MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. The absolute difference of the coefficient of x^7 and x^9 in the expansion of $\left(2x+\frac{1}{2x}\right)^{11}$ is

(1) 11×2^5 (2) 11×2^7

(3) 11×2^4 (4) 11×2^3

Answer (2)

Sol. $T_{r+1} = {}^{11}C_r\,(2x)^{11-r}\left(\frac{1}{2x}\right)^r$

$= {}^{11}C_r\,\frac{2^{11-r}}{2^r}\,x^{11-2r}$

$11 - 2r = 7$ and $11 - 2r = 9$

$r = 2$ $r = 1$

$\therefore$ Coefficient of x^7 is ${}^{11}C_2\frac{(2)^9}{2^2} = {}^{11}C_2(2)^7$

Coefficient of x^9 is ${}^{11}C_1\frac{(2)^{10}}{2} = {}^{11}C_1(2)^9$

${}^{11}C_2(2)^7 - 11\times(2)^9$

$= 11 \times 2^7$

2. Let $f(x) = \{1, 2, 3, 4, 5, 6, 7\}$ the relation $R = \{(x, y) \in A \times A, x + y = 7\}$ is

(1) Symmetric

(2) Reflexive

(3) Transitive

(4) Equivalence

Answer (1)

Sol. $x + y = 7$

$y = 7 - x$

$R = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$

$\because$ $(a, b) \in R \Rightarrow (b, a) \in R$.

$\therefore$ Relation is symmetric

3. The number of words with or without meaning can be formed from the word MATHEMATICS where C, S not come together is

(1) $\frac{9}{8}\times 10!$ (2) $\frac{1}{8}\times 10!$

(3) $\frac{5}{8}\times 10!$ (4) $\frac{1}{2}\times 10!$

Answer (1)

Sol. Total words = $\frac{11!}{2!2!2!}$

When C and S are together = $\frac{10!}{2!2!2!}\times 2!$

$\therefore$ Required number of words = $\frac{11!}{(2!)^3} - \frac{10!}{(2!)^3}\times 2!$

$= \frac{10!}{8}[11-2]$

$= \frac{9}{8}\times 10!$

4. Let $a_n = 5 + 8 + 14 + 23 + \ldots$ upto n terms. If $S_n = \sum_{k=1}^{n} a_k$, then $S_{30} - a_{40}$ is equal to

(1) 78025

(2) 12800

(3) 11600

(4) 12100

Answer (1)

Sol. $a_n = 5 + 8 + 14 + \ldots T_n$

$$\begin{array}{l} a_n = 5+8+14\ldots+T_{n-1}+T_n \\ \hline 0 = 5+\underbrace{3+6+9+\ldots}_{(n-1)\text{ terms}}-T_n \end{array}$$

$\Rightarrow$ $T_n = 5+\left(\frac{n-1}{2}\right)\left(6+(n-2)3\right) = 5+\frac{3}{2}(n-1)^n$

$5+\frac{3}{2}n^2-\frac{3}{2}n$

$\Rightarrow$ $\frac{1}{2}\left(10+3n^2-3n\right)$

$\therefore$ $T_n = \frac{1}{2}\left(3n^2-3n+10\right)$

$$a_n = \sum T_n = \frac{1}{2}\left[\frac{3\cdot(n)(n+1)(2n+1)}{6} - \frac{3\cdot(n)(n+1)}{2} + 10n\right]$$

$$= \frac{1}{2}(n)\left(\frac{(n+1)(2n+1)}{2} - \frac{3(n+1)}{2} + 100\right)$$

$$a_n = \frac{n}{4}\left(2n^2 + 3n + 1 - 3n - 3 + 20\right)$$

$$= \frac{n}{4}\left(2n^2 + 18\right) = \frac{n}{4}\left(n^2 + 9\right)$$

$$a_{40} = \frac{40}{2}(1600 + 9) = 1609 \times 20 = 32180$$

$$S_n = \sum a_n = \frac{1}{2}\left(\left(\frac{(n)(n+1)}{2}\right)^2 + \frac{9\cdot(n)(n+1)}{2}\right)$$

$$S_{30} = \frac{1}{2}\left(\left(\frac{30\times 3}{2}\right)^2 + \frac{9}{2}(30)(31)\right)$$

$$= \frac{1}{2}(216225 + 4185)$$

$= 110205$

$S_{30} - a_{40} = 78025$

5. The equation $ax^2 + bx + c = 0$ has roots α and β. Then find $\lim\limits_{x \to \frac{1}{\alpha}} \frac{1-\cos(cx^2 + bx + a)}{2(1-\alpha x)^2}$ is

(1) $\frac{c^2(\alpha-\beta)^2}{4\alpha^4\beta^2}$ (2) $\frac{c^2(\alpha-\beta)^2}{\alpha^4\beta^2}$

