JEE-Mains-08-04-2023 [Memory Based] [Evening Shift]

Physics

Question: If $y = x - \frac{x^2}{20}$ represents a projectile motion. Find maximum height.

Answer: 5.00

Question: Capacitor of capacity 600 pF is charged to 200 V then it is connected with identical uncharged capacitor find heat loss (in mJ)

Answer: 6.00

Question: s1 area under velocity time graph gives distance travelled
s2 area under acceleration time graph gives change in velocity

Options:

(a) S1 - True, S2 - False
(b) S1 - False, S2 - True
(c) S1 - True, S2 - True
(d) S1 - False, S2 - False

Answer: (b)

Question: A bullet of mass 0.1 kg moving with vel 400 m/s gets embedded in block of mass 3.9 kg at rest placed on rough surface. block comes to rest after travelling distance of 20 m. find coefficient of friction

Answer: 0.25

Question: Choose the correct relation between power radiated by signal and its wavelength λ.

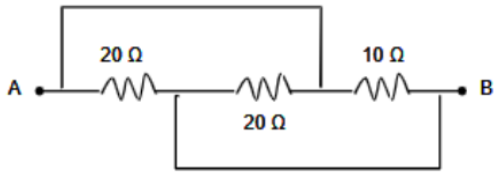
Options:

(a) $P \propto 1/\lambda^2$
(b) $P \propto 1/\lambda$
(c) $P \propto 1/\lambda^3$
(d) $P \propto \lambda^2$

Answer: (a)

Solution:

Question: Find equivalent resistance between A and B.



Options:

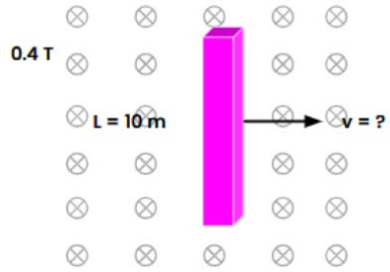
(a) 5Ω

(b) 10Ω
(c) 15Ω
(d) 20Ω
Answer: (a)
Solution:

$$\frac{1}{R_{eq}} = \frac{1}{20} + \frac{1}{20} + \frac{2}{20} = \frac{4}{20}$$

$$R_{eq} = 5\Omega$$

Question: Find the velocity of inductor if B is 0.4 T inwards.



Options:
(a) 1 m/s
(b) 2 m/s
(c) 3 m/s
(d) 4 m/s
Answer: (b)
Solution:

$$\varepsilon = Bvl$$

$$0.08 = 0.4 \times v \times 0.1$$

$$v = 2 \text{ m/s}$$

Question: Which of the following is the highest energy electromagnetic wave ?
Options:
(a) X-ray
(b) Infrared
(c) Microwaves
(d) Radiowaves
Answer: (a)

Question: Which of the following expressions give the value of acceleration due to gravity (g') at the altitude h above the surface of Earth. (R = radius of Earth, g = acceleration due to gravity at surface of Earth)
Options:

(a) $g' = g\frac{h^2}{R^2}$

(b) $g' = g\frac{R^2}{(R+h)^2}$

(c) $g' = g\left(1 - \frac{h}{R}\right)$

(d) $g' = g\left(1 - \frac{h^2}{R^2}\right)$

Answer: (b)

Question: Find the distance from a point charge of magnitude 5×10^{-9} C, where the electric potential is 50 V.

Options:

(a) 90 cm

(b) 70 cm

(c) 60 cm

(d) 50 cm

Answer: (a)

Solution:

$$V = \frac{kq}{r}$$

$$r = \frac{9 \times 10^9 \times 5 \times 10^{-9}}{50}$$

Question: A Carnot engine working between 27°C and 127°C performs 2 kJ of work. The amount of heat rejected is equal to:

Options:

(a) 4 kJ

(b) 6 kJ

(c) 8 kJ

(d) 12 kJ

Answer: (b)

Solution:

$$1 - \frac{Q_1}{Q_2} = 1 - \frac{T_1}{T_2}$$

$$\Rightarrow \frac{Q_2 - Q_1}{Q_2} = 1 - \frac{300}{400}$$

$$\Rightarrow \frac{2kJ}{Q_2} = \frac{1}{4}$$

$$Q_2 = 8kJ$$

$$Q_1 = 8kJ - 2kJ = 6kJ$$

Question: Match column I with column II. and choose the correct option.

Quantities	Dimensions
(i) Torque	(a) M^0LT^{-2}
(ii) Stress	(b) $ML^{-1}T^{-1}$
(iii) Coefficient of viscosity	(c) $ML^{-1}T^{-2}$

(iv) Gravitational Potential Gradient	(d) ML^2T^{-2}

Options:
(a) i-a; ii-c; iii-b; iv-d
(b) i-d; ii-b; iii-c; iv-a
(c) i-d; ii-c; iii-b; iv-a
(d) i-a; ii-c; iii-d; iv-b
Answer: (c)

Question: Statement-I: Electromagnets are made of soft iron.
Statement-II : Soft iron has lower permeability and high retentivity. Choose the correct option related to statements.
Options:
(a) Statement-I is true and Statement-II is also true
(b) Statement-I is true but Statement-II is false
(c) Statement-I is false but Statement-II is true
(d) Statement-I is false and Statement-II is also false
Answer: (b)

Question: Consider 2 statements:
Statement 1: We can get displacement from acceleration-time graph.
Statement 2: we can get acceleration from velocity-time graph.
Options:
(a) Both statements are true
(b) Both statement are false
(c) Statement 1 is true and statement 2 is false
(d) Statement 1 is false and statement 2 is true
Answer: (d)

Question: A block moving with speed 1 in/s comes to rest after moving for 20 cm over a rough surface. The coefficient of friction between the block and surface is
Answer: 0.25
Solution:

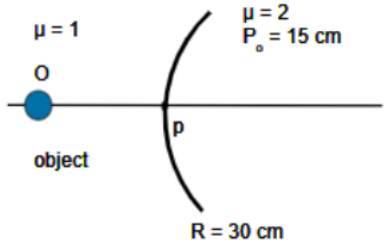
$$a = -\mu g$$

$$v^2 = u^2 + 2as$$

$$\Rightarrow 0 = 1 - 2 \times \mu \times 10 \times 0.2$$

$$\Rightarrow \mu = \frac{1}{4} = 0.25$$

Question: Find the distance of image from pole.



