# JEE-Mains-08-04-2023 [Memory Based] [Morning Shift]

## Physics

**Question:** Body weighs 400N on surface of earth find the weight of the body at a depth R/2 from the surface of earth (R = radius of earth)

**Options:**

(a) 100 N

(b) 200 N

(c) 300 N

(d) 400 N

**Answer: (b)**

**Solution:**

$$F_1 = \frac{400N}{F_2 N} = \frac{mgs}{mgs\left(1-\frac{d}{R}\right)}$$

where $d = R/2$

$$\frac{400}{F_2} = \frac{1}{\left(1-\frac{1}{2}\right)}$$

$$\Rightarrow F_2 = 200$$

**Question:** If m = 1/2 kg, $\vec{v} = (2t)\hat{i} + (3t^2)\hat{j}$. If at t = 1 sec $\vec{F} = \hat{i} + x\hat{j}$. Find x?

**Answer: 3.00**

**Solution:**

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$= \frac{1}{2}\left[\frac{d}{dt}\left(2t\hat{i} + 3t^2\hat{j}\right)\right]$$

$$= \frac{1}{2}\left(2\hat{i} + 6t\hat{j}\right)$$

$$\vec{F} = \hat{i} + 3t\hat{j} \text{ at } t = 1 \Rightarrow F = \hat{i} + 3\hat{j}$$

So, $x = 3$

**Question:** If momentum of body is increased by 50% find percentage increase in Kinetic Energy

**Options:**

(a) 80%

(b) 100%

(c) 50%

(d) 125%

**Answer: (d)**

**Question:** Statement 1: heat is given to a gas, its temp must increase
Statement 2: if positive work is done , volume must increase
**Options:**
(a) S1 - False, S2 - True
(b) S1 - True, S2 - True
(c) S1 - True, S2 - False
(d) S1 - False, S2 - False
**Answer: (a)**

**Question:** Two forces of magnitude A and $\frac{A}{2}$ act perpendicular to each other. The magnitude of the resultant force is equal to
**Options:**
(a) A/2
(b) $\sqrt{5}A/2$
(c) $3A/2$
(d) $5A/2$
**Answer: (b)**
**Solution:** $\sqrt{A^2+\frac{A^2}{4}}=\frac{\sqrt{5}}{2}A$

**Question:** An antenna has height 98m find distance upto which signal can be transmitted (R = 6400 km)
Options:
(a) 3642 $km^2$
(b) 3942 $km^2$
(c) 11200 $km^2$
(d) 22400 $km^2$
**Answer: (b)**
**Solution:**

**Question:** Two projectiles of speed $u_1=40m/s$ and $u_2=60$ m/s placed at a angle of 30° and 60° respectively. Find the ratio of range.
**Answer:**
**Solution:**

$$R_1=\frac{40^2\sin 60°}{g}$$

$$R_2=\frac{60^2\sin 120°}{g}$$

$$\frac{R_1}{R_2}=\left(\frac{40}{60}\right)^2=\frac{4}{9}$$

**Question:** An air bubble having volume 1 $cm^3$ at depth 40 m inside water comes to surface. What will be the volume of the bubble at the surface.
**Options:**

(a) 5 $cm^3$
(b) 2 $cm^3$
(c) 4 $cm^3$
(d) 3 $cm^3$
**Answer: (a)**
**Solution:**
Assuming isothermal condition,

$$P_1V_1 = P_2V_2$$

$$(P_0 + \rho gh)V_0 = P_0V_f$$

$$\left[10^5 + 10^3 \times 10 \times 40\right] 1\text{cm}^3 = 10^5 V_f$$

$$5 \times 10^5 \times 1\text{cm}^3 = 10^5 V_f$$

$$V_f = 5\text{cm}^3$$

**Question:** For an electron and a proton ($m_p$ = 1847 $m_e$) with same de-Broglie wavelength, the ratio of linear momentum is equal to:
**Options:**
(a) 1 : 2
(b) 2 : 1847
(c) 1 : 1
(d) $\sqrt{1847}:1$
**Answer: (a)**

**Question:** If Y = $7 \times 10^{11}$ $Nm^{-1}$ & $\Delta L/L = 0.04$. Find energy density.
**Solution:**

$$\left(\frac{U}{V}\right) = \frac{1}{2} Y \varepsilon^2$$

$$= \frac{1}{2} \times 7 \times 10^{11} \times (0.04)^2$$

$$= \frac{16 \times 10^{-4} \times 7 \times 10^{11}}{2} = 8 \times 7 \times 10^{-7}$$

$$= 5.6 \times 10^{-6} \left(\text{J/m}^3\right)$$

**Question:** The height of antenna is 98 m. The radius of Earth is 6400 km. The area up to which it will transmit signal is:
**Options:**
(a) 3642 $km^2$
(b) 3942 $km^2$
(c) 11200 $km^2$
(d) 22400 $km^2$
**Answer: (b)**
**Solution:**
Area

