08/04/2023
Evening

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Memory Based
Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2023 (Online) Phase-2

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

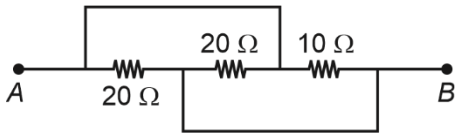
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. The effective resistance in the following circuit across terminal *A* and *B* is equal to

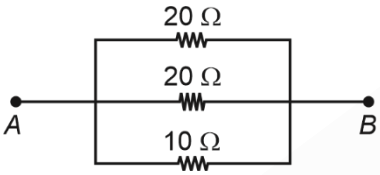


(1) 5 Ω (2) 10 Ω

(3) 20 Ω (4) 40 Ω

Answer (1)

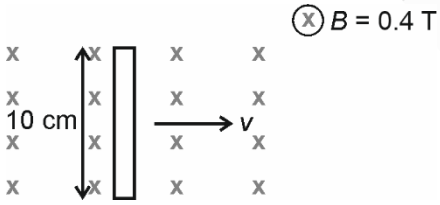
Sol. Equivalent circuit



$$\frac{1}{R}=\frac{1}{10}+\frac{1}{20}+\frac{1}{20}$$

$$\Rightarrow R = 5\ \Omega$$

2. If the emf generated in the moving rod in uniform magnetic field *B* is 0.08 V, then find the speed (*v*) of the rod.



(1) 1 m/s (2) 2 m/s

(3) 3 m/s (4) 4 m/s

Answer (2)

Sol. $\varepsilon = Blv$

$$v=\frac{\varepsilon}{Bl}=\frac{0.08\times100}{0.4\times10}=\ 2\ \text{m/s}$$

3. Which of the following expressions give the value of acceleration due to gravity (g') at the altitude *h* above the surface of earth. (*R*: radius of earth, *g*: acceleration due to gravity at surface of earth)

(1) $g'=g\frac{h^2}{R^2}$ (2) $g'=\frac{gR^2}{(R+h)^2}$

(3) $g'=g\left(1-\frac{h}{R}\right)$ (4) $g'=g\left(1-\frac{h^2}{R^2}\right)$

Answer (2)

Sol. $g'=\frac{GM_e}{(R+h)^2}$

$$g'=\frac{gR^2}{(R+h)^2}$$

4. Find the distance from a point charge of magnitude 5×10^{-9} C, where the electric potential is 50 V

(1) 90 cm (2) 70 cm

(3) 60 cm (4) 50 cm

Answer (1)

Sol. $V=\frac{kQ}{r}$

$$50=\frac{9\times10^9\times5\times10^{-9}}{r}$$

$r = 0.9$ m

5. Match column I with column II and choose the correct option.

	Column I		Column II
I.	Torque	a.	M^0LT^{-2}
II.	Stress	b.	$ML^{-1}T^{-1}$
III.	Coefficient of viscosity	c.	$ML^{-1}T^{-2}$
IV.	Gravitational potential gradient	d.	ML^2T^{-2}

(1) I → a, II → c, III →b, IV → d

(2) I → d, II → b, III → c, IV → a

(3) I → d, II → c, III → b, IV → a

(4) I → a, II → c, III → d, IV → b

Answer (3)

Sol. Torque = $r \times F = ML^2T^{-2}$

Stress = $\frac{F}{A}=ML^{-1}T^{-2}$

Coefficient of viscosity = $ML^{-1}T^{-1}$

Gravitational potential gradient = M^0LT^{-2}

6. Which of the following is the highest energy electromagnetic wave?

(1) X-rays (2) Infra Red

(3) Microwaves (4) Radiowave

Answer (1)

Sol. Since out of the given options, X-rays have the highest frequency.

$\Rightarrow$ Option (1) is correct

7. A carnot engine working between 27°C and 127° performs 2 kJ of work. The amount of heat energy rejected is equal to

(1) 4 kJ (2) 6 kJ

(3) 8 kJ (4) 12 kJ

Answer (2)

Sol. $2\text{ kJ} = x\left(1-\frac{300}{400}\right)$

$2\text{ kJ} = \frac{x}{4}$

$\Rightarrow x = 8\text{ kJ}$

$\Rightarrow$ Heat lost = 6 kJ

8. **Statement-I:** Electromagnet are made of soft iron.

Statement-II: Soft iron has lower permeability and high retentivity.

Choose the correct option related to statements.

(1) Statement-I is true and statement-II is true

(2) Statement-I is true and statement-II is false

(3) Statement-I is false and statement-II is true

(4) Statement-I is false and statement-II is false

Answer (2)

Sol. Soft iron has low retentivity and high permeability.

9. If a satellite is orbiting the earth at a height h has angular momentum 'L'. Then, the same satellite at a height 10 times 'h' will have angular momentum equal to

(1) $\sqrt{10}\,L$ (2) $\sqrt{5}\,L$

(3) $3L$ (4) $\sqrt{20}\,L$

Answer (1)

Sol. $\because \quad \frac{mv^2}{r} = \frac{GMm}{r^2}$

$\Rightarrow m^2v^2r^2 = GMmr$

$L^2 \propto r$

$\therefore \quad \frac{L_1}{L_2} = \sqrt{\frac{h}{10h}}$

$\Rightarrow L_2 = \sqrt{10}L$

10. Consider 2 statements:

Statement 1: We can get displacement from acceleration-time graph.

Statement 2: We can get acceleration from velocity-time graph.

Then

(1) Both statements are true

(2) Both statements are false

(3) Statement 1 is true and statement 2 is false

(4) Statement 1 is false and statement 2 is true

Answer (4)

Sol. To get displacement from acceleration-time graph, we will need 1 initial value (for velocity).