(3) $\frac{c^2(\alpha-\beta)^2}{2\alpha^4\beta^2}$ (4) $\frac{c^2(\alpha-\beta)^2}{4\alpha^2\beta^2}$

Answer (1)

Sol. $\lim\limits_{x \to \frac{1}{\alpha}} \frac{2\sin^2\left(\frac{cx^2 + bx + a}{2}\right)}{2\alpha^2\left(x - \frac{1}{\alpha}\right)^2}$

$$= \frac{c^2(\alpha-\beta)^2}{4\alpha^2\beta^2}$$

6. $\theta \in (0, 2\pi)$ and $\frac{1+2i\sin\theta}{1-i\sin\theta}$ is purely imaginary then the value of θ is

(1) π (2) 0

(3) 2π (4) $\frac{\pi}{4}$

Answer (4)

Sol. Real part has to be zero

$$\Rightarrow \frac{1-2\sin^2\theta}{1+\sin^2\theta} = 0$$

$$\sin^2\theta = \frac{1}{2}$$

$$\theta = n\pi \pm \frac{\pi}{4}, n \in I$$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{4}$$

7. The statement $(p \wedge (\sim q)) \vee (\sim p)$ is equivalent to

(1) $p \wedge q$ (2) $\sim p \vee \sim q$

(3) $p \vee q$ (4) $\sim p \wedge \sim q$

Answer (2)

Sol. $(p \wedge (\sim q)) \vee (\sim p)$

$= (p \vee \sim p) \wedge (\sim q \vee \sim p)$

$= T \wedge (\sim q \vee \sim p)$

$= \sim q \vee \sim p$

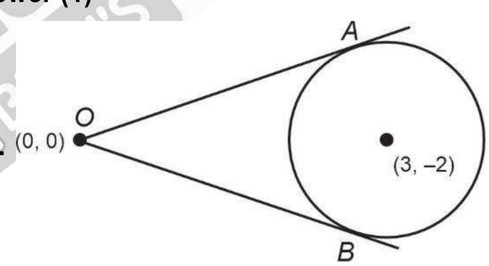
8. From $O(0, 0)$, two tangents OA and OB are drawn to a circle $x^2 + y^2 - 6x + 4y + 8 = 0$, then the equation of circumcircle of ΔOAB.

(1) $x^2 + y^2 - 3x + 2y = 0$ (2) $x^2 + y^2 + 3x - 2y = 0$

(3) $x^2 + y^2 + 3x + 2y = 0$ (4) $x^2 + y^2 - 3x - 2y = 0$

Answer (1)

Sol.



(0, 0) and (3, –2) are diametric end points

$\therefore$ $(x - 0)(x - 3) + (y - 0)(y + 2) = 0$

$$\boxed{x^2 + y^2 - 3x + 2y = 0}$$

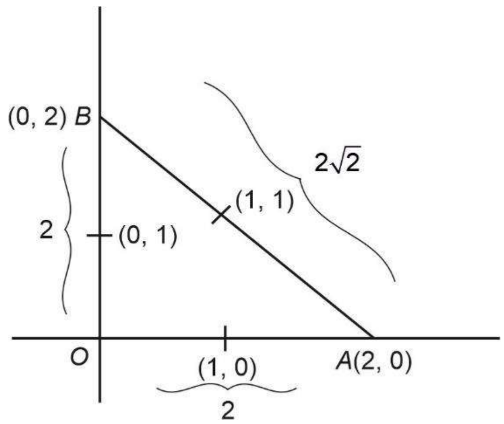
9. The mid points of side of a triangle are (0, 1), (1, 0), (1, 1), where incentre is D. A parabola $y^2 = 4ax$ passes through D whose focus is $\left(\alpha + \beta\sqrt{2}, 0\right)$ then $\frac{\beta^2}{\alpha}$ is

(1) $\frac{1}{2}$ (2) 2

(3) $\frac{1}{8}$ (4) 4

Answer (3)

Sol.



∵ Mid-point is (0, 1), (1, 0) and (1, 1)

$$I = \left(\frac{4}{4+2\sqrt{2}}, \frac{4}{4+2\sqrt{2}}\right)$$

$$y^2 = 4ax$$

∵ $y^2 = 4ax$ passes through I

$$\left(\frac{4}{4+2\sqrt{2}}\right)^2 = 4a\left(\frac{4}{4+2\sqrt{2}}\right) \Rightarrow a = \frac{1}{4+2\sqrt{2}}$$

Focus = $(a, 0)$

$$= \left(\frac{1}{4+2\sqrt{2}}, 0\right)$$

$$= \left(\frac{4-2\sqrt{2}}{8}, 0\right)$$

$$\therefore \quad \alpha = \frac{4}{8} = \frac{1}{2}, \ \beta = \frac{-2}{8} = \frac{-1}{4}$$

$$\frac{\beta^2}{\alpha} = \frac{1}{8}$$

10. Let $R = \{a, b, c, d, e\}$ and $S = \{1, 2, 3, 4\}$. Then number of onto functions $f(x) : R \to S$ such that $f(a) \neq 1$ is

