 4b^2 = 4$$

Question: If $2x^y + 3y^x = 20$. Find $\frac{dy}{dx}$ at $(2, 2)$.

Answer:

Solution:

$$2x^y + 3y^x = 20$$

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$$

$$= -\frac{2y x^{y-1} + 3y^x \ln y}{2x^y \cdot \ln x + 3xyx - 1}$$

$$= -\frac{2 + 3 \ln 2}{1 \ln 2 + 3}$$

$$= -\frac{2 + \ln 8}{\ln 4 + 3}$$

Question: If $a_1, a_2, a_3, \dots, a_n$ are in A.P.

$$\lim_{n \rightarrow \infty} \frac{\sqrt{d}}{\sqrt{n}} \left(\frac{1}{\sqrt{a_2} + \sqrt{a_1}} + \frac{1}{\sqrt{a_3} + \sqrt{a_2}} + \dots \right)$$

Answer: 1.00

Solution:

a_1, a_2, a_3 are in A.P.

$$\lim_{n \rightarrow \infty} \frac{\sqrt{d}}{\sqrt{n}} \left(\frac{1}{\sqrt{a_2} + \sqrt{a_1}} + \frac{1}{\sqrt{a_3} + \sqrt{a_2}} \dots \right)$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt{d}}{\sqrt{n}} \left[\frac{\sqrt{a_n} - \sqrt{a_1}}{d} \right]$$

$$= \lim_{n \rightarrow \infty} \left[\frac{\sqrt{a_1 + (n-1)d}}{\sqrt{n} \cdot \sqrt{d}} - \frac{\sqrt{d} \sqrt{a_1}}{\sqrt{nd}} \right]$$

$$= 1$$

Question: $f = [9 + 13 \sin x]$ when $x \in [0, \pi]$. Find the number of points where f is not differentiable.

Answer: 25.00

Solution:

$$[a + b \sin x]; x \in [0, \pi]$$

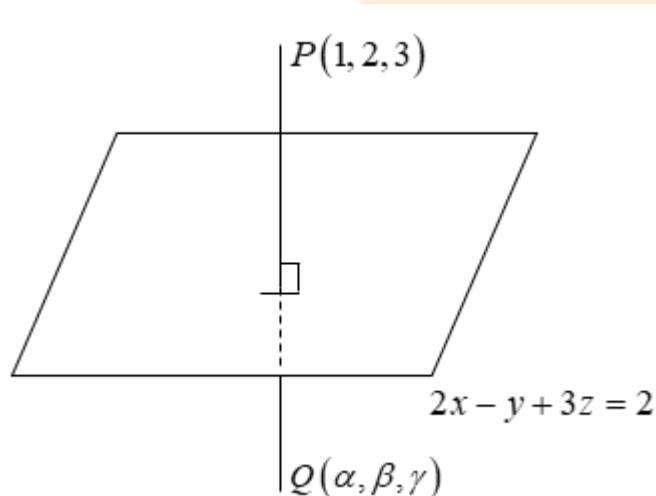
Non-differentiable at $2b - 1$ points

$$2b - 1 = 2 \times 13 - 1 = 25$$

Question: Image of point $P(1, 2, 3)$ about the plane $2x - y + 3z = 2$ is Q , then the area of $\Delta PQR = ?$ where $R = (4, 10, 12)$

Answer: $\frac{\sqrt{1531}}{2}$

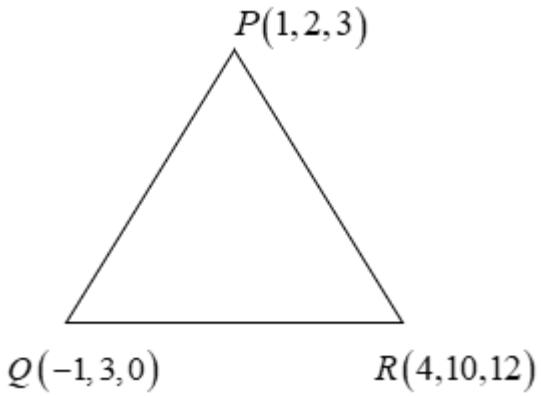
Solution:



$$\frac{\alpha-1}{2} = \frac{\beta-2}{-1} = \frac{\alpha-3}{3} = \frac{-2(7)}{14}$$

$$\frac{\alpha-1}{2} = \frac{\beta-2}{-1} = \frac{\alpha-3}{3} = -1$$

$$Q(-1, 3, 0)$$



$$\text{Area} = \frac{1}{2} \times |\overrightarrow{PQ} \times \overrightarrow{PR}|$$

$$= \frac{\sqrt{1531}}{2}$$

Question: If $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3$, then $18 \int_1^2 f(x) dx$ is:

Answer: $10 \log_e 2 - 6$

Solution:

$$5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3$$

$$5f(x) + 4f\left(\frac{1}{x}\right) \quad \dots(1)$$

$$\text{Take } x = \frac{1}{x}$$

$$5f\left(\frac{1}{x}\right) + 4f(x) = x + 3 \quad \dots(2)$$

$$(1) \times 5 - 4 \times (2)$$

$$9f(x) = \frac{5}{x} + 15 - 4x - 12$$

$$9f(x) = \frac{5}{x} - 4x + 3$$

By integrating

$$9 \int_1^2 f(x) dx = \int_1^2 \frac{5}{x} - 4x + 3 dx$$

$$2 \times 9 \int_1^2 f(x) dx = \int_1^2 \frac{10}{x} - 8x + 6 dx$$

$$= 10 \ln|x| - \frac{8x^2}{2} + 6x \Big|_1^2$$

$$= (10 \ln 2 - 16 + 12) - (0 - 4 + 6)$$

$$= 10 \log_e 2 - 6$$

Question: The sum of roots of $|x^2 - 8x + 15| - 2x + 7 = 0$ is

Answer: $9 + \sqrt{3}$

Solution:

$$|x^2 - 8x + 15| - 2x + 7 = 0$$

$$|(x-3)(x-5)| - 2x + 7 = 0$$

$$x \leq 3 \text{ or } x \geq 5$$

$$x^2 - 8x + 15 - 2x + 7 = 0$$

$$x^2 - 10x + 22 = 0$$

$$x = \frac{10 \pm \sqrt{100 - 88}}{2}$$

$$= 5 \pm \sqrt{3}$$

Take intersection

$$x = 5 + \sqrt{3}$$

$$3 < x < 5$$

$$-x^2 + 8x - 15 - 2x + 7 = 0$$

$$x^2 - 6x + 8 = 0$$

$$(x-2)(x-4) = 0$$

$$x = 2, 4$$

$$x = 4$$

So, sum of roots is $9 + \sqrt{3}$

Question: $(P \Rightarrow Q) \vee (R \Rightarrow Q)$ is equivalent to:

Options:

- (a) $(P \wedge R) \Rightarrow Q$
- (b) $(P \vee R) \Rightarrow Q$
- (c) $(Q \Rightarrow R) \vee (P \Rightarrow R)$
- (d) $(R \Rightarrow P) \vee (Q \Rightarrow R)$

Answer: (a)

Solution:

$$(P \Rightarrow Q) \vee (R \Rightarrow Q)$$

$$\equiv (\sim P \vee Q) \vee (\sim R \vee Q)$$

$$\equiv (\sim P \vee \sim R) \vee Q$$

$$\equiv \sim (P \wedge R) \vee Q$$

$$\equiv (P \wedge R) \Rightarrow Q$$

Question: Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}$, $\vec{c} = -\hat{i} + 4\hat{j} + 3\hat{k}$ and \vec{d} is a vector perpendicular to both \vec{b} and \vec{c} and $\vec{a} \cdot \vec{d} = 18$, then $|\vec{a} \times \vec{d}|^2$ is

Answer: 720.00

Solution:

$$\vec{d} = \lambda(\vec{b} \times \vec{c})$$

$$= \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -2 \\ -1 & 4 & 3 \end{vmatrix}$$

$$= \lambda(2\hat{i} - \hat{j} + 2\hat{k})$$

$$\vec{a} \cdot \vec{d} = 18$$

$$\Rightarrow \lambda(4-3+8)=18$$

$$\Rightarrow \lambda = 2$$

$$|\vec{a} \times \vec{d}|^2 = a^2 d^2 - (\vec{a} \cdot \vec{d})^2$$

$$= 29 \times 36 - 18^2$$

$$= 18(58 - 18)$$

$$= 18 \times 40$$

$$= 720$$

Question: If ${}^{2n}C_3 : {}^nC_3 = 10$, then $\frac{n^2 + 3n}{n^2 - 3n + 4}$ is equal to

Answer: 2.00

Solution:

$${}^{2n}C_3 : {}^nC_3 = 10$$

$$\frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)} = 10$$

$$\Rightarrow \frac{2(2n-1) \cdot 2(n-1)}{(n-1)(n-2)} = 10$$

$$4n-2 = 5n-10$$

$$\Rightarrow n = 8$$

$$\frac{n^2 + 3n}{n^2 - 3n + 4} = \frac{64 + 24}{64 - 24 + 4} = \frac{88}{44} = 2$$