Also, $a = \frac{dv}{dt}$

$\Rightarrow$ Slope will give a.

11. A projectile launched on a horizontal surface follows a trajectory given by $y = x - \frac{x^2}{20}$ where y-axis is in vertical upward direction. Maximum height attained by projectile is (All units are in SI)

(1) 10 m (2) 5 m

(3) 20 m (4) 40 m

Answer (2)

Sol. $y = x - \frac{x^2}{20}$

at maximum height $\frac{dy}{dx} = 0$

$\Rightarrow x = 10\text{ m}$

at $x = 10$ m, $y = 10 - 5 = 5$ m

12. An antenna of length l emits radiation of wavelength λ. The power emitted by the antenna is proportional to:

(1) $\left(\frac{l}{\lambda}\right)^2$ (2) $\frac{l}{\lambda}$

(3) $\frac{\lambda}{l}$ (4) $\frac{1}{l\lambda}$

Answer (1)

Sol. Since $P \propto \left(\frac{l}{\lambda}\right)^2$

$\Rightarrow$ Option (1) is correct.

13. In a radioactive process, $\frac{1}{8}$th of the initial amount of the element is decayed. If in 5 days further, 8×10^{-3} kg of the element decayed, find the original amount of element.

(1) 128 grams (2) 64 grams

(3) 256 grams (4) 32 grams

Answer (2)

Sol. $\frac{1}{8}=\frac{1}{2^3}$

$\Rightarrow$ 3 half lives = 3 days

$\Rightarrow \frac{b}{2}$ = 1 day

Let m: initial mass

$\Rightarrow \frac{m}{8}-\frac{m}{8\times32}$ =8 grams

$\Rightarrow m=\frac{64\times32}{32-1}\simeq 65$g.

14. Find the change in energy stored in a capacitor of 600 *pF* capacitance charged at 50 *V*, once connected with another 600 *pF* uncharged capacitor.

(1) 0.56 μJ (2) 0.4 μJ

(3) 0.86 μJ (4) 0.32 μJ

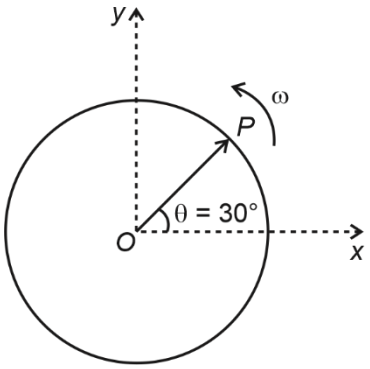
Answer (1)

Sol. $U_i=\frac{1}{2}Cv^2$, $U_f=\frac{1}{2}C\left(\frac{v}{2}\right)^2$

$\Delta U=\frac{3}{8}Cv^2$

$=\frac{3}{8}\times600\times10^{-12}\times(50)^2$

15. Phasor of a particle performing SHM is as shown in the diagram. The SHM has angular frequency ω and at t = 0 the phasor lies along *OP*. At any time t further the projection of phasor along *y*-axis is given by



(1) $R\sin\left(\omega t+\frac{\pi}{6}\right)$ (2) $R\cos\left(\omega t+\frac{\pi}{6}\right)$

(3) $R\sin\left(\omega t-\frac{\pi}{6}\right)$ (4) $R\cos\left(\omega t-\frac{\pi}{6}\right)$

Answer (1)

Sol. θ at any time t

$=\omega t=30°$

$\Rightarrow y_{projection}=R\sin\theta$

$=R\sin\left(\omega t+\frac{\pi}{6}\right)$

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. A body of mass 5 kg has the linear momentum of 100 kg ms^{-1} and acted upon by the force of 2 N for 2 seconds, then change in kinetic energy in Joule is

Answer (81.60)

Sol. $F\times t=\Delta P$

$\Rightarrow 2\times2=P_f-100$

$P_f=104$ kg ms^{-1}

$\Delta K=\frac{P_f^2}{2m}-\frac{P_i^2}{2m}=\frac{1}{2\times5}\times\left(104^2-100^2\right)$

$=\frac{1}{10}\times4\times204=81.6$ J

22. In a YDSE experiment, fringe width is 2 mm when wavelength of light used is λ = 400 nm. Find the fringe width (in mm) when wavelength is 600 nm.

Answer (3)

Sol. $\beta = \frac{\lambda D}{d}$

$\Rightarrow \frac{\beta'}{\beta} = \frac{600}{400} = 1.5$

$\Rightarrow \beta' = 3 \text{ mm}$

23. A block moving with speed 1 m/s comes to rest after moving for 20 cm over a rough surface. The coefficient of friction between the block and surface is___

Answer (00.25)

Sol. $\because \quad v^2 - u^2 = 2aS$

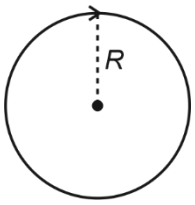
$0^2 - 1^2 = 2(-\mu g)\frac{20}{100}$

$\mu = \frac{1}{4} = 0.25$

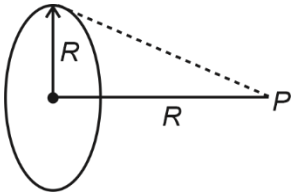
24. The ratio of magnetic field due to coil at centre and at a distance of R from the centre on the axis passing through the centre and perpendicular to the plane of ring is $\sqrt{x}:1$ (R is the radius of coil), find the value of x.

Answer (8)

Sol.