Answer: 30.00

Question: A projectile launched on a horizontal surface follows a trajectory given by $y = x - \frac{x^2}{20}$ where y axis is in vertical upward direction. Maximum height attained by projectile is (All units are in SI)

Options:

(a) 10 m

(b) 5 m

(c) 20 m

(d) 40 m

Answer: (b)

Solution:

$$y = \tan\theta\left\{1 - \frac{x}{R}\right\}$$

$$\frac{-D}{4a} = \frac{-1\times 20}{4\times(-1)} = 5 \text{ m}$$

Question: An antenna of length *l* emits radiation of wavelength λ. The power emitted by the antenna is proportional to

Options:

(a) $\left(\frac{l}{\lambda}\right)^2$

(b) $\frac{l}{\lambda}$

(c) $\frac{\lambda}{l}$

(d) $\frac{1}{l\lambda}$

Answer: (a)

Solution:

$$P \propto \left(\frac{l}{\lambda}\right)^2$$

Question: In YDSE if fringe width is β1 when wavelength of 400 nm is used, what will be the fringe width β2 when wavelength of 600 nm is used?

Answer: 3.00

Solution:

$$\beta = \lambda\frac{D}{d}$$

$$\beta \propto \lambda$$

Question: The ratio of magnetic field due to coil at centre and at a distance of R form the centre on the axis passing through the centre and perpendicular to the plane of ring is $\sqrt{x} : 1$ (R is the radius of coil), find the value of *x*.

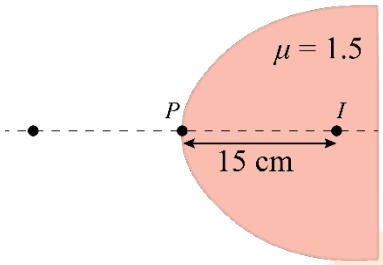
Answer: 8.00

Solution:

$$B_1 = \frac{\mu_0 i}{2R}$$

$$B_2 = \frac{\mu_0}{4\pi}\frac{2i \times \pi R^2}{\left(R^2 + R^2\right)^{3/2}}$$

Question: In the given diagram, image forms at a distance of 15 cm inside the medium of refractive index 1.5. Find the object distance (in cm) from the point P.



Answer: 12.00

Solution:

$$\frac{\mu_2}{u} - \frac{\mu_1}{u} = \frac{K_2 - K_1}{R}$$

$$\Rightarrow \frac{1.5}{15} = \frac{1}{u} = \frac{0.5}{30}$$

$$\Rightarrow \frac{1}{u} = \frac{2 \times 1.5}{2 \times 15} = \frac{0.5}{30}$$

$$\Rightarrow \frac{1}{u} = \frac{2.5}{30}$$

$$u = \frac{30}{2.5} = 12 \text{ cm}$$

Question: Find the charge in energy stored in a capacitor of 600pF capacitance charged at 50 V, once connected with another 600 pF uncharged capacitor

Options:

(a) 0.56 μJ

(b) 0.4 μJ

(c) 0.86 μJ

(d) 0.32 μJ

Answer: (a)

Solution: $\frac{1}{2} \times 600\,pF\left[50^2 - 25^2\right]$

Question: Ratio of wavelength of photons corresponding to first and second line of Balmer series in an emission spectrum is given by $x/20$ for a hydrogen like species. Value of x is equal to;

Answer: 27.00

Solution:

$$\frac{1}{\lambda_1} = RZ^2\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\frac{1}{\lambda_2} = RZ^2\left[\frac{1}{4} - \frac{1}{16}\right]$$

Question: Radioactive sample decays to 1/8th of its original value in 3 days. In 5 days amount left is 8×10^{-3} kg find the initial amount of sample in gram

Answer: 256.00

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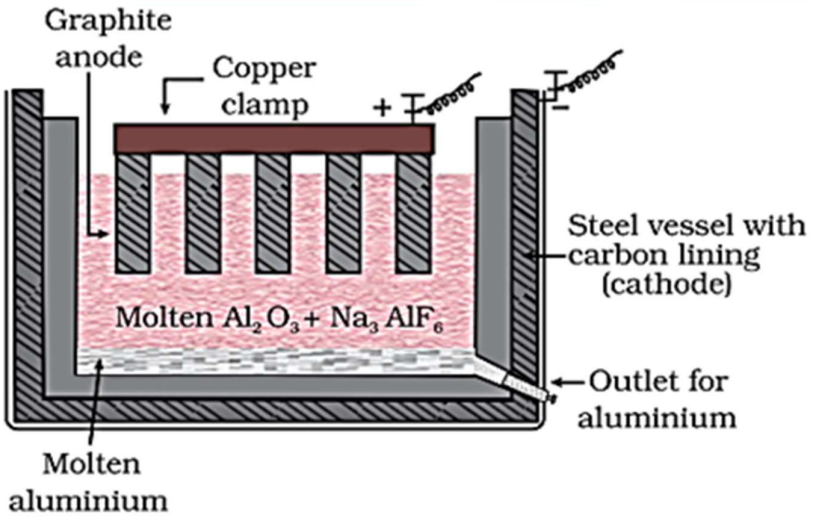
Chemistry

