JEE-Mains-06-04-2023 [Memory Based] [Evening Shift]

Mathematics

Question: How many 4 letter word can be made from UNIVERSE, such that it has two vowels and two consonants.
Answer: 504.00
Solution:

UNIVERSE
Vowels: U, I, E, E
Consonants: N, V, R, S

$${}^3C_2 \times {}^4C_2 \times 4! + {}^4C_2 \times \frac{4!}{2!}$$
$$= 18 \times 24 + 6 \times 12$$
$$= 72[6+1]$$
$$= 504$$

Question: If all the letters of the word PUBLIC are arranged in dictionary, then the rank of PUBLIC is
Answer: 582.00
Solution:

PUBLIC
B, C, I, L, P, U

$$4 \times 5! + 4 \times 4! + 2 \times 2! + 1 + 1$$
$$= 480 + 96 + 4 + 2$$
$$= 582$$

Question: $\tan 9° - \tan 27° - \tan 63° + \tan 81° =$ ___
Answer: 4.00
Solution:

$$\tan 9° - \tan 27° - \tan 63° + \tan 81°$$
$$= \tan 9° + \cot 9° - [\tan 27° + \cot 27°]$$
$$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°}$$
$$= 2\left[\frac{1}{\frac{\sqrt{5}-1}{4}} - \frac{1}{\frac{\sqrt{5}+1}{4}}\right]$$

$=8\left[\frac{2}{4}\right]$
$=4$

Question: $1^2-2^2+3^2-4^2+....(2022)^2+(2023)^2=$ ____

Answer: 2023×2012

Solution:

$(1-2)(1+2)+(3-4)(3+4)+...+(2021-2022)(2021+2022)+(2023)^2$

$-(1+2+3+4+....+2021+2022)+(2023)^2$

$-\frac{2022\times 2023}{2}+(2023)^2$

$(2023)(1012)$

Question: If $f(x)+f(\pi-x)=\pi^2$, then $\int_0^{\pi} f(x)\times \sin x\,dx=$ ____

Answer: π^2

Solution:

$\int_0^{\pi} f(x)\times \sin x\,dx$

$2I=\int_0^{\pi}(f(x)+f(\pi-x))\sin x\,dx$

$2I=\pi^2\times 2$

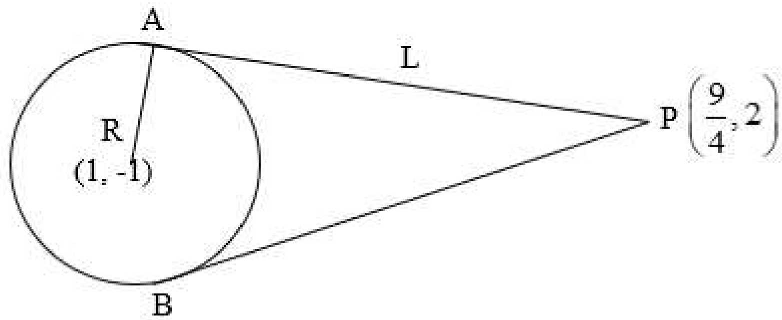
$I=\pi^2$

Question: $x^2+y^2-2x+y=5$. Two tangents are drawn from $\left(\frac{9}{4},2\right)$. AB is chord of contact. Find area of ΔPAB.

Answer: $\frac{5}{8}$

Solution:

$x^2+y^2-2x+y-5=0$



Area of $\Delta PAB = \frac{RL^3}{R^2 + L^2}$

$R = \sqrt{g^2 + f^2 - c}$

$= \sqrt{1 + \frac{1}{4} - (-5)}$

$= \sqrt{\frac{25}{4}}$

$= \frac{5}{2}$

$L = \sqrt{S_1}$

$= \sqrt{\frac{81}{16} + 4 - \frac{9}{2} + 2 - 5}$

$= \sqrt{\frac{81}{16} - \frac{9}{2} + 1}$

$= \sqrt{\frac{81 - 72 + 16}{16}}$

$= \sqrt{\frac{25}{16}}$

$= \frac{5}{4}$

$$\text{Area of } \Delta PAB = \frac{\frac{5}{2} \times \left(\frac{5}{4}\right)^3}{\left(\frac{5}{2}\right)^2 + \left(\frac{5}{4}\right)^2} = \frac{\frac{5^4}{2 \times 2^6}}{5^2\left[\frac{1}{4} + \frac{1}{16}\right]} = \frac{5}{8}$$

Question: 3 dice are thrown. Find probability that none of them shows the same number.