$B_C = \left(\frac{\mu_0 i}{2R}\right)$

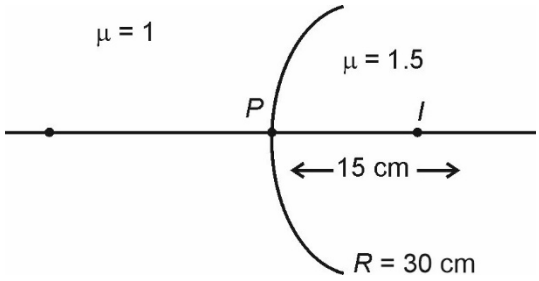


$B_P = \frac{\mu_0}{4\pi} \times \frac{2 \times i \times \pi R^2}{(R^2 + R^2)^{3/2}}$

$= \frac{\mu_0}{2R}\frac{i}{2\sqrt{2}} = \left(\frac{\mu_0 i}{4\sqrt{2}R}\right)$

$\frac{B_C}{B_P} = \frac{4\sqrt{2}}{2} = \sqrt{8}:1$

25. In the given diagram image forms at a distance of 15 cm inside the



medium of refractive index 1.5. Find the object distance (in cm) from point P.

Answer (12.00)

Sol. $\frac{1.5}{15} - \frac{1}{u} = \left(\frac{1.5-1}{30}\right) = \frac{0.5}{30} = \frac{1}{60}$

$\frac{1}{10} - \frac{1}{u} = \frac{1}{60} \Rightarrow \frac{1}{10} - \frac{1}{60} = \frac{1}{u}$

$\frac{1}{u} = \frac{5}{60} \Rightarrow u = \frac{60}{5} = 12 \text{ cm}$

26. Ratio of wavelengths of photons corresponding to first and second line of Balmer series in an emission spectrum is given by $\frac{x}{20}$ for a hydrogen like species. Value of x is equal to

Answer (27)

Sol. $\frac{1}{\lambda_1} = -R\left(\frac{1}{9} - \frac{1}{4}\right)$

$\frac{1}{\lambda_2} = -R\left(\frac{1}{16} - \frac{1}{4}\right)$

$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{36}{5} \times \frac{3}{16} = \frac{27}{20}$

$\Rightarrow x = 27$

27.

28.

29.

30.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Which of the following acts as a stabilizer in the decomposition of H_2O_2.

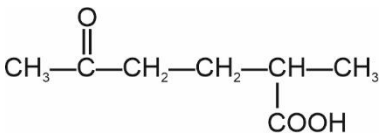
(1) Urea (2) Alkali

(3) Glass (4) Dust

Answer (1)

Sol. Urea acts as a stabilizer in the decomposition of H_2O_2

2. IUPAC name of given compound is



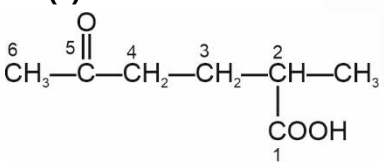
(1) 5-oxo-2-methyl hexanoic acid

(2) 2-methyl-5-oxohexanoic acid

(3) 5-oxo-2-methyl pentatonic acid

(4) 5-carboxy-2-oxohexane

Answer (2)

Sol. 

3. Order of van der waals constant a for Ar, CH_4, H_2O, and C_6H_6

(1) $H_2O > C_6H_6 > Ar > CH_4$

(2) $Ar > H_2O > CH_4 > C_6H_6$

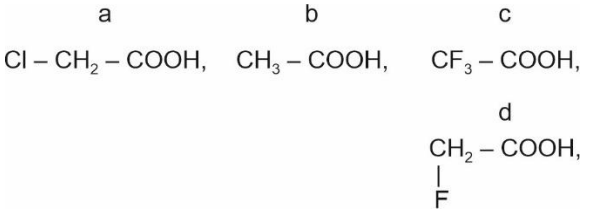
(3) $Ar > C_6H_6 > H_2O > CH_4$

(4) $H_2O > C_6H_6 > CH_4 > Ar$

Answer (4)

Sol. H_2O has hydrogen bonding.

4. Find the correct order of acidity for the following



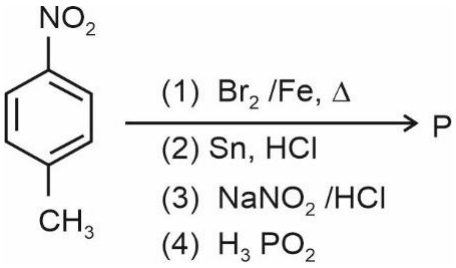
(1) c > d > a > b (2) b > a > d > c

(3) a > b > c > d (4) b > c > a > d

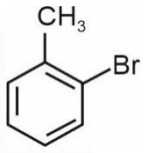
Answer (1)

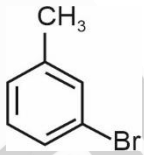
Sol. Correct order of acidity is

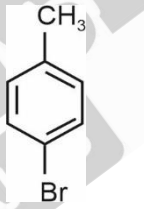
c > d > a > b (based on –I effect of –F & –Cl group)

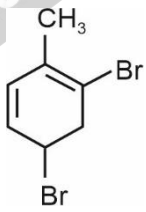
5. 

Find out final product of this reaction

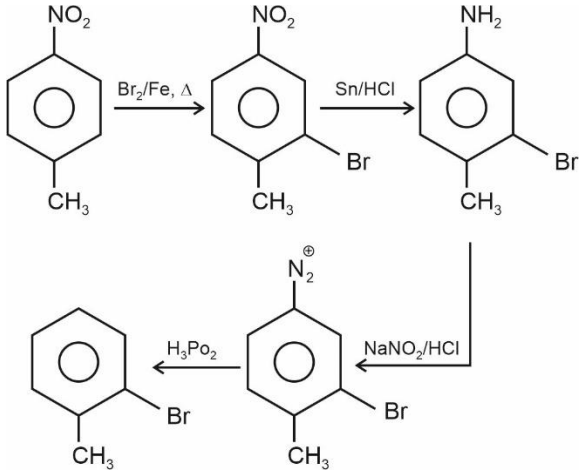
(1) 

(2) 

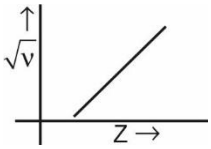
(3) 

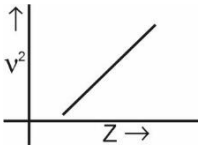
(4) 

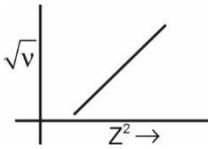
Answer (1)

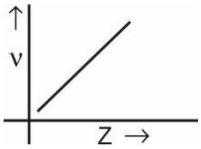
Sol. 