Question: In which of the following has maximum Vander wall forces of attraction?
Options:
(a) CH_4
(b) Hexane
(c) Ar
(d) Water
Answer: (d)
Solution: Hydroygen bond

Question: Which of the following is used for the reduction of Al_2O_3
Options:
(a) Na_3AlF_6
(b) CaF_2
(c) Graphite
(d) Mg
Answer: (c)
Solution:



Electrolytic cell for the extraction of aluminium

Aluminium: In the metallurgy of aluminium, purified Al_2O_3 is mixed with Na_3AlF_6 or CaF_2 which lowers the melting point of the mixture and brings conductivity. The fused matrix is electrolysed. Steel vessel with lining of carbon acts as cathode and graphite anode is used. The overall reaction may be written as:
$2Al_2O_3 + 3C - 4Al + 3CO_2$
This process of electrolysis is widely know as Hall-Heroult process.
Thus electrolysis of the molten mass is carried out in an electrolytic cell using carbon electrodes. The oxygen liberated at anode reacts with the carbon of anode producing

CO and CO_2. This way for each kg of aluminium produced, about 0.5 kg of carbon anode is burnt away. The electrolytic reactions are:
Cathode: Al^{3+} (melt) + $3e^-$ → Al(l)
Anode: C(s) + O^{2-} (melt) → CO(g) + $2e^-$
C(s) + $2O^{2-}$ (melt) → CO_2 (g) + $4e^-$

Question: Ratio of silica in cement
Options:
(a) 50% – 60%
(b) 10%
(c) 20%
(d) 5%
Answer: (a)
Solution: aluminium, iron and magnesium. The average composition of Portland cement is: CaO, 50-60%; SiO_2, 20-25%; Al_2O_3, 5-10%; MgO, 2-3%; Fe_2O_3, 1-2% and SO_3, 1-2%. For a good quality cement, the ratio of silica (SiO_2) to alumina (Al_2O_3) should be between 2.5 and 4 and the ratio of lime (CaO) to the total of the oxides of silicon (SiO_2) aluminium (Al_2O_3) and iron (Fe_2O_3) should be as close as possible to 2.

Ratio of Silica in cement = 50% - 60% SiO_2

Question: Which of following reaction is incorrect ?

$CF_2Cl_2(g) \xrightarrow{UV} \dot{C}l(g) + \dot{C}F_2Cl(g)$ **(i)**

$\dot{C}l(g) + O_3(g) \rightarrow Cl\dot{O}(g) + O_2(g)$ **(ii)**

$Cl\dot{O}(g) + O(g) \rightarrow \dot{C}l(g) + O_2(g)$ **(iii)**

$Cl\dot{O}(g) + NO_2(g) \rightarrow ClONO_2(g)$ **(iv)**

$\dot{C}l(g) + CH_4(g) \rightarrow \dot{C}H_3(g) + HCl(g)$ **(v)**

$ClONO_2(g) + H_2O(g) \rightarrow HOCl(g) + HNO_3(g)$ **(vi)**

$ClONO_2(g) + HCl(g) \rightarrow Cl_2(g) + HNO_3(g)$ **(vii)**

$HOCl(g) \xrightarrow{h\nu} \dot{O}H(g) + \dot{C}l(g)$ **(viii)**

$Cl_2(g) \xrightarrow{h\nu} 2\dot{C}l(g)$ **(ix)**

Options:
(a) (i), (ii), (iv), (vi) are correct
(b) (ii), (iii), (v), (vii) are correct
(c) (i), (vii), (viii), (ix) are correct
(d) All are correct
Answer: (d)
Solution: All are correct

Question:

Aspartic acid	**D**
Glutamine	**E**
Lysine	**Q**
Glutamic acid	**K**

Options:

(a) Aspartic acid – D; Glutamine – Q; Lysine – K; Glutamic acid – E
(b) Glutamine – Q; Lysine – K; Aspartic acid – D; Glutamic acid – E
(c) Aspartic acid – D; Glutamic acid – E; Lysine – K; Glutamine – Q
(d) Glutamic acid – E; Glutamine – Q; Lysine – K; Aspartic acid – D

Answer: (a)

Solution:

Aspartic acid	**D**
Glutamine	**Q**
Lysine	**K**
Glutamic acid	**E**

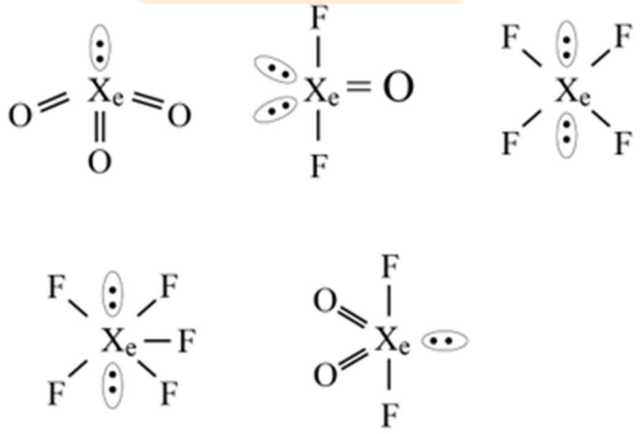
Question: Compound of Xenon having one electron pair on central atom:
XeO_3, $XeOF_2$, XeF_4, XeF_5^-, XeO_2F_2

Options:

(a) 2
(b) 3
(c) 4
(d) 5

Answer: (a)

Solution:



Question: Which of the following acts as a stabilizer in the decomposition of H_2O_2?