(1) 240 (2) 180

(3) 204 (4) 216

Answer (2)

Sol. Total number of onto functions

$$= \frac{5!}{3!2!} \times 4!$$

Now, when $f(a) = 1$

$$\frac{4!}{2!2!} \times 3! + 4!$$

∴ Required functions = 240 – 36 – 24

= 180

11. A parabola with focus (3, 0) and directrix $x = -3$. Points P and Q lie on the parabola and their ordinates are in the ratio 3 : 1. The point of intersection of tangents drawn at points P and Q lies on the parabola

(1) $y^2 = 16x$ (2) $y^2 = 4x$

(3) $y^2 = 8x$ (4) $x^2 = 4y$

Answer (1)

Sol. Given parabola $y^2 = 12x$

$$P\left(3t_1^2, 6t_1\right), Q\left(3t_2^2, 6t_2\right)$$

$$\frac{t_1}{t_2} = 3 \Rightarrow t_1 = 3t_2 \quad \text{...(i)}$$

Let point of intersection be (h, k)

$h = 3t_1t_2$ ...(ii)

and $k = 3(t_1 + t_2)$ ...(iii)

(i) and (iii) $\Rightarrow t_2 = \frac{k}{12}$

(ii) $\Rightarrow h = 9t_2^2 = 9 \times \frac{k^2}{144} \Rightarrow k^2 = 16h$

$\Rightarrow y^2 = 16x$

12. In probability distribution for discrete variable $x = 0, 1, 2 \ldots$ $P(x = x) = k(x + 1).3^{-x}$. The probability of $P(x \geq 2)$ is equal to

(1) $\frac{5}{18}$ (2) $\frac{10}{18}$

(3) $\frac{20}{27}$ (4) $\frac{7}{27}$

Answer (4)

Sol. $\Sigma P = 1$

$\Rightarrow k(1 + 2.3^{-1} + 3.3^{-2} + \ldots) = 1$

$$\text{Let } S = 1 + \frac{2}{3} + \frac{3}{3^2} + \ldots$$

$$\frac{S}{3} = \frac{1}{3} + \frac{2}{3^2} + \ldots\ldots$$

$$\frac{2S}{3} = 1 + \frac{1}{3} + \frac{1}{3^2} + \ldots = \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$$

$$\Rightarrow S = \frac{9}{4}$$

$$\therefore \quad k \cdot \frac{9}{4} = 1 \Rightarrow k = \frac{4}{9}$$

Now $P(x \geq 2) = 1 - P(x = 0, 1)$

$$= 1 - \left(k + k.\frac{2}{3}\right)$$

$$= 1 - \frac{5k}{3}$$

$$= 1 - \frac{5}{3}.\frac{4}{9}$$

$$= \frac{7}{27}$$

13. If $f(x) = \begin{cases} 3x^2 + k\sqrt{x+1} & 0 < x < 1 \\ 3mx^2 + k^2 & x \geq 1 \end{cases}$ is differentiable at $x > 1$ then $\dfrac{8f'(8)}{f'\left(\frac{1}{8}\right)}$ is for $k \neq 0$

(1) 309 (2) 311

(3) 306 (4) 305

Answer (1)

Sol. $f(x) = \begin{cases} 3x^2 + k\sqrt{x+1} & 0 < x < 1 \\ 3mx^2 + k^2 & x \geq 1 \end{cases}$

$$3 + k\sqrt{2} = 3m + k^2 \quad \ldots\text{(i)}$$

$$f'(x) = \begin{cases} 6x + \dfrac{k}{2\sqrt{x+1}} & 0 < x < 1 \\ 6mx & x \geq 1 \end{cases}$$

$$6 + \frac{k}{2\sqrt{2}} = 6m \quad \ldots\text{(ii)}$$

$$3 + k\sqrt{2} = 3 + \frac{k}{4\sqrt{2}} + k^2$$

$$k = 0 \text{ or } \frac{7\sqrt{2}}{8}$$

If $k = 0$ | If $k = \dfrac{7\sqrt{2}}{8}$

$m = 1$ | $m = \dfrac{103}{96}$

(Rejected)

Now, $\dfrac{8f'(8)}{f'\left(\frac{1}{8}\right)} = \dfrac{48m}{\frac{6}{8} + \frac{k}{2\sqrt{\frac{9}{8}}}} = \dfrac{48m}{\frac{6}{8} + \frac{\sqrt{2}k}{3}}$

$$\frac{8f'(8)}{f'\left(\frac{1}{8}\right)} = 309$$

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. The area of quadrilateral having vertices as (1, 2), (5, 6), (7, 6), (–1, –6)