6. Find the correct plot

(1) 

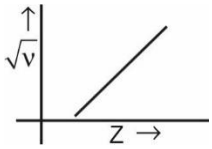
(2) 

(3) 

(4) 

Answer (1)

Sol. As per Moseley's law, cannot plot is $\left(\sqrt{\nu} = a(Z - b)\right]$



7. Total spin only magnetic moment of the ion $[Mn(SCN)_6]^{x-}$ is 5.92 B.M. Find out the value of x.

(1) 5

(2) 3

(3) 2

(4) 4

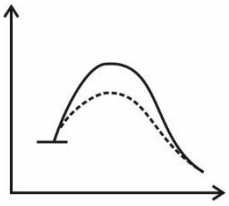
Answer (4)

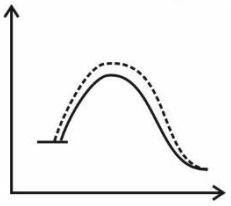
Sol. The value of magnetic moment showing the presence of five unpaired electrons hence the central atom Mn will be at +2.

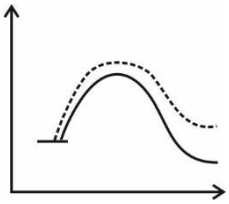
8. Find out the correct option by using +ve catalyst.

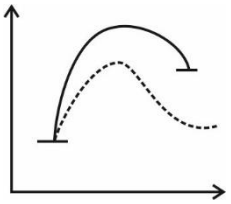
_____ without catalyst

-------- with catalyst

(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. $\frac{\Delta H \text{ doesn't change}}{E_a \text{ will decrease}}$

9. Match Column-I with Column-II

	Column-I		**Column-II (Unpaired Electrons)**
A	$[Ni(NH_3)_6]^{2+}$	P	O
B	$[Co(NH_3)_6]^{3+}$	Q	2
C	$[Fe(CN)_6]^{3-}$	R	4
D	$[CoF_6]^{3-}$	S	1

(1) A-Q; B-P; C-R; D-S

(2) A-P; B-Q; C-S; D-R

(3) A-Q; B-P; C-S; D-R

(4) A-S; B-Q; C-P; D-R

Answer (3)

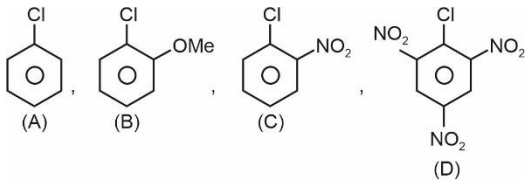
Sol. $[Ni(NH_3)_6]^{2+}$: sp^3d^2 n = 2

$[Co(NH_3)_6]^{3+}$: d^2sp^3 n = 0

$[Fe(CN)_6]^{3-}$: d^2sp^3 n = 1

$[CoF_6]^{3-}$: sp^3d^2 n = 4

10. The correct order of nucleophilic substitution of following compounds with NaOH



(1) A > B > C > D

(2) D > C > A > B

(3) D > C > B > A

(4) A > B > D > C

Answer (2)

Sol. Nucleophilic of substitution rate depends on the presence of E.W.G at ortho and para position of benzene ring. Hence the correct order of nucleophilic substitution will be D > C > A > B.

11. Statement-1 : Methyl orange is a weak acid

Statement-2 : Benzenoid form of methyl orange is deeply coloured than quinonoid form

(1) Statement-1 is correct and Statement-2 is wrong

(2) Both the Statements-1 and Statement-2 are correct

(3) Statement-1 is wrong and Statement-2 is correct

(4) None of them

Answer (1)

Sol. Methyl orange is a weak acid. So, statement-1 is correct. In acidic medium, it exists in quinonoid form which is red in colour and in alkaline medium it exists in benzenoid form which is yellow in colour. Since red is more deeply coloured than yellow, Statement-2 is wrong.

12. Which of the following is correct?

(I) Photocurrent α Intensity of photoelectrons

(II) Kinetic energy is dependent on frequency

(III) Kinetic energy is independent of frequency

(1) I, II only

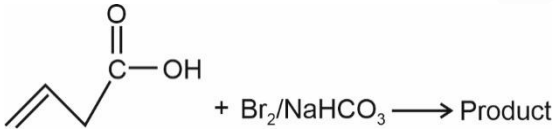
(2) III, I only

(3) II only

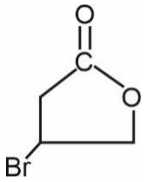
(4) III only

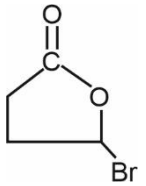
Answer (1)

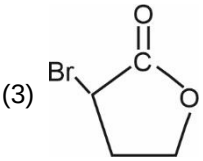
Sol. Photocurrent α Intensity of incident light. Kinetic energy of electron is dependent on frequency of incident light.

13. 