$= \pi(\text{ Range })^2$

$= \pi\left[\sqrt{2hR_E}\right]^2$

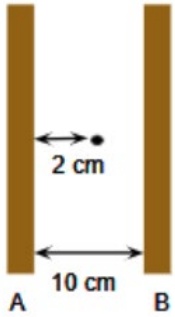
$= 2\pi hR_E$

$= 2 \times 3.14 \times 98 \times 6400 \times 10^3$

$\approx 3940 \times 10^6 \left(\text{m}^2\right)$

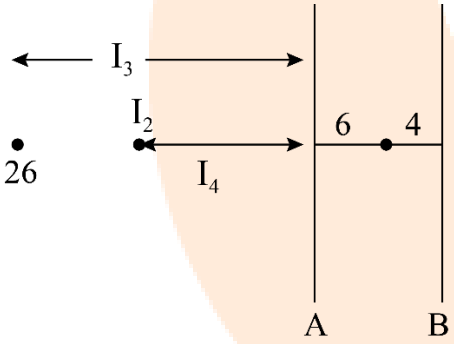
or $3942\left(\text{km}^2\right)$

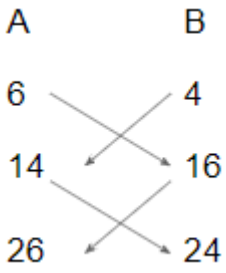
**Question:** In the given diagram, find the distance between 2nd and 3rd image formed left of mirror A.




**Answer: 12.00**

**Solution:**




Gap = 26 – 14 = 12

**Question:** If mass radius of cross-section and height of a cylinder are (0.4 + 0.01) g, (6 + 0.03) m and (8 + 0.04) m. The maximum percentage of error in the measurement of density of cylinder is:

**Options:**

(a) 1%

(b) 4%

(c) 8%
(d) 7%
**Answer: (b)**
**Solution:**

$$\rho = \frac{M}{\pi r^2} \cdot h$$

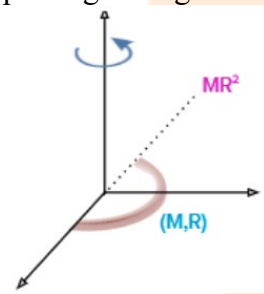
$$\Rightarrow \frac{\Delta\rho}{\rho} \times 100 = \frac{\Delta M}{M} \times 100 + 2\frac{\Delta r}{r} \times 100 + \frac{\Delta h}{h} \times 100$$

$$= \frac{0.01}{0.4} \times 100 + 2 \times \frac{0.03}{6} \times 100 + \frac{0.04}{8} \times 100$$

$$= 2.5 + 1 + 0.5 = 4\%$$

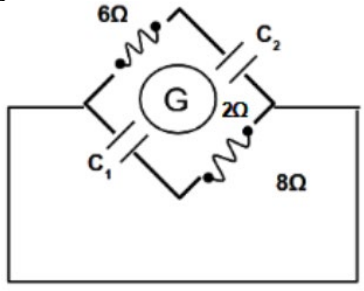
**Question:** If velocity of charged particle has the component both in and perpendicular to the direction of magnetic field then the path traced by the charged particle will be
**Options:**
(a) Circular
(b) Straight line
(c) Cycloid
(d) Helical
**Answer: (d)**

**Question:** The moment of inertia of semi-circular ring of mass m and radius R about an axis passing through centre and perpendicular to the plane of ring is



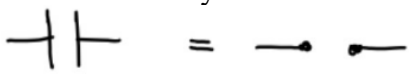

**Options:**
(a) $MR^2$
(b) 1/2 $MR^2$
(c) $2MR^2$
(d) 3/4 $MR^2$
**Answer: (a)**

**Question:** What is the ratio of potential difference across $C_1$ and $C_2$ at steady state for the given circuit




**Options:**

(a) 4 : 5
(b) 2 : 5
(c) 1 : 4
(d) 3 : 1
**Answer: (a)**
**Solution:** Steady State



So, $i = \frac{4}{6+2+8} = \frac{1}{4}$

So $V_B = 4 - iR$

$= 4 - \frac{1}{4} \times 6$

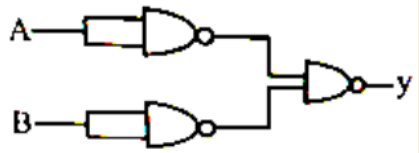
$V_B = 2.5$

$V_C = V_B - iR$

$= 2.5 - \frac{1}{4} \times 2 = 2$

$\frac{V_{AC}}{V_{BD}} = \frac{4-2}{2.5-0} = \frac{2}{\frac{5}{2}} = \frac{4}{5}$

**Question:** Which gate is this?




**Options:**
(a) AND
(b) OR
(c) NOT
(d) NOR
**Answer: (b)**

**Question:** In an LC oscillating circuit with L = 75 mH and C = 30 μF, the maximum charge of capacitor is $2.7 \times 10^{-4}$ C. Maximum current through the circuit will be
**Options:**
(a) 0.18 Amp
(b) 0.24 Amp
(c) 0.72 Amp
(d) 0.92 Amp
**Answer:** (a)
**Solution:**