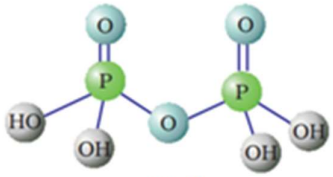
Options:

(a) Urea
(b) Alkali
(c) Glass
(d) Dust

Answer: (a)

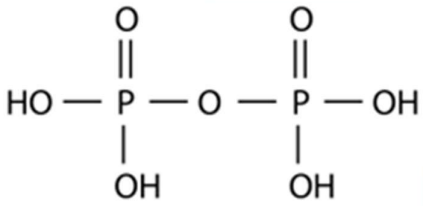
Solution: reaction is catalysed. It is, therefore, stored in wax-lined glass or plastic vessels in dark. Urea can be added as a stabilizer. It is kept away from dust because dust can induce explosive decomposition of the compound.

Question: What is the ratio of σ and pi bonds in pyrophosphoric acid?



$H_4P_2O_7$
Pyrophosphoric acid

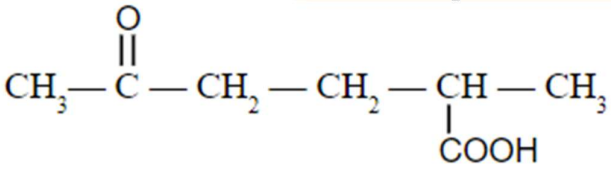
Options:
(a) 1 : 2
(b) 2 : 1
(c) 6 : 1
(d) 5 : 1
Answer: (c)
Solution:



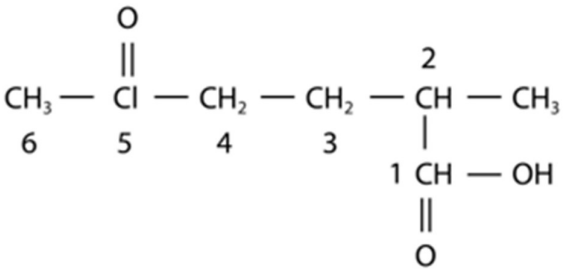
Pyrophosphoric Acid

σ	:	π
12		2
6	:	1

Question: IUPAC Name of the compound



Options:
(a) 5 Oxo 2 Methyl Hexanoic acid
(b) 2 Methyl 5 Oxo Hexanoic Acid
(c) 3 Methyl 4 Oxo Hexanoic acid
(d) 1 Oxo 3 Methyl Hexanoic acid
Answer: (b)
Solution:



2 Methyl 5 Oxo Hexanoic Acid

Question: Find out the sum of oxidation no. of central metal atom of $Fe(CO)_5$, VO^{2+} & WO_3

Options:

(a) 9

(b) 10

(c) 13

(d) 11

Answer: (b)

Solution: $[Fe(CO)_5]$

Fe = O.S = 0

VO^{+2} = O.S = +4

x – 2 = +2

WO_3 = O.S = +6

x – 6 = 0

x = +6

O + 4 + 6 = 10

Question: Number of orbitals having five radial Nodes 7s, 7p, 7d, 6s and 5s

Options:

(a) 7p, 6s

(b) 7p, 7d

(c) 7d, 5s

(d) 6s – 5s

Answer: (a)

Solution: Total Node = n – 1

Radial Node

n – l – 1

7s = 6 Radial Node

7p = 5 Radial Node

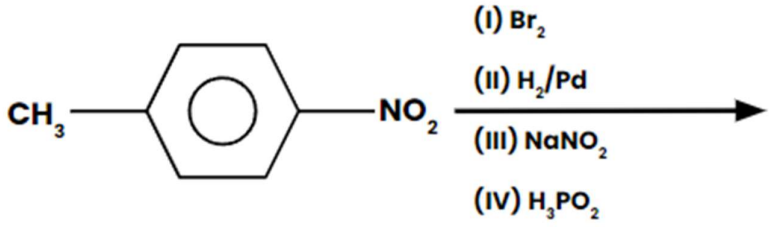
7d = 4 Radial Node

6s = 5 Radial Node

5s = 4 Radial Node

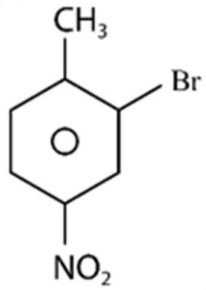
2 have 5 Radial Node (7p, 6s).

Question:

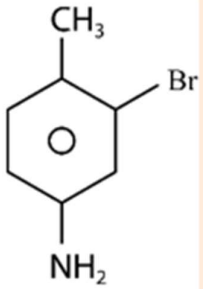


Options:

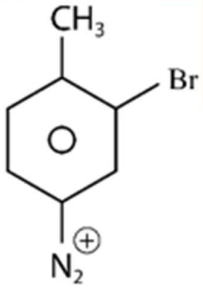
(a)



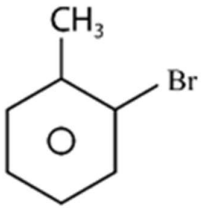
(b)



(c)

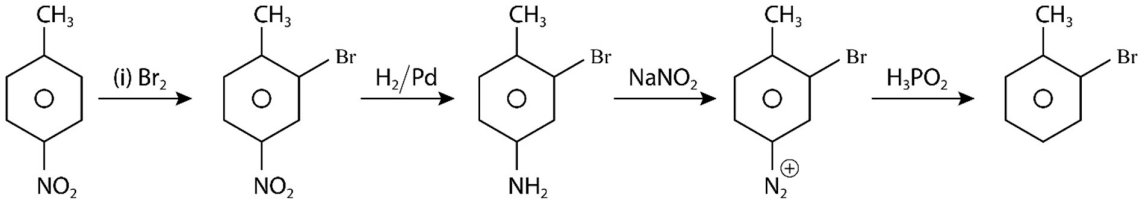


(d)