Find out final product of this reaction

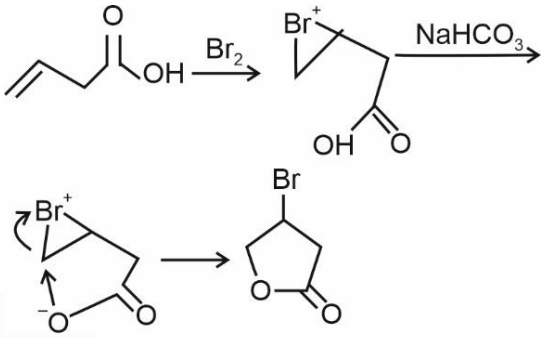
(1) 

(2) 

(3) 

(4) None

Answer (1)

Sol. 

14.

15.

16.

17.

18.

19.

20.

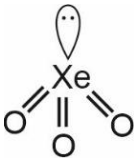
SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Compounds of Xenon having one electron pair on central atom

XeO_3 $XeOF_2$ XeF_4 XeF_5^-

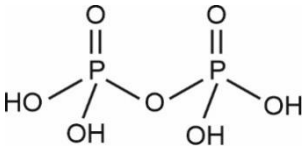
Answer (01.00)

Sol.  have 1 lone pair on central atom

22. What is the ratio of σ and π bonds in pyrophosphoric acid?

Answer (06)

Sol. Pyrophosphoric acid is $H_4P_2O_7$



σ bonds = 12

π bonds = 2

Ratio of $\frac{\sigma}{\pi} = \frac{12}{2} = 6$

23. Find out oxidation number of central metal atom of $Fe(CO)_5$, VO^{2+} and WO_3. Then calculate the sum of their oxidation states.

Answer (10.00)

Sol.

Compound	Oxidation state of central metal atoms
$Fe(CO)_5$	0
VO^{2+}	+4
WO_3	+6

Sum of oxidation states = 0 + 4 + 6 = 10

24. How many of the following have five radial nodes?

5*s*, 6*s*, 7*s*, 6*p* and 4*p*

Answer (01)

Sol. Radial nodes is given by (n – l – 1)

For 5*s*, Radial node = 4

For 6*s*, Radial node = 5

For 7*s*, Radial node = 6

For 6*p*, Radial node = 4

For 4*p*, Radial node = 2

25. In good quality cement ratio of lime total oxides of Si(SiO_2), Aluminium(Al_2O_3) and Iron(Fe_2O_3) should be as close as possible to_____.

Answer (2)

Sol. Fact

Reference NCERT Page-304 NCERT.

26. The boiling points of two solvents X and Y are in the ratio 2 : 1 (in K) and their enthalpy of vaporisation is in the ratio 1 : 2. Find the ratio of elevation in boiling point when same moles of solute are added to same mass of both the solvents, if the molar mass of X is twice that of Y

Answer (16.00)

Sol. $K_b = \frac{RTb^2M}{1000\Delta H}$

$$\frac{(K_b)_X}{(K_b)_Y} = \frac{(Tb)_X^2}{(Tb)_Y^2} \times \frac{M_X}{M_Y} \times \frac{(\Delta H)_Y}{(\Delta H)_X}$$

$$= \frac{4}{1} \times 2 \times 2 = 16$$

27. K_{sp} of $BaSO_4$ is 8×10^{-11}. If the solubility in presence of 0.1 M $CaSO_4$ is

Answer (8)

Sol. 'X' × 10^{-10}M, X is :

$BaSO_4 \rightleftharpoons Ba^{+2} + SO_4^{-2}$

S S + 0.1

≈ 0.1

$S \times 0.1 = 8 \times 10^{-11}$

$S = 8 \times 10^{-10}$

$\therefore \quad X = 8$

28. For As_2S_3 colloidal solution, the coagulation value of $AlCl_3$ & NaCl are 0.09 and 50.04 respectively. If coagulation power of $AlCl_3$ is x times of NaCl then tell the value of x.

Answer (556)

Sol. For a given colloid

$$\frac{\text{Coagulation value of NaCl}}{\text{Coagulation value of } AlCl_3} = \frac{\text{Coagulation power of } AlCl_3}{\text{Coagulation power of NaCl}}$$

$$\frac{50.04}{0.09} = x$$

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. The absolute difference of the coefficient of x^7 and x^9 in the expansion of $\left(2x+\frac{1}{2x}\right)^{11}$ is

(1) 11×2^5 (2) 11×2^7

(3) 11×2^4 (4) 11×2^3

Answer (2)

Sol. $T_{r+1} = {}^{11}C_r (2x)^{11-r}\left(\frac{1}{2x}\right)^r$

$= {}^{11}C_r \frac{2^{11-r}}{2^r} x^{11-2r}$

$11 - 2r = 7$ and $11 - 2r = 9$

$r = 2$ $r = 1$

$\therefore$ Coefficient of x^7 is ${}^{11}C_2 \frac{(2)^9}{2^2} = {}^{11}C_2 (2)^7$

Coefficient of x^9 is ${}^{11}C_1 \frac{(2)^{10}}{2} = {}^{11}C_1 (2)^9$

${}^{11}C_2 (2)^7 - 11 \times (2)^9$

$= 11 \times 2^7$

2. Let $f(x) = \{1, 2, 3, 4, 5, 6, 7\}$ the relation $R = \{(x, y) \in A \times A, x + y = 7\}$ is

(1) Symmetric

(2) Reflexive

(3) Transitive

(4) Equivalence

Answer (1)

Sol. $x + y = 7$

$y = 7 - x$

$R = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$

$\because$ $(a, b) \in R \Rightarrow (b, a) \in R$.