$i_{max} = \frac{Q_{max}}{\sqrt{LC}} = \frac{2.7 \times 10^{-4} \text{C}}{\sqrt{75 \times 10^{-3} \times 30 \times 10^{-6}}}$

$= \frac{2.7 \times 10^{-4}}{15 \times 10^{-4}} = 0.18$

# JEE-Mains-08-04-2023 [Memory Based] [Morning Shift]

## Chemistry

**Question: The correct order of electronegativity B, S, C and At**
**Options:**
(a) B > C > S > At
(b) S > C > B > At
(c) C > B > S > At
(d) S > C > At > B
**Answer: (d)**
**Solution: Fact based**

**Question: Match the following:**

| Parameter | Maximum Prescribed conc. |
|---|---|
| (P)Fluorine | (1) 50 ppb |
| (Q)Lead | (2) 0.2 ppm |
| (R)Fe | (3) 1 ppm |
| (S)Nitrate | (4) 50 ppm |

**Options:**
(a) P-3, Q-1, R-2, S-4
(b) P-3, Q-2, R-1, S-4
(c) P-2, Q-1, R-3, S-4
(d) P-1, Q-2, R-4, S-3
**Answer: (a)**
**Solution: Fact based**
**Maximum Prescribed Concentration of Some Metals in Drinking Water.**

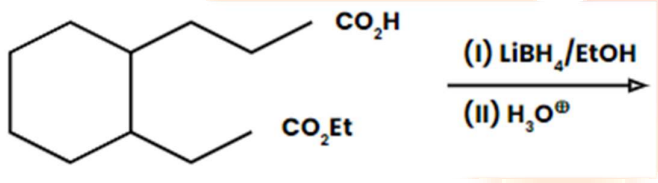
| Metal | Maximum concentration (ppm or mg $dm^{-3}$) |
|---|---|
| Fe | 0.2 |
| Mn | 0.05 |
| Al | 0.2 |
| Cu | 3.0 |
| Zn | 5.0 |
| Cd | 0.005 |

**Question: Assertion: Butanol has highest boiling point than ethoxyethane.**
**Reason: Because butanol has more hydrogen bonding.**
**Options:**
(a) Both assertion and reason are correct and reason is correct explanation
(b) Both assertion and reason are correct and reason is incorrect explanation
(c) Both assertion and reason are incorrect
(d) Assertion is correct but reason is incorrect
**Answer: (a)**
**Solution: fact based**

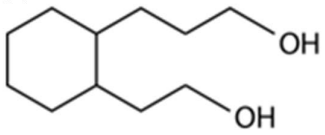
**Question: Which element is purified by leaching**
**Options:**
(a) Pb
(b) Sn
(c) Cu
(d) Au
**Answer: (d)**
**Solution:** In the metallurgy of silver and gold, the respective metal is leached with a dilute solution of NaCN or KCN in the presence of air, which supplies $O_2$. The metla is obtained later by replacement reaction.
**$4M(s) + 8CN^-(aq) + 2H_2O(aq) + O_2(g) \rightarrow 4[M(CN)_2]^-(aq) + 4OH^-(aq)$ (M = Ag or Au)**
**$2[M(CN)_2]^-(aq) + Zn(s) \rightarrow [Zn(CN)_4]^{2-}(aq) + 2M(s)$**

**Question: $2Cu^{+2} + 4X_2 \rightarrow Cu_2X_2 + X_2$**
**What is X here?**
**Options:**
(a) Iodine
(b) Bromine
(c) Chlorine
(d) All of above
**Answer: (a)**
**Solution: $2Cu^{+2} + 4X_2 \rightarrow Cu_2X_2 + X_2$ and the same applies to CuX. On the other hand, all Cu(II) halides are known except the iodide. In this case, Cu2+ oxidises I– to I2:**
**$2Cu^{2+} + 4I^- \rightarrow Cu_2I_2(s) + I_2$**
**However, many copper (I) compounds are unstable in aqueous solution and undergo disproportionation.**
**$2Cu^+ \rightarrow Cu^{2+} + Cu$**
**The stability of $Cu^{2+}(aq)$ rather than $Cu^+(aq)$ is due to the much more negative $\Delta_{hyd}H^{\ominus}$ of $Cu^{2+}(aq)$ than $Cu^+$, which more than compensates for the second ionization enthalpy of Cu.**

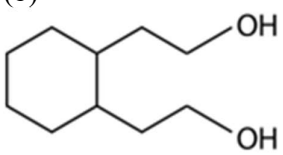
**Question:**



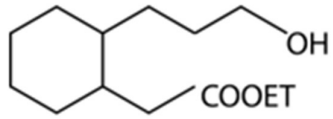

**Options:**
(a)



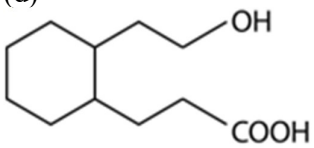

(b)



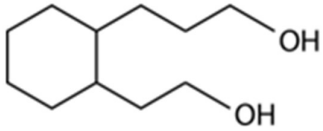

(c)