Answer: $\frac{5}{9}$

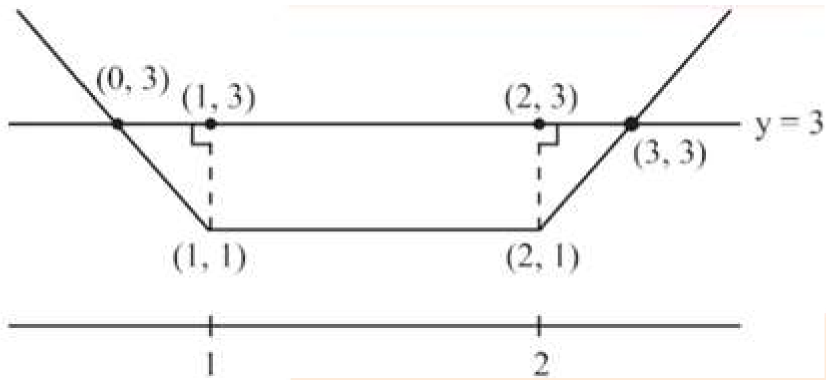
Solution:

$$\frac{^6C_3 \times 3!}{216} = \frac{20}{36} = \frac{5}{9}$$

Question: Find area bounded by the curve $y=|x-1|+|x-2|$ and the line $y=3$

Answer: 4.00

Solution:



$$A=\frac{1}{2}(1+3)\times 2=4$$

Question: $20^{19}+2\times 20^{18}\times 21+3\times 20^{17}\times 21^{2}+\ldots+20\times 21^{19}=k\times 20^{19}$. Find k.

Answer: 400.00

Solution:

$$20^{19}+2\times 20^{18}\times 21+3\times 20^{17}\times 21^{2}+\ldots+20\times 21^{19}$$

$$k=1+2\times\left(\frac{21}{20}\right)^{1}+3\times\left(\frac{21}{20}\right)^{2}+\ldots.20\left(\frac{21}{20}\right)^{19}$$

$$\frac{21}{20}k=\quad 1\times\left(\frac{21}{20}\right)+2\times\left(\frac{21}{20}\right)^{2}+\ldots.+\quad +20\left(\frac{21}{20}\right)^{20}$$

$$\frac{k}{20}=20\times\left(\frac{21}{20}\right)^{20}-\left[1+\frac{21}{20}+\ldots+\left(\frac{21}{20}\right)^{19}\right]$$

$$=20\left(\frac{21}{20}\right)^{20}-\left[\frac{\left(\frac{21}{20}\right)^{20}-1}{\frac{21}{20}-1}\right]$$

$$=20\times\left(\frac{21}{20}\right)^{20}-20\left(\frac{21}{20}\right)^{20}+20$$

$$k=400$$

Question: Eccentricity of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is e_1 & eccentricity of $2x^2-2y^2=1$ is e_2 such that $e_1=\frac{1}{e_2}$. Ellipse intersect Hyperbola orthogonally. Find $(LR)^2$ of ellipse.

Answer: 2.00

Solution:

They are confocal foci at (1, 0) & (-1, 0)

$$e_1=\frac{1}{\sqrt{2}}$$
$$e_2=\sqrt{2}$$
$$ae_1=1\Rightarrow a=\sqrt{2}$$
$$(LR)^2=\left(\frac{2b^2}{a}\right)^2=\left[2a\left(1-e^2\right)\right]^2$$
$$=8\times\frac{1}{4}=2$$

Question: Solve: $(1+\ln x)\frac{dx}{dy}-x\ln x=e^y$

Answer: ()

Solution:

$$(1+\ln x)\frac{dx}{dy}-x\ln x=e^y$$
$$e^{-y}(1+\ln x)dx-e^{-y}x\ln x\,dy=dy$$
$$d\left[e^{-y}(x\ln x)\right]=dy$$
$$e^{-y}x\ln x=y+C$$

Question: Find the square of the distance of (12, 12, 18) from the plane containing the line of intersection of $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=6$ & $\vec{r}\cdot\left(2\hat{i}+3\hat{j}+4\hat{k}\right)=-5$ and passing through (0, 2, -2).