$\therefore$ Relation is symmetric

3. The number of words with or without meaning can be formed from the word MATHEMATICS where C, S not come together is

(1) $\frac{9}{8} \times 10!$ (2) $\frac{1}{8} \times 10!$

(3) $\frac{5}{8} \times 10!$ (4) $\frac{1}{2} \times 10!$

Answer (1)

Sol. Total words = $\frac{11!}{2!2!2!}$

When C and S are together = $\frac{10!}{2!2!2!} \times 2!$

$\therefore$ Required number of words = $\frac{11!}{(2!)^3} - \frac{10!}{(2!)^3} \times 2!$

$= \frac{10!}{8}[11-2]$

$= \frac{9}{8} \times 10!$

4. Let $a_n = 5 + 8 + 14 + 23 + \ldots$. upto n terms. If $S_n = \sum_{k=1}^{n} a_k$, then $S_{30} - a_{40}$ is equal to

(1) 78025

(2) 12800

(3) 11600

(4) 12100

Answer (1)

Sol. $a_n = 5 + 8 + 14 + \ldots. T_n$

$$\begin{array}{l} a_n = 5+8+14\ldots+T_{n-1}+T_n \\ 0 = 5+\underbrace{3+6+9+\ldots.}_{(n-1)\text{ terms}}-T_n \end{array}$$

$\Rightarrow T_n = 5 + \left(\frac{n-1}{2}\right)\left(6 + (n-2)3\right) = 5 + \frac{3}{2}(n-1)^n$

$5 + \frac{3}{2}n^2 - \frac{3}{2}n$

$\Rightarrow \frac{1}{2}\left(10 + 3n^2 - 3n\right)$

$\therefore$ $T_n = \frac{1}{2}\left(3n^2 - 3n + 10\right)$

$$a_n = \sum T_n = \frac{1}{2}\left[\frac{3\cdot(n)(n+1)(2n+1)}{6} - \frac{3\cdot(n)(n+1)}{2} + 10n\right]$$

$$= \frac{1}{2}(n)\left(\frac{(n+1)(2n+1)}{2} - \frac{3(n+1)}{2} + 100\right)$$

$$a_n = \frac{n}{4}\left(2n^2 + 3n + 1 - 3n - 3 + 20\right)$$

$$= \frac{n}{4}\left(2n^2 + 18\right) = \frac{n}{4}\left(n^2 + 9\right)$$

$$a_{40} = \frac{40}{2}(1600 + 9) = 1609 \times 20 = 32180$$

$$S_n = \sum a_n = \frac{1}{2}\left(\left(\frac{(n)(n+1)}{2}\right)^2 + \frac{9\cdot(n)(n+1)}{2}\right)$$

$$S_{30} = \frac{1}{2}\left(\left(\frac{30\times 3}{2}\right)^2 + \frac{9}{2}(30)(31)\right)$$

$$= \frac{1}{2}(216225 + 4185)$$

$$= 110205$$

$S_{30} - a_{40} = 78025$

5. The equation $ax^2 + bx + c = 0$ has roots α and β. Then find $\lim\limits_{x\to\frac{1}{\alpha}} \frac{1-\cos(cx^2+bx+a)}{2(1-\alpha x)^2}$ is

(1) $\frac{c^2(\alpha-\beta)^2}{4\alpha^4\beta^2}$ (2) $\frac{c^2(\alpha-\beta)^2}{\alpha^4\beta^2}$

(3) $\frac{c^2(\alpha-\beta)^2}{2\alpha^4\beta^2}$ (4) $\frac{c^2(\alpha-\beta)^2}{4\alpha^2\beta^2}$

Answer (1)

Sol. $$\lim_{x\to\frac{1}{\alpha}} \frac{2\sin^2\left(\frac{cx^2+bx+a}{2}\right)}{2\alpha^2\left(x-\frac{1}{\alpha}\right)^2}$$

$$= \frac{c^2(\alpha-\beta)^2}{4\alpha^2\beta^2}$$

6. $\theta \in (0, 2\pi)$ and $\frac{1+2i\sin\theta}{1-i\sin\theta}$ is purely imaginary then the value of θ is

(1) π (2) 0

(3) 2π (4) $\frac{\pi}{4}$

Answer (4)

Sol. Real part has to be zero

$$\Rightarrow \frac{1-2\sin^2\theta}{1+\sin^2\theta} = 0$$

$$\sin^2\theta = \frac{1}{2}$$

$$\theta = n\pi \pm \frac{\pi}{4}, n \in I$$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{4}$$

7. The statement $(p \wedge (\sim q)) \vee (\sim p)$ is equivalent to

(1) $p \wedge q$ (2) $\sim p \vee \sim q$

(3) $p \vee q$ (4) $\sim p \wedge \sim q$

Answer (2)

Sol. $(p \wedge (\sim q)) \vee (\sim p)$

$= (p \vee \sim p) \wedge (\sim q \vee \sim p)$

$= T \wedge (\sim q \vee \sim p)$

$= \sim q \vee \sim p$

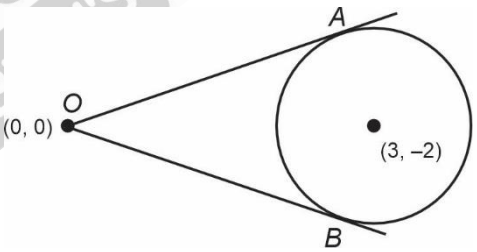
8. From $O(0, 0)$, two tangents OA and OB are drawn to a circle $x^2 + y^2 - 6x + 4y + 8 = 0$, then the equation of circumcircle of ΔOAB.

(1) $x^2 + y^2 - 3x + 2y = 0$ (2) $x^2 + y^2 + 3x - 2y = 0$

(3) $x^2 + y^2 + 3x + 2y = 0$ (4) $x^2 + y^2 - 3x - 2y = 0$

Answer (1)

Sol.



