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OF EXCELLENCE



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JEE MAIN (APRIL) 2023 (06-04-2023-AN)

Memory Based Question Paper
MATHEMATICS



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MATHEMATICS
SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. If $f(x) + f(\pi - x) = \pi^2$ then $\int_0^{\pi} f(x) \sin x dx$

- (1) π^2 (2) $\frac{\pi^2}{4}$
 (3) $2\pi^2$ (4) $\frac{\pi^2}{8}$

Answer (1)

Sol. $I = \int_0^{\pi} f(x) \sin x dx$

$I = \int_0^{\pi} f(\pi - x) \sin x dx$

$2I = \int_0^{\pi} (\sin x)(f(x) + f(\pi - x)) dx$

$2I = \pi^2 \int_0^{\pi} \sin x dx$

$2I = 2\pi^2 \int_0^{\frac{\pi}{2}} \sin x dx$

$I = \pi^2 (-\cos x)_0^{\frac{\pi}{2}}$
 $= \pi^2$

2. The system of the equations

$x + y + z = 6$

$x + 2y + \alpha z = 5$ has

$x + 2y + 6z = \beta$

- (1) Infinitely many solution for $\alpha = 6, \beta = 3$
 (2) Infinitely many solution for $\alpha = 6, \beta = 5$
 (3) Unique solution for $\alpha = 6, \beta = 5$
 (4) No solution for $\alpha = 6, \beta = 5$

Answer (2)

Sol. Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \alpha \\ 1 & 2 & 6 \end{vmatrix} = 6 - \alpha$

\therefore for $\alpha \neq 6$ system has unique solution

Now, when $\alpha = 6$

$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 5 & 2 & 6 \\ \beta & 2 & 6 \end{vmatrix} = 0 - (30 - 6\beta) + (10 - 2\beta)$
 $= 4(\beta - 5)$

$\Delta_2 = \begin{vmatrix} 1 & 6 & 2 \\ 1 & 5 & 6 \\ 1 & \beta & 6 \end{vmatrix} = -4(\beta - 5)$

$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 5 \\ 1 & 2 & \beta \end{vmatrix} = \begin{vmatrix} 1 & 1 & 6 \\ 0 & 1 & -1 \\ 0 & 1 & \beta - 6 \end{vmatrix} = \beta - 5$

Clearly at $\beta = 5, \Delta_i = 0$ for $i = 1, 2, 3$

\therefore at $\alpha = 6, \beta = 5$ system has infinite solutions.

3. Let statement 1 : $(2002)^{2023} - (1919)^{2002}$ is divisible by 8.

Statement 2 : $13 \cdot 13^n - 12n - 13$ is divisible by 144
 $\forall n \in \mathbb{N}$, then

- (1) Statement-1 and statement-2 both are true
 (2) Only statement-1 is true
 (3) Only statement-2 is true
 (4) Neither statement-1 nor statement-2 are true

Answer (3)

Sol. $\therefore (2002)^{2023} = 8m$

$\therefore (2002)^{2023}$ is divisible by 8
 and $(1919)^{2002}$ is not divisible by 8

$\therefore (2002)^{2023} - (1919)^{2002}$ is not divisible by 8.

Now

$13 \cdot (13)^n - 12n - 13$
 $= 13(1 + 12)^n - 12n - 13$
 $= 13[1 + 12n + {}^{n}C_2 12^2 + \dots] - 12n - 13$
 $= 144n + 144n {}^{n}C_2 + \dots$
 $= 144[n + n {}^{n}C_2 + \dots]$
 $= 144K$

\therefore Statement-2 is correct

4. If the coefficient of x^7 in $\left(\alpha x^2 + \frac{1}{2\beta x}\right)^{11}$ and x^{-7} in

$\left(x + \frac{1}{3\beta x^2}\right)^{11}$ are equal then

(1) $\alpha^6\beta = \frac{2^5}{3^6}$ (2) $\alpha^6\beta = \frac{2^6}{3^5}$

(3) $\alpha\beta^6 = \frac{2^5}{3^6}$ (4) $\alpha\beta^6 = \frac{2^6}{3^5}$

Answer (1)