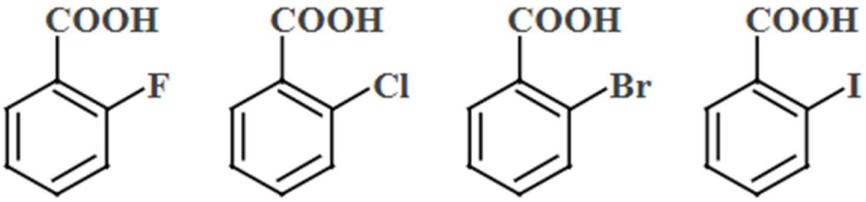


Answer: (d)

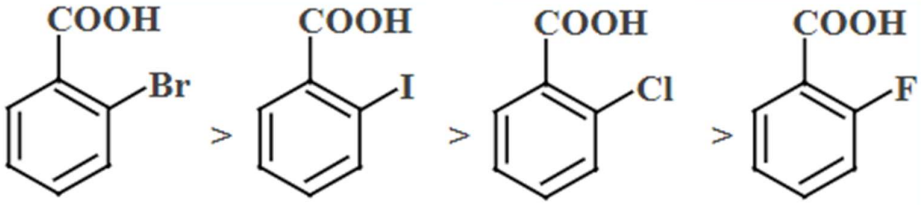
Solution:



Question: Compare the acidic strength :
1. o-fluoro benzoic acid
2. o-chloro benzoic acid
3. o-bromo benzoic acid
4. o-iodo benzoic acid



Options:
(a) 3 > 4 > 2 > 1
(b) 1 > 2 > 3 > 4
(c) 2 > 4 > 1 > 3
(d) 3 > 1 > 2 > 4
Answer: (a)
Solution:



Question: K_{sp} of $BaSO_4$ is 8×10^{-11}. If the solubility in presence of 0.1 M $CaSO_4$ is X × 10^{-10} M, X is :
Options:
(a) 3
(b) 5
(c) 6
(d) 8
Answer: (d)
Solution:

Question: Match the Column I with Column II

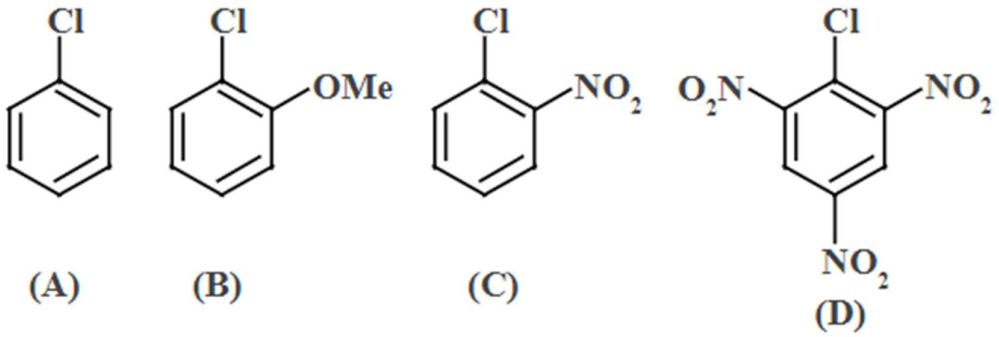
Column I	Column II
(a) $[Ni(NH_3)_6]^{2+}$	(p) 0
(b) $[Co(NH_3)_6]^{3+}$	(q) 2
(c) $[Fe(CN)_6]^{3+}$	(r) 4
(d) $[CoF_6]^{3-}$	(s) 1

Options:
(a) A – q; B – p; C – r; D – s
(b) A – p; B – q; C – s; D – r
(c) A – q; B – p; C – s; D – r
(d) A – s; B – q; C – p; D – r
Answer: (c)
Solution:
$[Ni(NH_3)_6]^{2+}$ = 2 unpaired $e^{\ominus}$
$[Co(NH_3)_6]^{3+}$ = 0 unpaired $e^{\ominus}$
$[Fe(CN)_6]^{3+}$ = 1 unpaired $e^{\ominus}$
$[CoF_6]^{3-}$ = 4 unpaired $e^{\ominus}$

Question: Total spin only magnetic moment of the ion $[Mn(SCN)_6]X^-$ is 5.92 B.M. Find out the value of X
Options:
(a) 5
(b) 3
(c) 2
(d) 4
Answer: (d)
Solution:

Question: Statement-1: Methyl orange is weak acid.
Statement-2: Benzenoid form of methyl orange is deeply coloured than quinonoid form.
Options:
(a) Statement -1 is true, statement -2 is true.
(b) Statement -1 is false, statement -2 is true.
(c) Statement -1 is true, statement -2 is false.
(d) Statement -1 is false, Statement -2 is false.
Answer: (a)
Solution:

Question: The correct order of nucleophilic substitution of following compound with NaOH



(A) (B) (C) (D)

Options:
(a) A > B > C > D
(b) D > C > A > B
(c) D > C > B > A

(d) A > B > D > C
Answer: (b)
Solution:

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Mathematics

Question: If α and β are the roots of $ax^2+bx+1=0$, and $\lim\limits_{x\to\frac{1}{\alpha}}\left(\frac{1-\cos\left(x^2+bx+a\right)}{2\left(1-\alpha x\right)^2}\right)\frac{1}{2}=\frac{1}{\alpha k}\left(\frac{1}{\beta}-\frac{1}{\alpha}\right)$, find k.