(d)




**Answer: (a)**
**Solution:**




**Question: The chromium in chromyl chloride has the same oxidation state as which of the following**
**Options:**
(a) $Fe^{3+}$
(b) $V^{4+}$
(c) $Ti^{+2}$
(d) $Mn^{+6}$
**Answer: (d)**
**Solution: Chromyl chloride reaction is not redox reaction**

**Question: Match the following:**

| Column I | Column II |
|---|---|
| (P) Saccharine | (1) Sweetest sugar |
| (Q) Alitame | (2) Unstable on cooking |
| (R) Aspartame | (3) Stable at cooking temperature |
| (S) Sucrolose | (4) First artificial sweetner |

**Options:**
(a) P – 4; Q – 1; R – 2; S – 3
(b) P – 4; Q – 3; R – 2; S – 1
(c) P – 1; Q – 2; R – 3; S – 4
(d) P – 2; Q – 3; R – 4; S – 1
**Answer: (a)**
**Solution: Some other commonly marketed artificial sweeteners are given in table.**

| | |
|---|---|
| Saccharin | First Artificial Sweetener |
| Alitame | More stable than aspartame |
| Aspartame | Unstable at cooking temperature |
| Sucralose | Stable at Cooking temperature |

**Question: How many statements are correct ?**
**Statement-1 : $Li^+$ has higher polarizing power than $Mg^{2+}$**
**Statement-2: $Mg^{2+}$ ion is smaller than than $Li^{2+}$ ion.**
**Options:**
(a) statement 1 is correct and statement 2 is incorrect

(b) statement 1 is incorrect and statement 2 is correct
(c) statement 1 is incorrect and statement 2 is also incorrect
(d) statement 1 is correct and statement 2 is also correct
**Answer: (b)**
**Solution: Mg has more polarizing power than Li,**

**Question: Which of the following contain Sulphur**
**Lysine, Methionine, glutamic acid , threonine, Arginine, Cystine, tyrosine, isoleucine**
**Options:**
(a) 1
(b) 2
(c) 3
(d) 4
**Answer: (b)**
**Solution: Methionine and Cystine**

**Question: What is formed when water gas reacts in presence of Cobalt as catalyst**
**Options:**
(a) Ethanol
(b) Methanal
(c) Methanoic acid
(d) Ethanoic acid
**Answer: (b)**
**Solution: fact based**

**Question: How many of the following factors results in covalent character in a compound?**
**Polarising power of cation.**
**Polarisibility of anion**
**Electron cloud distortion of anion**
**Electron cloud distortion of Cation**
**Options:**
(a) 3
(b) 4
(c) 2
(d) 1
**Answer: (a)**
**Solution: Polarisibility of anion causes cloud distortion and polarizing power of cation causes covalent nature. FAJAN RULE**

**Question: Which Cell representation is correct for**
**$H_2 + 2AgCl \rightarrow 2H^+ + 2Ag + 2Cl^-$**
**Options:**
(a) Pt $|H_2|$ HCl | AgCl | Ag
(b) Pt | $H_2$ | HCl | AgCl | Pt
(c) Ag | AgCl | HCl| $H_2$| Pt
(d) Pt |AgCl| HCl| $H_2$| Pt
**Answer: (a)**
**Solution: Fact based**

**Question: Find the value of 'n' in the following redox reaction.**

$IO_3^- + H^+ + nI^- \rightarrow 6I_2 + H_2O$

**Options:**

(a) 10

(b) 12

(c) 9

(d) 5

**Answer: (a)**

**Solution:** $2IO_3^- + 12H^+ + 10I^- \rightarrow 6I_2 + 6H_2O$

**Question: For an electron and proton of same de-Broglie wavelength, the ratio of linear momentum is:**

**Options:**

(a) 1 : 2

(b) 2 : 1847

(c) 1 : 1

(d) $\sqrt{1847}$ :1

**Answer: (c)**

**Solution: calculation based** $\lambda = \frac{h}{p}$

**Question: How many statements are correct:**

**Options:**

(a) If there is no relation between rate constant and temperature, then activation energy is negative

(b) If the activation energy is zero, rate constant is temperature independent

(c) IF rate constant increases with increase of temperature, activation energy is positive

(d) IF rate constant decreases with increase in temperature, activation energy is negative.