Sol. Coefficient of x^7 in $\left(\alpha x^2 + \frac{1}{2\beta x}\right)^{11}$

$$T_{r+1} = {}^{11}C_r (\alpha x^2)^{11-r} \left(\frac{1}{2\beta x}\right)^r$$

Now, $22 - 2r - r = 7$

$r = 5$

Coeff. = ${}^{11}C_5 \frac{\alpha^6}{(2\beta)^5}$

Coeff. of x^{-7} in $\left(x + \frac{1}{3\beta x^2}\right)^{11}$ will be, if $r = 6$ is $\frac{{}^{11}C_6}{3^6\beta^6}$

$${}^{11}C_5 \frac{\alpha^6}{2^5\beta^5} = \frac{{}^{11}C_6}{3^6\beta^6}$$

$$\alpha^6\beta = \frac{2^5}{3^6}$$

5. If $(21)^{18} + 20 \cdot (21)^{17} + (20)^2 \cdot (21)^{16} + \dots + (20)^{18} = k(21^{19} - 20^{19})$ then $k =$

(1) $\frac{21}{20}$ (2) 1

(3) $\frac{21}{20}$ (4) $\frac{20}{21}$

Answer (2)

Sol. $a = (21)^{18}, r = \frac{20}{21}, n = 19$

$$S = (21)^{18} \frac{\left(1 - \left(\frac{20}{21}\right)^{19}\right)}{1 - \frac{20}{21}}$$

$$\Rightarrow \frac{(21)^{19}}{(21)^{19}} \left((21)^{19} - (20)^{19}\right) = (21)^{19} - (20)^{19}$$

6. If $1^2 - 2^2 + 3^2 - 4^2 + \dots - (2022)^2 + (2023)^2 = m^2n$, where $m, n \in N$ and $m > 19$ then $n^2 - m$ is

- (1) 615
- (2) 562
- (3) 812
- (4) 264

Answer (1)

Sol. $1^2 - 2^2 + 3^2 - 4^2 + \dots - (2021)^2 - (2022)^2 + (2023)^2$

$$= \underbrace{-3 - 7 - 11 \dots}_{1011 \text{ terms}} + (2023)^2$$

$$= -\frac{1011}{2}(6 + (1010)4) + (2023)^2$$

$$= -1011(3 + 2020) + (2023)^2$$

$$= (2023)(-1011) + (2023)^2$$

$$= (2023)(2023 - 1011)$$

$$= 2023(1012)$$

$$= (17)^2 \cdot 7(2^2 \cdot 253) = (34)^2(1771)$$

$$= m = 34, n = 1771$$

$$\therefore n - m^2$$

$$= 1771 - 1156$$

$$= 615$$

7. If $a \neq b$ and are purely real, $z \in$ complex number, $\text{Re}(az^2 + bz) = a$ and $\text{Re}(bz^2 + az) = b$ then number of value of z possible is

- (1) 0
- (2) 1
- (3) 2
- (4) 3

Answer (1)

Sol. $a(x^2 - y^2) + bx = a$... (i)

$b(x^2 - y^2) + ax = b$... (ii)

(i) - (ii)

$$(a - b)(x^2 - y^2) + (b - a)x = a - b \quad (a \neq b)$$

$$\Rightarrow x^2 - y^2 - x = 1$$

(i) + (ii)

$$(a + b)(x^2 - y^2) + x(a + b) = a + b \quad (a \neq -b)$$

$$\Rightarrow x^2 - y^2 + x = 1$$

$$\Rightarrow x = 0$$

$$\Rightarrow y^2 = -1 \quad (\text{not possible } \because y \in R)$$

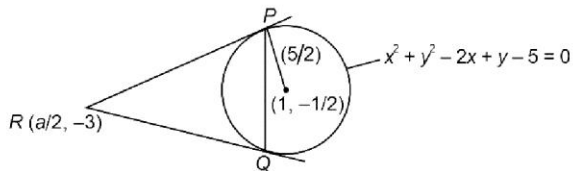
\therefore No complex number possible.