Answer: 2.00

Solution:

$x\to\frac{1}{x}$,

$x^2+bx+a=0$

$\frac{1}{\alpha},\frac{1}{\beta}$ are roots

$$\lim_{x\to\frac{1}{\alpha}}\left[\frac{2\sin^2\left(\frac{x^2+bx+a}{2}\right)}{2\left(1-\alpha x\right)^2}\right]$$

$$\lim_{x\to\frac{1}{\alpha}}\left[\frac{2\sin^2\left(\frac{\left(\frac{x-1}{\alpha}\right)\left(\frac{x-1}{\beta}\right)}{2}\right)}{4\times2\alpha^2\frac{\left(x-\frac{1}{\alpha}\right)^2\left(x-\frac{1}{\beta}\right)^2}{4}}\right]\times\left|x-\frac{1}{\beta}\right|$$

$$=\frac{1}{2\alpha}\left|\frac{1}{\alpha}-\frac{1}{\beta}\right|$$

$\therefore k=2$

Question: Number of ways letters of MATHEMATICS can be arranged such that 'S' & 'C' are not together is $(6!)k$. Find k.

Answer: 5670.00

Solution:

MATHEMATICS

M-2, A-2, T-2, H, E, I, C, S

Favorable cases

= Total cases – unfavorable cases

$$= \frac{11!}{(2!)^3} - \underbrace{\frac{10!}{(2!)^3} \times 2}_{\text{when S,C together}}$$

$$= \frac{10!}{8}(11-2)$$

$$= \frac{10!}{8} \times 9$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6!}{8} \times 9$$

$$= \frac{10 \times 9 \times 9 \times 7 \times 6!}{k}$$

$$= 5670$$

Question: Find the number of onto functions from $B = \{a,b,c,d,e\}$ to $A = \{1,2,3,4\}$, $f(a) \neq 1$.

Answer: 180.00
Solution:

Total number of onto functions $= \frac{5!}{3!2!} \times 4!$

Now,

when $f(a) = 1$

$$\frac{4!}{2!2!} \times 3! + 4!$$

$\therefore$ Required functions $= 240 - 36 - 24 = 180$

Question: Find the absolute difference of coefficients of x^7 & x^{10} in $\left(2x^2 + \frac{1}{2x}\right)^{11}$.

Answer: 396.00
Solution:

$${}^{11}C_r \left(2x^2\right)^{11-2} \left(\frac{1}{2x}\right)^r$$

$22-2r-r=7$

$3r=15$

$r=5$

$22-2r-r=10$

$r=4$

$\left|{}^{11}C_5(2)^6\left(\frac{1}{2}\right)^5-{}^{11}C_4 2^7\left(\frac{1}{2}\right)^4\right|$

$\left|{}^{11}C_5\times 2-{}^{11}C_4\times 8\right|$

$=2\left|462-264\right|$

$=2\left|198\right|$

$=396$

Question: The relation R defined on $\{1,2,3,4,5,6,7\}$ such that $xRy \Leftrightarrow x+y=7$ is

Options:
(a) Reflexive
(b) Symmetric
(c) Transitive
(d)Equivalence

Answer: (b)

Solution:

$A=[1,2,3,4,5,6,7]$

(A) (x,x)

$\Rightarrow x+x=7$

$2x=7$

$x=\frac{7}{2}\notin A$

Hence not reflexive

(B) $(x,y)\rightarrow x+y=7$

$(y,x)\rightarrow y+x=7$

$(x,y)\Rightarrow(y,x)$

Hence, symmetric

$(x,y)\rightarrow x+y=7 \quad ...(1)$

$(y,z)\rightarrow y+z=7 \quad ....(2)$

(1) – (2)

$x-z=0$

$(x,y)\ \&\ (y,z) \not\Rightarrow (x,z)$

Hence not transitive

Question: $\frac{1}{x},\frac{1}{y},\frac{1}{z}$ are in A.P. and $x,\sqrt{2}y,z$ are in G.P., provided $x,y,z>0;\ x>y>z$. If $xy+yz+zx=\frac{3}{\sqrt{2}}xyz$, then find $3(x+y+z)^2$.

Answer: 150.00

Solution:

$2y^2=xz \quad ....(1)$

$\frac{2}{y}=\frac{1}{x}+\frac{1}{z}=\frac{x+z}{xz}$

$4y=x+z \quad ....(2)$

$xy+yz+zx=\frac{3}{\sqrt{2}}xyz$

$4y^2+2y^2=\frac{3}{\sqrt{2}}(2y^3)$

$6y^2=3\sqrt{2}y^3$

$y=0,\ y=\sqrt{2}$

$x+z=4\sqrt{2},\ xz=4$

$3(x+y+z)^2=3\left(4\sqrt{2}+\sqrt{2}\right)^2$

$=3\times 50$

$=150$

Question: $S_n=5+8+14+23+....$. Find $S_{30}-a_{40}$.