**Answer: (c)**

**Solution: fact based**

**Question: Which of the following is most stable, diamagnetic and octahedral shaped:**

**Options:**

(a) $K_3\left[Co(CN)_6\right]$

(b) $\left[Co(H_2O)_6\right]Cl_3$

(c) $Na_3\left[CoF_6\right]$

(d) All have exact equal stability

**Answer: (a)**

**Solution: Hybridization is $d^2sp^3$**

**Question: For the ions: $[MnF_6]^{4-}$, $[Fe(CN)_6]^{3-}$ and $[Co(NH_3)_6]^{3+}$. The order of the spin magnetic moment is correct in which of the following option.**

**Options:**

(a) **$[MnF_6]^{4-} > [Co(NH_3)_6]^{3+} > [Fe(CN)_6]^{3-}$**

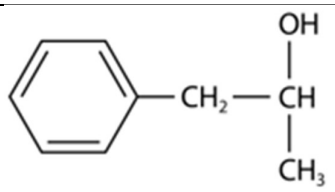
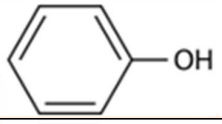
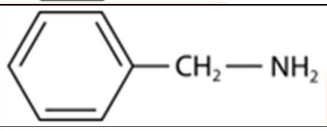
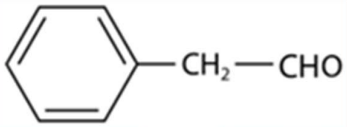
(b) **$[Fe(CN)_6]^{3-} > [MnF_6]^{4-} > [Co(NH_3)_6]^{3+}$**

(c) **$[MnF_6]^{4-} > [Fe(CN)_6]^{3-} > [Co(NH_3)_6]^{3+}$**

(d) **$[Co(NH_3)_6]^{3+} > [Fe(CN)_6]^{3-} > [MnF_6]^{4-}$**

**Answer: (c)**
**Solution:**

**Question: Match the column**

| Column I | | Column II | |
|---|---|---|---|
| **(a)** | **Neutral $FeCl_3$** | **(p)** |  |
| **(b)** | **Iodoform** | **(q)** |  |
| **(c)** | **Carbylamine test** | **(r)** |  |
| **(d)** | **[$CuSO_4$ + Sodim potassium tartarate (Rochecle's salt)]** | **(s)** |  |

**Options:**
(a) A – Q; B – R; C – P; D – S
(b) A – P; B – R; C – Q; D – S
(c) A – Q; B – P; C – R; D – S
(d) A – Q; B – P; C – S; D – R
**Answer: (c)**
**Solution:**

**Question: 0.5 g of an organic compound with 60% carbon will produce___g of $CO_2$ upon complete combustion?**
**Options:**
(a) 11 g
(b) 12 g
(c) 15 g
(d) 20 g
**Answer: (a)**
**Solution:** $0.5 \times 0.6 =$ **0.3 g of carbon**

**Moles of carbon** $= \frac{0.3}{12} = 0.025$

$\therefore CO_2 = 0.025$

$wt = 0.025 \times 44 = 11g$

# JEE-Mains-08-04-2023 [Memory Based] [Morning Shift]

## Mathematics

**Question:** Find the maximum value of $n$ such that $66!$ is divisible by $3^n$.
**Answer: 31.00**
**Solution:**

Exponent of 3 in 66!

$$\left[\frac{66}{3}\right]+\left[\frac{66}{3^2}\right]+\left[\frac{66}{3^3}\right]+\left[\frac{66}{3^4}\right]$$

$$\left[\frac{66}{3}\right]+\left[\frac{66}{3^2}\right]+\left[\frac{66}{3^3}\right]+0$$

$$=22+7+2$$

$$=31$$

**Question:** Let $A=\begin{bmatrix}2 & 1 & 0\\ 1 & 2 & -1\\ 0 & 1 & 2\end{bmatrix}$ and $\left|adj\left(adj\left(adj\,2A\right)\right)\right|=16^n$. Find $n$.

**Answer: 12.00**
**Solution:**

Given $A=\begin{bmatrix}2 & 1 & 0\\ 1 & 2 & -1\\ 0 & 1 & 2\end{bmatrix}$

$$|A|=2\times5-1\times2=8$$

$$\left|adj\left(adj\left(adj\,2A\right)\right)\right|=\left|2A\right|^{2^3}=\left|2A\right|^8$$

$$=\left|2^3|A|\right|^8=\left(8^2\right)^8=8^{16}$$

$$=2^{48}=\left(2^4\right)^{12}=16^{12}$$

$$\Rightarrow n=12$$

**Question:** $x, y, 12, 12, 10, 4, 8, 6$; Variance $= 9.25$ & mean $= 9$. If $x > y$, then find $3x - 2y$.

**Answer:** 25.00

**Solution:**

$$\frac{x + y + 52}{8} = 9$$

$$x + y = 20$$

$$x - 9, y - 9, 3, 3, 1, -5, -1, -3$$

$$\bar{x} = 0$$

Now, Variance, $\frac{(x-9)^2 + (y-9)^2 + 54}{8} - 0^2 = 9.25$

$$(x-9)^2 + (y-9)^2 = 20$$

$$(x-9)^2 + (11-x)^2 = 20$$

$$x = 7, 13$$

$$y = 13, 7$$

$$3x - 2y = 3 \times 13 - 2 \times 7 = 25$$

**Question:** Find the fourth term in the expansion of $(1+x)^n$, if the coefficient of three consecutive terms are in the ratio $1:5:20$.