(0, 0) and (3, –2) are diametric end points

$\therefore$ $(x - 0)(x - 3) + (y - 0)(y + 2) = 0$

$$\boxed{x^2 + y^2 - 3x + 2y = 0}$$

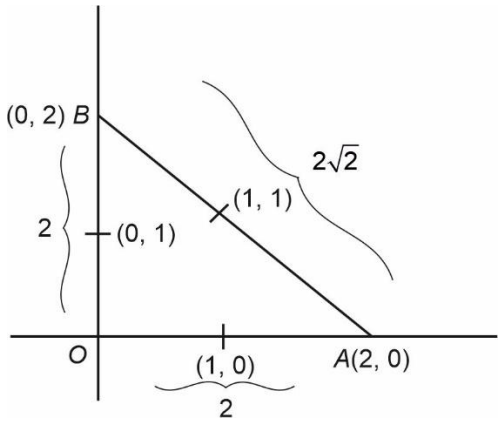
9. The mid points of side of a triangle are (0, 1), (1, 0), (1, 1), where incentre is D. A parabola $y^2 = 4ax$ passes through D whose focus is $(\alpha + \beta\sqrt{2}, 0)$ then $\frac{\beta^2}{\alpha}$ is

(1) $\frac{1}{2}$ (2) 2

(3) $\frac{1}{8}$ (4) 4

Answer (3)

Sol.



$\because$ Mid-point is (0, 1), (1, 0) and (1, 1)

$$I = \left(\frac{4}{4+2\sqrt{2}}, \frac{4}{4+2\sqrt{2}}\right)$$

$$y^2 = 4ax$$

$\because$ $y^2 = 4ax$ passes through I

$$\left(\frac{4}{4+2\sqrt{2}}\right)^2 = 4a\left(\frac{4}{4+2\sqrt{2}}\right) \Rightarrow a = \frac{1}{4+2\sqrt{2}}$$

Focus = $(a, 0)$

$$= \left(\frac{1}{4+2\sqrt{2}}, 0\right)$$

$$= \left(\frac{4-2\sqrt{2}}{8}, 0\right)$$

$$\therefore \quad \alpha = \frac{4}{8} = \frac{1}{2}, \ \beta = \frac{-2}{8} = \frac{-1}{4}$$

$$\frac{\beta^2}{\alpha} = \frac{1}{8}$$

10. Let $R = \{a, b, c, d, e\}$ and $S = \{1, 2, 3, 4\}$. Then number of onto functions $f(x) : R \to S$ such that $f(a) \neq 1$ is

(1) 240 (2) 180

(3) 204 (4) 216

Answer (2)

Sol. Total number of onto functions

$$= \frac{5!}{3!2!} \times 4!$$

Now, when $f(a) = 1$

$$\frac{4!}{2!2!} \times 3! + 4!$$

$\therefore$ Required functions = 240 – 36 – 24

= 180

11. A parabola with focus (3, 0) and directrix $x = -3$. Points P and Q lie on the parabola and their ordinates are in the ratio 3 : 1. The point of intersection of tangents drawn at points P and Q lies on the parabola

(1) $y^2 = 16x$ (2) $y^2 = 4x$

(3) $y^2 = 8x$ (4) $x^2 = 4y$

Answer (1)

Sol. Given parabola $y^2 = 12x$

$$P\left(3t_1^2, 6t_1\right), Q\left(3t_2^2, 6t_2\right)$$

$$\frac{t_1}{t_2} = 3 \Rightarrow t_1 = 3t_2 \qquad ...(i)$$

Let point of intersection be (h, k)

$h = 3t_1t_2$ ...(ii)

and $k = 3(t_1 + t_2)$ ...(iii)

(i) and (iii) $\Rightarrow t_2 = \frac{k}{12}$

(ii) $\Rightarrow h = 9t_2^2 = 9 \times \frac{k^2}{144} \Rightarrow k^2 = 16h$

$\Rightarrow$ $y^2 = 16x$

12. In probability distribution for discrete variable $x = 0, 1, 2 \ldots$ $P(x = x) = k(x + 1).3^{-x}$. The probability of $P(x \geq 2)$ is equal to

(1) $\frac{5}{18}$ (2) $\frac{10}{18}$

(3) $\frac{20}{27}$ (4) $\frac{7}{27}$

Answer (4)

Sol. $\Sigma P = 1$

$\Rightarrow k(1 + 2.3^{-1} + 3.3^{-2} + \ldots.) = 1$

Let $S = 1 + \frac{2}{3} + \frac{3}{3^2} + \ldots.$

$$\frac{S}{3} = \frac{1}{3} + \frac{2}{3^2} + \ldots.....$$

$$\frac{2S}{3} = 1 + \frac{1}{3} + \frac{1}{3^2} + \ldots. = \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$$

$$\Rightarrow S = \frac{9}{4}$$

$$\therefore \quad k \cdot \frac{9}{4} = 1 \Rightarrow k = \frac{4}{9}$$

Now $P(x \geq 2) = 1 - P(x = 0, 1)$

$$= 1 - \left(k + k.\frac{2}{3}\right)$$

$$= 1 - \frac{5k}{3}$$

$$= 1 - \frac{5}{3}.\frac{4}{9}$$

$$= \frac{7}{27}$$

13. If $f(x) = \begin{cases} 3x^2 + k\sqrt{x+1} & 0 < x < 1 \\ 3mx^2 + k^2 & x \geq 1 \end{cases}$ is differentiable at $x > 1$ then $\frac{8f'(8)}{f'\left(\frac{1}{8}\right)}$ is for $k \neq 0$

(1) 309 (2) 311

(3) 306 (4) 305

Answer (1)

Sol. $f(x) = \begin{cases} 3x^2 + k\sqrt{x+1} & 0 < x < 1 \\ 3mx^2 + k^2 & x \geq 1 \end{cases}$