8. Two tangents are drawn from point $R\left(\frac{a}{2}, -3\right)$ intersect the circle $x^2 + y^2 - 2x + y = 5$ at P and Q then the area of ΔPQR is

- (1) $\frac{1710}{290}$ (2) $\frac{1715}{296}$
(3) $\frac{296}{1715}$ (4) $\frac{290}{1710}$

Answer (2)

Sol.



$$\text{Area} = \frac{RL^3}{R^2 + L^2} r$$

$$L = \frac{7}{2}$$

$$\text{Area} = \frac{\frac{5}{2} \times \left(\frac{7}{2}\right)^3}{\left(\frac{5}{2}\right)^2 + \left(\frac{7}{2}\right)^2}$$

$$= \frac{1715}{16} = \frac{1715 \times 4}{16 \times 4} = \frac{1715}{296}$$

9. Equation of plane passing through intersection of $P_1 : \vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$ and $P_2 : \vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$ and $(0, 2, -2)$ is P . Then square of distance of $(12, 12, 18)$ from P is

- (1) 310 (2) 1240
(3) 155 (4) 620

Answer (4)

Sol. $P_1 + \lambda P_2 = 0$

$$(x + y + z - 6) + \lambda (2x + 3y + 4z + 5) = 0$$

$$(1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z + 5\lambda - 6 = 0$$

Passing through $(0, 2, -2)$

$$\Rightarrow \lambda = 2$$

$$\therefore \text{Plane} : 5x + 7y + 9z = -4$$

$$\text{Distance} = \frac{|5(12) + 7(12) + 9(18) + 4|}{\sqrt{5^2 + 7^2 + 9^2}}$$

$$= 620$$

10. If V is volume of parallelopiped whose three coterminous edges are $\vec{a}, \vec{b}, \vec{c}$, then volume of a parallelopiped whose coterminous edges are $\vec{a}, \vec{b} + \vec{c}, \vec{a} + 2\vec{b} + 3\vec{c}$ is

- (1) $6V$ (2) V
(3) $2V$ (4) $3V$

Answer (2)

Sol. $[\vec{a} \ \vec{b} + \vec{c} \ \vec{a} + 2\vec{b} + 3\vec{c}]$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} [\vec{a} \ \vec{b} \ \vec{c}]$$

$$= [\vec{a} \ \vec{b} \ \vec{c}]$$

$$= V$$

11. $S_1 : (p \Rightarrow q) \vee (\sim p \wedge q)$ is a tautology

$S_2 : (q \Rightarrow p) \Rightarrow (\sim p \wedge q)$ is a contradiction

- (1) Both S_1 and S_2 are true
(2) Neither S_1 nor S_2 are true
(3) Only S_1 are true
(4) Only S_2 are true

Answer (2)

Sol. $(p \Rightarrow q) \vee (\sim p \wedge q)$

$$\equiv (\sim p \vee q) \vee \sim p \wedge q$$

$$\equiv \sim p \vee q \vee \sim p q$$

$$\equiv \sim p \vee q$$

Which is not a tautology

$$(q \Rightarrow p) \Rightarrow (\sim p \wedge q)$$

$$\equiv (\sim q \vee p) \Rightarrow (\sim p \wedge q)$$

$$\equiv \sim(\sim q \vee p) \vee (\sim p \wedge q)$$

$$\equiv (q \wedge \sim p) \vee (\sim p \wedge q)$$

$$\equiv \sim p \wedge q$$

Which is not a contradiction.