**Answer:** ${}^{29}C_3 x^3$

**Solution:**

$${}^nC_{r-1} = 1,\ {}^nC_r = 5,\ {}^nC_{r+1} = 20$$

$$\frac{{}^nC_r}{{}^nC_{r-1}} = 5 \Rightarrow \frac{n-r+1}{r} = 5 \Rightarrow n = 6r - 1$$

$$\frac{{}^nC_{r+1}}{{}^nC_r} = 4 \Rightarrow \frac{n-(r+1)+1}{r+1} = 4 \Rightarrow n = 5r + 4$$

$$\Rightarrow 6r - 1 = 5r + 4$$

$$\Rightarrow r = 5$$

$$\Rightarrow n = 29$$

$$t_4 = {}^nC_3 x^{n-3} = {}^{29}C_3 x^3$$

**Question:** $P=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}, Q=PAP^T, A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$, then $P^TQ^{2007}P=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$. Find $2a+b+3c-4d$.

**Answer: 2005.00**

**Solution:**

Given, $P=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$

$$PP'=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}\begin{bmatrix}\frac{\sqrt{3}}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}=I$$

$\Rightarrow P'=P^{-1}$

$Q=PAP^{-1}$

$\Rightarrow Q^{2007}=PA^{2007}P^{-1}$

$P'Q^{2007}P=P^{-1}PA^{2007}P^{-1}P=A^{2007}$

Now, $A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}=I+B$

$$B^2=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$\Rightarrow B^n=0\,\forall\, n\geq 2$

So, $A^{2007}=(I+B)^{2007}=I+2007B$

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2007\\ 0 & 0\end{bmatrix}=\begin{bmatrix}1 & 2007\\ 0 & 1\end{bmatrix}$$

$2a+b+3c-4d=2+2007+0-4=2005$

**Question:** $f(x)=\frac{\sin x+\cos-\sqrt{2}}{\sin x-\cos x}$. Find $f\left(\frac{\pi}{12}\right)+f''\left(\frac{\pi}{12}\right)$.

**Answer:** $30-17\sqrt{3}$

**Solution:**

$$f(x)=\frac{\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right)-\sqrt{2}}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x\right)}$$

$$f(x)=\frac{-\sqrt{2}\left[1-\cos\left(x-\frac{\pi}{4}\right)\right]}{\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)}$$

$$f(x)=-\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)$$

$$f'(x)=-\frac{1}{2}\sec^2\left(\frac{x}{2}-\frac{\pi}{8}\right)$$

$$f''(x)=\frac{-1}{4}\times 2\sec^2\left(\frac{x}{2}-\frac{\pi}{8}\right)\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)$$

$$f''\left(\frac{\pi}{12}\right)=\frac{1}{2}\left(1+\tan^2\frac{\pi}{12}\right)\tan\frac{\pi}{12}$$

$$=\frac{1}{2}\left(1+\left(2-\sqrt{3}\right)^2\right)\left(2-\sqrt{3}\right)$$

$$=\frac{1}{2}\left(1+4+3-4\sqrt{3}\right)\left(2-\sqrt{3}\right)$$

$$=\frac{1}{2}\left(8-4\sqrt{3}\right)\left(2-\sqrt{3}\right)$$

$$=\left(4-2\sqrt{3}\right)\left(2-\sqrt{3}\right)$$

$$=4\left(2-\sqrt{3}\right)^2$$

$$=4\left(4+3-4\sqrt{3}\right)$$

$$=28-16\sqrt{3}$$

$$f\left(\frac{\pi}{12}\right)+f''\left(\frac{\pi}{12}\right)=2-\sqrt{3}+\left(28-16\sqrt{3}\right)$$

$$=2-\sqrt{3}+28-16\sqrt{3}$$

$$=30-17\sqrt{3}$$

**Question:** If all the letters of the word INDEPENDENCE are arranged randomly, the find the probability that all vowels are together.

**Answer:** $\frac{1}{99}$

**Solution:**

Given word INDEPENDENCE

I E E E E

N D P N D N C

I-1 E-4 N-3 D-2 P-1 C-1

$$P=\frac{\frac{8!}{3!2!}\times\frac{5!}{4!}}{\frac{12!}{4!3!2!}}$$

$$=\frac{120}{12\times11\times10\times9}$$

$$=\frac{1}{99}$$

**Question:** How many subsets of $A\times B$ are possible such that it has no more than 6 elements and atleast 3 elements if 'A' has 5 elements 'B' has 2 elements?

**Answer: 792.00**

**Solution:**

$A\to5$

$B\to2$

$n(A\times B)=10$

$^{10}C_3+{}^{10}C_4+{}^{10}C_5+{}^{10}C_6=792$

**Question:** If $\left(e,\frac{4}{3}\right)$ & $\left(e^4,\alpha\right)$ satisfies the differential equation $\frac{dy}{dx}+\frac{y}{2x\ln x}=\frac{1}{x}$, then find $\alpha$.