Answer: 11000.00

Solution:

$S_n=5+8+14+23+....+T_n$

$S_n=\quad 5+8+14+....+T_{n-1}+T_n$

$0=5+3+6+9+....+....-T_n$

$\Rightarrow T_n=5+\left(\frac{n-1}{2}\right)(6+(n-2)3)=5+\frac{3}{2}(n-1)n$

$$\therefore T_n = \frac{1}{2}\left(3n^2 - 3n + 10\right)$$

$$a_n = \sum T_n = \frac{1}{2}\left[\frac{3n(n+1)(2n+1)}{6} - \frac{3n(n+1)}{2} + 10n\right]$$

$$a_n = \sum T_n = \frac{1}{2}n\left(\frac{(n+1)(2n+1)}{2} - \frac{3(n+1)}{2} + 10\right)$$

$$a_n = \frac{n}{4}\left(2n^2 + 3n + 1 - 2n - 3 + 20\right)$$

$$a_n = \frac{n}{2}\left(n^2 + 9\right)$$

$$a_{40} = \frac{40}{2}(1600 + 9) = 32180$$

$$S_n = \sum a_n = \frac{1}{2}\left(\left(\frac{n(n+1)}{2}\right)^2 + 9\frac{n(n+1)}{2}\right)$$

$$S_{30} = \frac{1}{2}\left(\left(\frac{30\times 31}{2}\right)^2 + 9\frac{30\times 31}{2}\right)$$

$$= \frac{1}{2}(216225 + 4185)$$

$$S_{30} = 110205$$

$$S_{30} - a_{40} = 110205 - 32180$$

$$\Rightarrow S_{30} - a_{40} = 78025$$

Question: $36\left(4\cos^2 9° - 1\right)\left(4\cos^2 27° - 1\right)\left(4\cos^2 81° - 1\right)\left(4\cos^2 153° - 1\right) = ?$

Answer: 36.00

Solution:

$$4\cos^2 9° - 1 = 2\left(1 + \cos 18°\right) - 1$$

$$= 2\cos 18° + 1$$

$$4\cos^2 27° - 1 = 2\left(1 + \cos 54°\right) - 1$$

$$= 2\cos 54° + 1$$

$$4\cos^2 81° - 1 = 2\left(1 + \cos 162°\right) - 1$$

$$= -2\sin 72° + 1$$

$$= -2\cos 18° + 1$$

$$4\cos^2 243^\circ - 1 = 2\left(1+\cos 486^\circ\right) - 1$$
$$= -2\sin 36^\circ + 1$$
$$= -2\cos 54^\circ + 1$$
$$\Rightarrow 36\left(1-4\cos^2 18^\circ\right)\left(1-4\cos^2 54^\circ\right)$$
$$= 36\left(1-\frac{10+2\sqrt{5}}{4}\right)\left(1-\frac{10-2\sqrt{5}}{4}\right)$$
$$= 36\left(\frac{4-10-2\sqrt{5}}{4}\right)\left(\frac{4-10+2\sqrt{5}}{4}\right)$$
$$= \frac{9}{4}\left(-6-2\sqrt{5}\right)\left(-6+2\sqrt{5}\right)$$
$$= \frac{9}{4}\left(36-20\right)$$
$$= \frac{9}{4}\times 16$$
$$= 36$$

Question: Mean and variance of 12 observations is $\frac{9}{2}$ and 4. Two observations 9 and 10 were wrongly taken as 7 and 14. If the new variance is $\frac{m}{n}$. Find value of $m+n$ where m and n coprime.

Answer: 41.00

Solution:

$$\bar{x} = \frac{9}{2},\ a^2 = 4,\ n = 12$$

$$\bar{X} = \frac{12\times\frac{9}{2}+7-14+9+10}{12} = \frac{54-2}{12} = \frac{13}{3}$$

$$\sigma^2 = \frac{\sum x_i^2}{12} - \left(50\right)^2$$

$$\Rightarrow 4 = \frac{\sum x_i^2}{12} = \frac{81}{4}$$

$$\Rightarrow \sum\left(x_i\right)^2 = 291$$

Now,

$$\sum (X_i)^2 = 291 - 7^2 - 14^2 + 9^2 + 10^2$$
$$= 291 - 49 - 196 + 81 + 100$$
$$= 227$$

$$\sigma^2_{new} = \frac{227}{12} - \left(\frac{13}{3}\right)^2 = \frac{227}{12} - \frac{169}{9}$$

$$= \frac{681 - 676}{36} = \frac{5}{36}$$

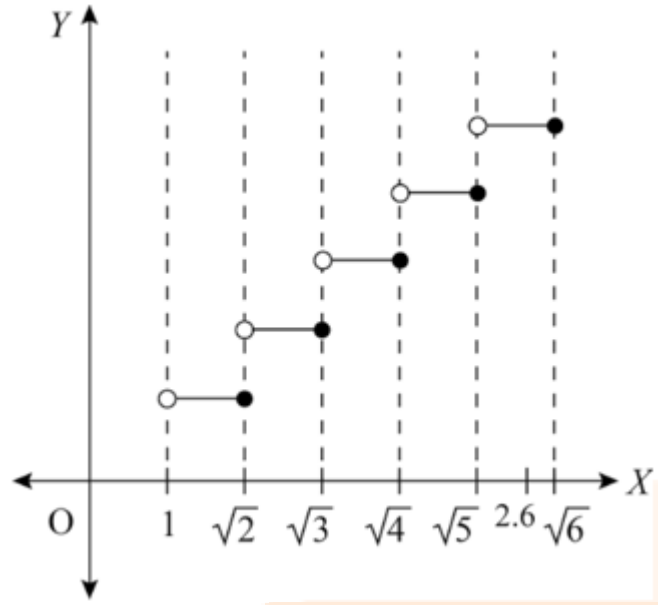
Sum = $m + n = 5 + 36 = 41$

Question: $25^{190} - 19^{190} - 8^{190} + 2^{190}$ is divisible by ____.
Answer: 6.00
Solution:

$$25^{190} - 19^{190} - 8^{190} + 2^{190}$$
$$= (19 + 6)^{190} - 19^{190} - (2 + 6)^{190} + 2^{190}$$
$$= {}^{190}C_0 19^{190} + 6k_1 - 19^{190} - {}^{190}C_0 2^{190} - 6k_2 + 2^{190}$$
$$= 6k_1 - 6k_2$$
$$= 6(k_1 - k_2)$$
$$= 6k$$

Hence divisible by 6.