Answer (24)

Sol. Area $= \dfrac{1}{2}\begin{vmatrix} 1 & 2 \\ 5 & 6 \\ 7 & 6 \\ -1 & -6 \\ 1 & 2 \end{vmatrix}$

$$= \frac{1}{2}[6 + 30 - 42 - 2 - 10 - 42 + 6 + 6]$$

$$= \frac{1}{2}[48] = 24$$

22. The value of $\int_0^{2.4} [x^2]dx$ is $\alpha + \beta\sqrt{2} + \gamma\sqrt{3} + \delta\sqrt{5}$ then $(a + b + c + d + e)$ is equal to

Answer (06)

Sol. $\int_0^{2.4} [x^2]dx = \int_0^{1} 0\,dx + \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dx + \int_{\sqrt{3}}^{\sqrt{4}} 3\,dx + \int_{\sqrt{4}}^{\sqrt{5}} 4\,dx + \int_{\sqrt{5}}^{2.4} 5\,dx$

$$= \left(\sqrt{2} - 1\right) + 2\left(\sqrt{3} - \sqrt{2}\right) + 3\left(\sqrt{4} - \sqrt{3}\right) + 4\left(\sqrt{5} - \sqrt{4}\right) + 5\left(2.4 - \sqrt{5}\right)$$

$$= 9 - \sqrt{2} - \sqrt{3} - \sqrt{5}$$

$\therefore$ $\alpha = 9$, $\beta = -1$, $\gamma = -1$, $\delta = -1$

$\therefore$ $\alpha + \beta + \gamma + \delta = 6$

23. $\dfrac{dx}{dy} - \dfrac{3\sin y}{\cos y(\ln\cos y)}x = \dfrac{\sin y}{(\ln\cos y)^2\cos y}$ and $x\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2\ln 2}, x\left(\dfrac{\pi}{6}\right) = \dfrac{1}{\ln(m) - \ln(n)}$ then the value of mn is

Answer (12)

Sol. $I = e^{\int \frac{-3\sin y}{\cos y(\ln\cos y)}dy}$

Put $\ln(\cos y) = t$

$$\frac{-1}{\cos y}\sin y\ dy = dt$$

$$= e^{\int \frac{3}{t}dt}$$

$$= (\ln\cos y)^3$$

$$x(\ln\cos y)^3 = \int \frac{\sin y}{\cos y}\ln\cos y\ dy$$

$$x(\ln\cos y)^3 = \frac{-(\ln(\cos y))^2}{2} + C$$

$$x\left(\frac{\pi}{3}\right) = \frac{1}{2\ln 2}$$

$\Rightarrow$ $C = 0$

$\therefore$ $x = -\dfrac{1}{2\ln(\cos y)}$

$$x\left(\frac{\pi}{6}\right) = \frac{1}{\ln 4 - \ln 3}$$

$m = 4$

$n = 3$

24. If m is the number of solution of $x^2 - 12x + 31 + [x] = 0$ and n be the number of solution of $x^2 - 5|x+2| - 4 = 0$, then the value of $m^2 + mn + n^2$ is

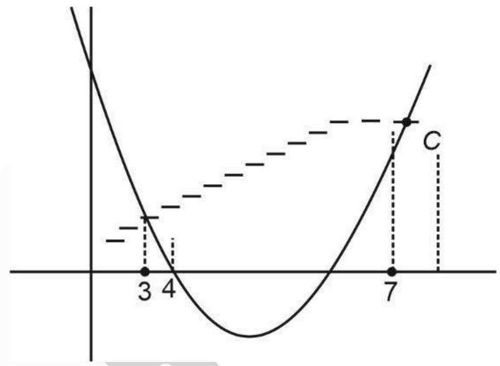
Answer (19)

Sol. $x^2 - 12x + 31 - [x] = 0$

$x^2 - 12x + 31 = [x]$

$(x - 6)^2 - 5 = [x]$

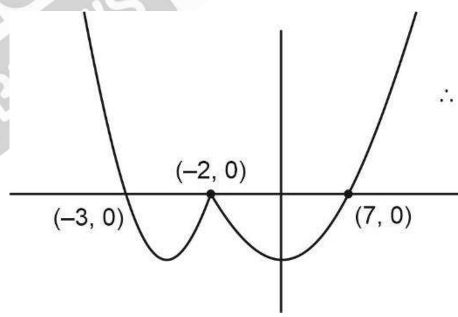
So, by graph



$\therefore$ Two points of intersects

$\therefore$ $m = 2$

$x^2 - 5|x-2| - 4 = 0$



$\therefore n = 3$

$m^2 + mn + n^2 = 4 + 6 + 9 = 19$

25.

26.

27.

28.

29.

30.

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