$3 + k\sqrt{2} = 3m + k^2$ ...(i)

$f'(x) = \begin{cases} 6x + \frac{k}{2\sqrt{x+1}} & 0 < x < 1 \\ 6mx & x \geq 1 \end{cases}$

$6 + \frac{k}{2\sqrt{2}} = 6m$ ...(ii)

$3 + k\sqrt{2} = 3 + \frac{k}{4\sqrt{2}} + k^2$

$k = 0$ or $\frac{7\sqrt{2}}{8}$

If $k = 0$ If $k = \frac{7\sqrt{2}}{8}$

$m = 1$ $m = \frac{103}{96}$

(Rejected)

Now, $\frac{8f'(8)}{f'\left(\frac{1}{8}\right)} = \frac{48m}{\frac{6}{8} + \frac{k}{2\sqrt{\frac{9}{8}}}} = \frac{48m}{\frac{6}{8} + \frac{\sqrt{2}k}{3}}$

$\frac{8f'(8)}{f'\left(\frac{1}{8}\right)} = 309$

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. The area of quadrilateral having vertices as (1, 2), (5, 6), (7, 6), (–1, –6)

Answer (24)

Sol. Area $= \frac{1}{2}\begin{vmatrix} 1 & 2 \\ 5 & 6 \\ 7 & 6 \\ -1 & -6 \\ 1 & 2 \end{vmatrix}$

$= \frac{1}{2}[6 + 30 - 42 - 2 - 10 - 42 + 6 + 6]$

$= \frac{1}{2}[48] = 24$

22. The value of $\int_0^{2.4} [x^2]dx$ is $\alpha + \beta\sqrt{2} + \gamma\sqrt{3} + \delta\sqrt{5}$ then $(a + b + c + d + e)$ is equal to

Answer (06)

Sol. $\int_0^{2.4} [x^2]dx = \int_0^1 0\,dx + \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dx + \int_{\sqrt{3}}^{\sqrt{4}} 3\,dx + \int_{\sqrt{4}}^{\sqrt{5}} 4\,dx + \int_{\sqrt{5}}^{2.4} 5\,dx$

$= \left(\sqrt{2} - 1\right) + 2\left(\sqrt{3} - \sqrt{2}\right) + 3\left(\sqrt{4} - \sqrt{3}\right) + 4\left(\sqrt{5} - \sqrt{4}\right) + 5\left(2.4 - \sqrt{5}\right)$

$= 9 - \sqrt{2} - \sqrt{3} - \sqrt{5}$

$\therefore \quad \alpha = 9, \beta = -1, \gamma = -1, \delta = -1$

$\therefore \quad \alpha + \beta + \gamma + \delta = 6$

23. $\frac{dx}{dy}-\frac{3\sin y}{\cos y(\ln\cos y)}x=\frac{\sin y}{(\ln\cos y)^2\cos y}$ and $x\left(\frac{\pi}{3}\right)=\frac{1}{2\ln 2}, x\left(\frac{\pi}{6}\right)=\frac{1}{\ln(m)-\ln(n)}$ then the value of mn is

Answer (12)

Sol. $I=e^{\int\frac{-3\sin y}{\cos y(\ln\cos y)}dy}$

Put $\ln(\cos y)=t$

$\frac{-1}{\cos y}\sin y\,dy=dt$

$=e^{\int\frac{3}{t}dt}$

$=(\ln\cos y)^3$

$x(\ln\cos y)^3=\int\frac{\sin y}{\cos y}\ln\cos y\,dy$

$x(\ln\cos y)^3=\frac{-(\ln(\cos y))^2}{2}+C$

$x\left(\frac{\pi}{3}\right)=\frac{1}{2\ln 2}$

$\Rightarrow\ C=0$

$\therefore\quad x=-\frac{1}{2\ln(\cos y)}$

$x\left(\frac{\pi}{6}\right)=\frac{1}{\ln 4-\ln 3}$

$m=4$

$n=3$

24. If m is the number of solution of $x^2-12x+31+[x]=0$ and n be the number of solution of $x^2-5|x+2|-4=0$, then the value of m^2+mn+n^2 is

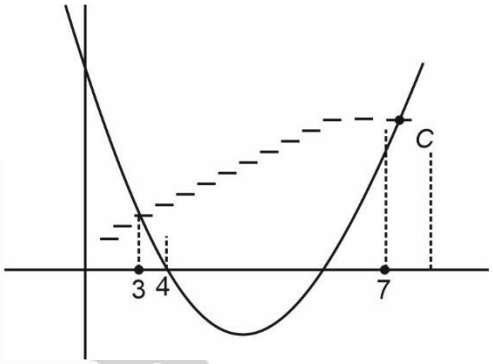
Answer (19)

Sol. $x^2-12x+31-[x]=0$

$x^2-12x+31=[x]$

$(x-6)^2-5=[x]$

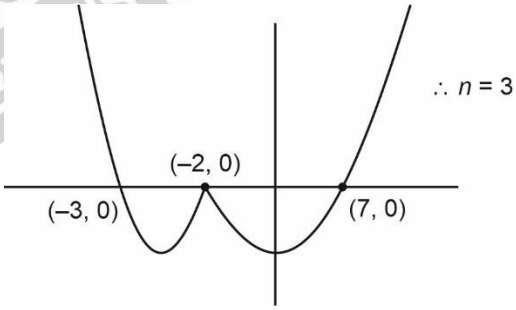
So, by graph



$\therefore$ Two points of intersects

$\therefore\quad m=2$

$x^2-5|x-2|-4=0$



$m^2+mn+n^2=4+6+9=19$

25.

26.

27.

28.

29.

30.

❑ ❑ ❑