Answer: 620.00

Solution:

The planes are

$$x+y+z-6=0$$
$$2x+3y+4z+5=0$$

Required plane $(x+y+z-6)+\lambda(2x+3y+4z+5)=0$

$$(0,2,-2)\Rightarrow -6+\lambda(3)=0$$
$$\lambda=2$$

$\therefore$ Plane is

$$5x+7y+9z+4=0$$

Square of distance $=\left|\dfrac{5(12)+7(12)+9(18)+4}{\sqrt{5^2+7^2+9^2}}\right|^2=620$

Question: Sum of all values of α for which $\hat{i}-2\hat{j}+3\hat{k}, 2\hat{i}-3\hat{j}+4\hat{k}, (\alpha+1)\hat{i}+2\hat{k}$ and $9\hat{i}+(\alpha-8)\hat{j}+6\hat{k}$ are coplanar

Options:
(a) 6
(b) 2
(c) -2
(d) 4
Answer: (b)
Solution:

$\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$

$\vec{b}=2\hat{i}-3\hat{j}+4\hat{k}$

$\vec{c}=(\alpha+1)\hat{i}+2\hat{k}$

$\vec{d}=9\hat{i}+(\alpha-8)\hat{j}+6\hat{k}$

$\overrightarrow{AB}=\hat{i}-\hat{j}+\hat{k}$

$\overrightarrow{AC}=\alpha\hat{i}+2\hat{j}-\hat{k}$

$\overrightarrow{AB}=8\hat{i}+(\alpha-6)\hat{j}+3\hat{k}$

$$\begin{vmatrix}1 & -1 & 1\\ \alpha & 2 & -1\\ 8 & \alpha-6 & 3\end{vmatrix}=(6+\alpha-6)+(3\alpha+8)+(\alpha^2-6\alpha-16)=0$$

$\Rightarrow \alpha^2-2\alpha-8=0$

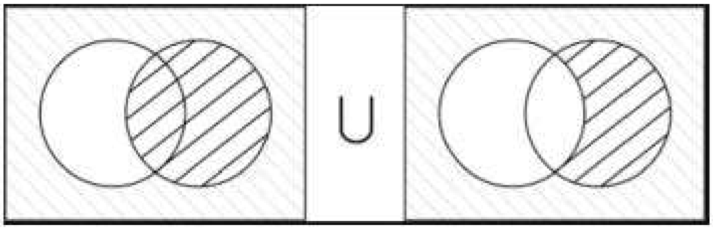
Question: $(p\Rightarrow q)\vee(\sim p\wedge q)$: tautology

$(q\Rightarrow p)\Rightarrow(\sim p\wedge q)$: contradiction

Options:
(a) Both are true
(b) Neither true
(c) Only first one is true
(d) Only second is true
Answer: (b)
Solution:

$(p\Rightarrow q)\vee(\sim p\wedge q)$

$(\sim p \vee q) \vee (\sim p \wedge q)$



Not universal set
Hence not tautology

$(q \Rightarrow p) \Rightarrow (\sim p \wedge q)$
$(\sim q \vee p) \Rightarrow (\sim p \wedge q)$
$a \Rightarrow \sim a$
$\sim a \vee \sim a$
Not contradiction

Question: Parallelopiped with coinitial edges $\vec{a}, \vec{b}$ and $\vec{c}$ is V. Then volume of parallelopiped with coinitial edges $\vec{a}, \vec{b}+\vec{c}, \vec{a}+2\vec{b}+3\vec{c}$.
Answer: V
Solution:

$V = [\vec{a} \quad \vec{b} \quad \vec{c}]$
$V' = [\vec{a} \quad \vec{b}+\vec{c} \quad \vec{a}+2\vec{b}+3\vec{c}]$
$= [\vec{a} \quad \vec{b} \quad \vec{a}+2\vec{b}+3\vec{c}] + [\vec{a} \quad \vec{c} \quad \vec{a}+2\vec{b}+3\vec{c}]$
$= [\vec{a} \quad \vec{b} \quad 3\vec{c}] + [\vec{a} \quad \vec{c} \quad 2\vec{b}]$
$= 3[\vec{a} \quad \vec{b} \quad \vec{c}] - 2[\vec{a} \quad \vec{b} \quad \vec{c}]$
$= [\vec{a} \quad \vec{b} \quad \vec{c}] = V$