**Answer: 3.00**

**Solution:**

$$\frac{dy}{dx}+\frac{y}{2x\ln x}=\frac{1}{x}$$

$$\text{IF}=e^{\frac{1}{x}\int\frac{1}{x\ln x}}$$

$$=e^{\frac{1}{2}\ln(\ln x)}$$

$$=\sqrt{\ln x}$$

$$y\times\left(\sqrt{\ln x}\right)=\int\frac{\sqrt{\ln x}}{x}=\frac{2\left(\ln x\right)^{\frac{3}{2}}}{3}+C$$

$$\frac{4}{3}=\frac{2}{3}+C\Rightarrow C=\frac{2}{3}$$

$$\left(e^4,\alpha\right)\Rightarrow\alpha\times2=\frac{2}{3}\times8+\frac{2}{3}$$

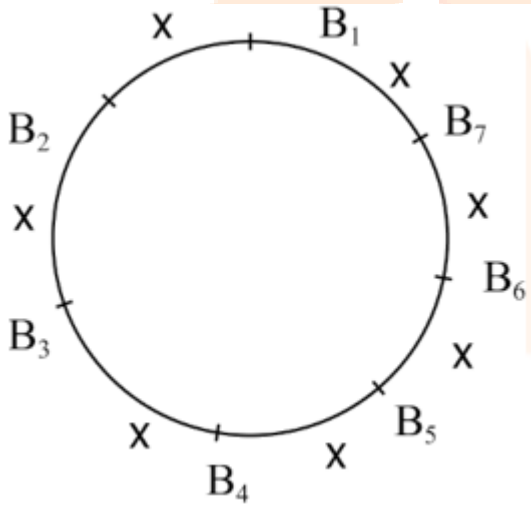
$$2\alpha=6$$

$$\Rightarrow\alpha=3$$

**Question:** How many ways are there to arrange 5 girls and 7 boys on a circular table such that no two girls are together?

**Answer:** $6!\times{}^{7}C_{5}\times5!$

**Solution:**




Arrangement of boys $=6!$

Arrangement of girls $={}^{7}C_{5}\times5!$

The number of ways $=6!\times{}^{7}C_{5}\times5!$

**Question:** If $t_n=\frac{n^3}{n^4+147};n\in N$ . Find maximum value of $t_n$ .

**Answer:** $\frac{125}{772}$

**Solution:**

$$f\left(x\right)=\frac{x^3}{x^4+147}$$

$$f'(x)=\frac{\left(x^4+147\right)\times 3x^2-x^3\left(4x^3\right)}{\left(x^4+147\right)^2}$$

$$=\frac{x^2\left[3x^4+441-4x^4\right]}{\left(x^4+147\right)^2}$$

$$=\frac{x^2\left((21)^2-x^4\right)}{\left(x^4+147\right)^2}$$

Maximum at $x=\sqrt{21}$

Ans is $x=4$ or 5

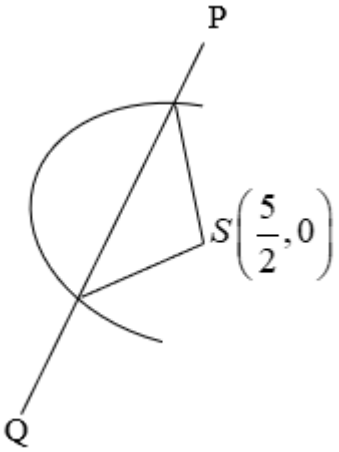
$$f(4)<f(5)$$

$$f(5)=\frac{125}{772}$$

**Question:** Line $y=mx+c$ intersects $y^2=bx$ at P and Q. Focus of $y^2=10x$ is at S. Centroid of $\Delta PQS$ is $(10,10)$. If $c-m=6$, then the value of $|PQ|^2=?$

**Answer: 1400.00**

**Solution:**




$$y^2=10\left(\frac{y-c}{m}\right)$$

$$y^2-\frac{10}{m}y+\frac{10c}{m}=0$$

$$\frac{\frac{10}{m}+0}{3}=10$$

$m = \frac{1}{3}$

$c = \frac{19}{3}$

So $y^2 - 30y + 190 = 0$

$|y_2 - y_1| = \sqrt{140}$

$|PQ|^2 = (y_2 - y_1)^2 \left(1 + \frac{1}{m^2}\right)$

$= 140 \times 10 = 1400$

**Question:** If $S_k = \frac{1 + 2 + .... + k}{k}$, then $\sum_{k=1}^{n} (S_k)^2 = ?$

**Answer: ()**
**Solution:**

$S_k = \frac{1 + 2 + .... + k}{k}$

$\sum_{k=1}^{n} (S_k)^2 = \sum_{k=1}^{n} \left(\frac{k+1}{2}\right)^2 = \frac{1}{4}\left[\frac{(n+1)(n+2)(2n+3)}{6} - 1\right]$

**Question:** $\lim_{x \to 0} \frac{1 - \cos^2 3x}{\cos^3 4x} \times \frac{\sin^3 4x}{\left[\ln(1+2x)\right]^5} =$

**Answer: 18.00**
**Solution:**

$\lim_{x \to 0} \frac{1 - \cos^2 3x}{\cos^3 4x} \times \frac{\sin^3 4x}{\left[\ln(1+2x)\right]^5}$