12. $(1 + \ln x) \frac{dx}{dy} + x \ln x = e^y$

Solution of this differential equation satisfies $(1, 90)$ and $(\alpha, 92)$ then α^α is

(1) $\frac{e^{90}}{90}$ (2) $\frac{e^{92} - e^{90}}{45}$

(3) $e^{\left(\frac{e^{92} - e^{90}}{90}\right)}$ (4) $e^{92} - e^{90}$

Answer (3)

Sol. $(1 + \ln x)dx + x \ln x dy = e^y dy$

$$d(y \cdot x \ln x) = d(e^y)$$

$$\Rightarrow \boxed{xy \ln x = e^y + C}$$

Through $(1, 90) \Rightarrow C = -e^{90}$

$$xy \ln x = e^y - e^{90}$$

$$\therefore \alpha \cdot 92 \ln \alpha = e^{92} - e^{90}$$

$$\ln \alpha^\alpha = \frac{e^{92} - e^{90}}{92}$$

$$\alpha^\alpha = e^{\left(\frac{e^{92} - e^{90}}{92}\right)}$$

- 13.
- 14.
- 15.
- 16.
- 17.
- 18.
- 19.
- 20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. The rank of the word "PUBLIC" is

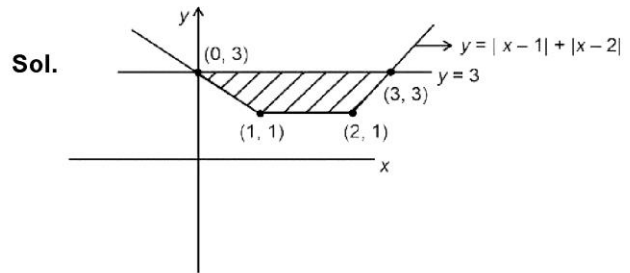
Answer (198)

	5	6	1	4	3	2
	P	U	B	L	I	C
Sol. \therefore	4	4	0	2	1	0
	4!	4!	3!	2!	1!	0!

$$\begin{aligned} \therefore \text{Rank} &= (1 \times 1! + 2 \times 2! + 4 \times 4! + 4 \times 4!) + 1 \\ &= (1 + 4 + 96 + 96) + 1 \\ &= 198 \end{aligned}$$

22. The area enclosed by $y = |x - 1| + |x - 2|$ and $y = 3$ is

Answer (04)



$$\begin{aligned} &= \frac{1}{2} [1 + 3] \times 2 \\ &= 4 \end{aligned}$$

23. If the number of all 4 letter words with 2 vowels and 2 consonants from the word UNIVERSE is n , then $n - 500$ is

Answer (4)

Sol. Vowels \rightarrow I, U, E, E

Consonants \rightarrow N, V, R, S

(I) 2 Vowels same (II) 2 Vowels different

$$\begin{aligned} 4C_2 \times \frac{4!}{2!} &= 72 & 3C_2 \times 4C_2 \times 4! &= 432 \\ 72 + 432 &= 504 \end{aligned}$$

24. Three dice are thrown. Then the probability that no outcomes is similar is $\frac{p}{q}$ then $q - p$ is (where p and q are co-prime)

Answer (04)

Sol. $P(E) = \frac{6 \times 5 \times 4}{6 \times 6 \times 6}$

$$= \frac{20}{36} = \frac{5}{9} = \frac{p}{q}$$

$$q - p = 9 - 5 = 4$$

25. $P^2 = I - P$

$$P^\alpha + P^\beta = \gamma I - 2qP$$

$$P^\alpha - P^\beta = \delta I - 13P$$

Then find the value of $\alpha + \beta + \gamma - \delta$

Answer (24)

Sol. $P^3 = P - P^2$
 $= P - (I - P) = 2P - I$
 $P^4 = 2P^2 - P$
 $= 2(I - P) - P = 2I - 3P$
Similarly
 $P^6 = 5I - 8P$
and $P^8 = 13I - 21P$
 $P^8 + P^6 = 18I - 29P$

$$P^8 - P^6 = 8I - 13P$$
$$\alpha = 8, \beta = 6, \gamma = 18, \delta = 8$$
$$\alpha + \beta + \gamma - \delta = 8 + 6 + 18 - 8 = 24$$

26.
27.
28.
29.
30.