Question: Coefficient of x^{-7} in $\left(ax - \frac{1}{3bx^2}\right)^{11}$ is equal to coefficient of x^7 in $\left(ax^2 + \frac{1}{2bx}\right)^{11}$.
Find relation in a & b.
Options:
(a) $243ab = 64$
(b) $32ab = 729$
(c) $64ab = 243$
(d) $729ab = 32$
Answer: (d)
Solution:

$x^7 \to \left(ax^2 + \frac{1}{2bx}\right)^{11}$

$T_{r+1} = {}^{11}C_r \left(ax^2\right)^{11-r} \left(\frac{1}{2bx}\right)^r$

$= {}^{11}C_r \frac{a^{11-r}}{(2b)^r} x^{22-3r}$

$22 - 3r = 7$

$\Rightarrow r = 5$

$\frac{{}^{11}C_5 a^6}{32b^5}$ ....(1)

$x^{-7} \to \left(ax - \frac{1}{3bx^2}\right)^{11}$

$r = \frac{np - m}{p + q} = \frac{11(1) - (-7)}{1 + 2} = 6$

$T_7 = {}^{11}C_6 (a)^{11-6} \left(\frac{-1}{3b}\right)^6$

$= \frac{{}^{11}C_6 a^5}{729b^6}$ ....(2)

Equalising (1) and (2) we will get

$729ab = 32$

Question: $\lim_{n\to\infty}\left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}} - 2^{\frac{1}{5}}\right) \times .... \times \left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right) = ?$

Answer: 0.00
Solution:

Zero

Question: The system of the equations: $x + y + z = 6$; $x + 2y + \alpha z = 5$ and $x + 2y + 6z = \beta$ has

Options:
(a) infinitely many solutions for $\alpha = 6, \beta = 3$
(b) infinitely many solutions for $\alpha = 6, \beta = 5$
(c) unique solution for $\alpha = 6, \beta = 5$
(d) No solution for $\alpha = 6, \beta = 5$

Answer: (b)
Solution:

$x+y+z=6$

$x+2y+\alpha z=5$

$x+2y+6z=\beta$

$$\Delta=\begin{vmatrix}1 & 1 & 1\\1 & 2 & \alpha\\1 & 2 & 6\end{vmatrix}=0$$

$\Rightarrow 12-2\alpha-1(6-\alpha)+0=0$

$6-\alpha=0$

$\alpha=6$

$$\Delta_x=\begin{vmatrix}6 & 1 & 1\\5 & 2 & 6\\\beta & 2 & 6\end{vmatrix}$$

$6(0)-1(30-6\beta)+(10-2\beta)=0$

$4\beta-20=0$

$\beta=5$

Substituting in given system of equations we get,

$x+y+z=6$

$x+2y+6z=5$

Which are non parallel planes

Hence infinitely many solutions.

Question: $S(1)=(2002)^{2023}-(1919)^{2002}$ is divisible by 8.

$S(2):13\times13^n-12n-13$ is divisible by 144 (where $n\in N$), then

Options:

(a) $S(1)$ and $S(2)$ both are true

(b) Only $S(1)$ is true

(c) Only $S(2)$ is true

(d) Neither $S(1)$ and $S(2)$ are true.

Answer: (c)

Solution:

Statement 1:

$\because (2002)^{2023}=8m$

$\because (2002)^{2023}$ is divisible by 8.

Also, $(1919)^{2002}$ is not divisible by 8.

$\therefore (2002)^{2023}-(1919)^{2002}$ is not divisible by 8.

Statement 2:

$13\times13^n-12n-13$

$$=13(1+12)^n-12n-13$$
$$=13\left(1+12n+{}^nC_2 12^2+\ldots\right)-12n-13$$
$$=144n+144\,{}^nC_2+\ldots$$
$$=144\left(n+{}^nC_2+\ldots\right)$$
$$=144k$$