$= \lim_{x \to 0} \frac{1 - \cos^2 3x}{9x^2} \times \frac{\sin^3 4x}{64x^3} \times \frac{32x^5}{\left[\ln(1+2x)\right]^5} \times \frac{18}{\cos^3 4x}$

$= 2 \times \frac{1}{2} \times 18$

$= 18$

**Question:** Find the coefficient of $x^0$ in the expansion of $\left(3x^2-\frac{1}{2x^5}\right)^7$.

**Answer: ()**
**Solution:**

Coefficient $={}^7C_2\left(3x^2\right)^5\times\left(-\frac{1}{2x^5}\right)^2$

$=\frac{{}^7C_2\times 3^5}{2^2}$

$=\frac{5103}{4}$

**Question:** A bolt manufacturing factory has three products A, B & C. 50% and 30% of the product are A and B type respectively and remaining are C type. Then probability that the product A is defective is 4%, that of B is 3% and that of C is 2%. A product is picked randomly and found to be defective, then the probability that it is of type C is:

**Answer:** $\frac{4}{33}$

**Solution:**

$P(A)=0.5$

$P(B)=0.3$

$P(C)=0.2$

$P\left(\frac{D}{A}\right)=0.04$

$P\left(\frac{D}{A}\right)=0.03$

$P\left(\frac{D}{C}\right)=0.02$

$$\left(\frac{C}{D}\right)=\frac{P(C)\times P\left(\frac{D}{C}\right)}{\sum P(A)\times P\left(\frac{D}{A}\right)}$$

Substituting values we get

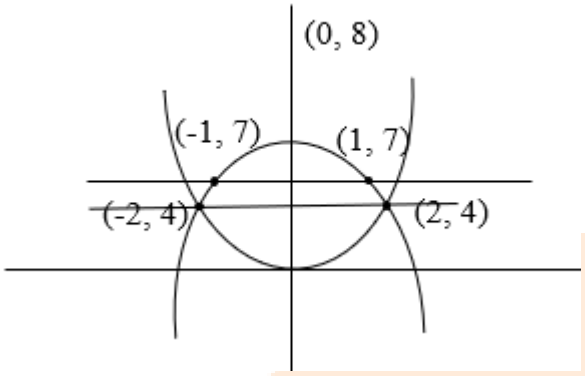
$\frac{4}{33}$

**Question:** The area under the curves $x^2\le y, y\le 8-x^2$ and $y\le 7$ is _____.

**Answer: 20.00**
**Solution:**

$x^2 = y$

$x^2 = -(y-8)$




$\frac{64}{6} + \frac{64}{6} - \frac{8}{6} = 20$

**Question:** $\int \frac{x+1}{x\left(1+xe^x\right)^2} dx$

**Answer:**

**Solution:**

$I = \int \frac{x+1}{x\left(1+xe^x\right)^2} dx$

$\frac{d}{dx}\left(xe^x\right) = e^x\left(x+1\right)$

$\left(1+xe^x\right) = t$

$e^x\left(1+x\right)dx = dt$

$I = \int \frac{dt}{(t-1)t^2}$

$= \int \frac{dt}{t^2(t-1)}$

$$\left[\frac{1}{t}\times\left(\frac{1}{t(t-1)}\right)\right]=\frac{1}{t}\left(\frac{1}{t-1}-\frac{1}{t}\right)$$

$$=\frac{1}{t(t-1)}-\frac{1}{t^2}$$

$$=\frac{1}{t-1}-\frac{1}{t}-\frac{1}{t^2}$$

$$I=\ln|t-1|-\ln|t|+\frac{1}{t}+C$$

$$=\ln|xe^x|-\ln|1+xe^x|+\frac{1}{1+xe^x}+C$$

$$I(x)=\ln\left|\frac{xe^x}{1+xe^x}\right|+\frac{1}{1+xe^x}+1$$

$$\therefore\ I(1)=\ln\left|\frac{e}{1+e}\right|+\frac{1}{1+e}+1$$

$$=2-\ln(1+e)+\frac{1}{1+e}$$

**Question:** $\sim((q\to p)\to(p\to q))$ is equivalent to

**Options:**

(a)

(b)

(c)

(d)

**Answer: ()**

**Solution:**

$$\equiv\sim(\sim(q\to p)\vee(p\to q))$$

$$\equiv(q\to p)\wedge(p\to q)$$

$$\equiv(\sim q\vee p)\wedge\sim(\sim p\vee q)$$

$$\equiv(\sim q\vee p)\wedge(p\wedge\sim q)$$

$$\equiv p\wedge\sim q$$

**Question:** If $\alpha,\beta,\gamma$ are the roots of the equation $x^3+cx+d=0$ and $\gamma\beta=1=-\alpha$, then find relation which $b$ satisfies.

**Answer:** $b = 0$

**Solution:**

$\alpha = -1,\ \alpha\beta = 1$

$\alpha + \beta + \gamma = 0$

$\beta + \alpha = 1$

$x^2 - x + 1 = 0$

Now, $S_2 = b$

$-\beta + \beta\gamma + (-\alpha) = b$

$b = -1 + 1$

$b = 0$