Question: The value of $\int_0^{2.4} \left[x^2\right] dx$ is $\alpha + \beta\sqrt{2} + \gamma\sqrt{3} + \delta\sqrt{5} + \phi$, then $(\alpha + \beta + \gamma + \delta + \phi)$ is equal to ____.
Answer: 6.00
Solution:



$$\int_0^{2.4}\left[x^2\right]dx=\int_0^1 0\,dx+\int_1^{\sqrt{2}}1dx+\int_{\sqrt{2}}^{\sqrt{3}}2\,dx+\int_{\sqrt{3}}^{\sqrt{4}}3dx+\int_{\sqrt{4}}^{\sqrt{5}}4\,dx+\int_{\sqrt{5}}^{2.4}5\,dx$$

$$=0+\left(\sqrt{2}-1\right)+2\left(\sqrt{3}-\sqrt{2}\right)+3\left(2-\sqrt{3}\right)+4\left(\sqrt{5}-2\right)+5\left(2.4-\sqrt{5}\right)$$

$$=0-1-\sqrt{2}-\sqrt{3}-2-\sqrt{5}+12$$

$$=\sqrt{2}-\sqrt{3}-\sqrt{5}+9$$

$$\Rightarrow \beta=\gamma=\Delta=-1$$

$$\alpha+\phi=9$$

$$\Rightarrow \alpha+\beta+\gamma+\delta+\phi=9-1-1-1=6$$

Question: $\theta\in(0,2\pi)$ and $\dfrac{1+2i\sin\theta}{1-i\sin\theta}$ is purely imaginary then the value of θ is

Options:

(a) π

(b) 0

(c) 2π

(d) $\dfrac{\pi}{4}$

Answer: (d)

Solution:

$$\frac{1+2i\sin\theta}{1-i\sin\theta}\times\frac{1+i\sin\theta}{1+i\sin\theta}=\frac{(1+2i\sin\theta)(1+i\sin\theta)}{1+\sin^2\theta}$$

Real part has to be zero, $\mathrm{Re}(z)=0$

$$\Rightarrow \text{Real}\left(\frac{(1+2i\sin\theta)(1+i\sin\theta)}{1+\sin^2\theta}\right)=0$$

$$\Rightarrow \frac{1-2\sin^2\theta}{1+\sin^2\theta}=0$$

$$\Rightarrow \sin^2\theta=\frac{1}{2}$$

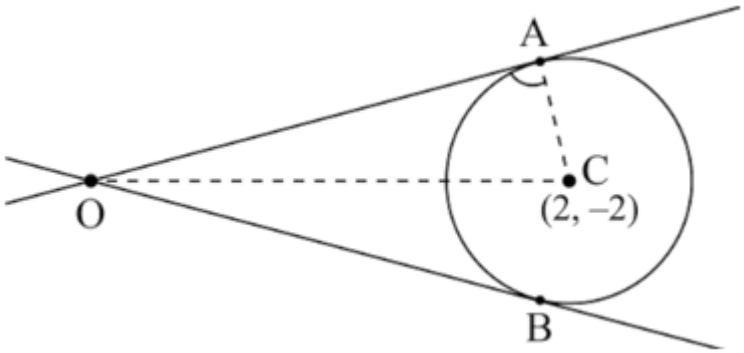
$$\Rightarrow \sin\theta=\pm\frac{1}{\sqrt{2}}$$

$$\therefore\ \theta=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$$

Question: Two tangents OA & OB are drawn to the circle $x^2+y^2-6x+4y+8=0$ when O is origin. If the circumcircle of ΔOAB passes through $\left(\alpha,\frac{1}{2}\right)$. Find α.

Answer:
Solution:

OC is diagonal



Center $\left(\frac{3}{2},-1\right)$

$$r=\frac{\sqrt{9+4}}{2}=\frac{\sqrt{13}}{2}$$

$$\left(x-\frac{3}{2}\right)^2+(y+1)^2=\frac{13}{4}$$

$$\left(\alpha-\frac{3}{2}\right)^2+\left(\frac{1}{2}+1\right)^2=\frac{13}{4}$$

$$\left(\alpha-\frac{3}{2}\right)^2+\frac{9}{4}=\frac{13}{4}$$

$$\left(\alpha-\frac{3}{2}\right)^2=1$$

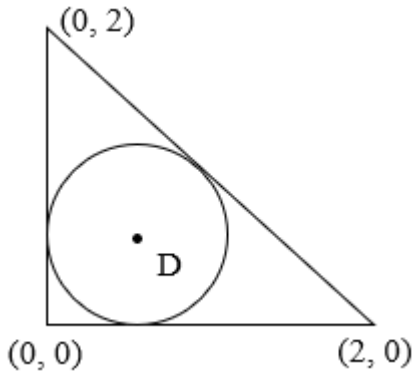
$$\alpha-\frac{3}{2}=1 \text{ or } \alpha-\frac{3}{2}=-1$$

$$\alpha=\frac{5}{2} \text{ or } \alpha=\frac{1}{2}$$

Question: Midpoint of sides of a triangle are (0, 1), (1, 0) and (1, 1). Incentre of triangle is D. A parabola $y^2=4ax$ passes through D, whose focus is $\left(\alpha+\sqrt{2}\beta,0\right)$. Find $\frac{\beta^2}{\alpha}$.

Answer: $\frac{1}{8}$

Solution:



$$x=\frac{2(2)}{2+2+2\sqrt{2}}=\frac{2}{\sqrt{2}+2}=\frac{\sqrt{2}}{1+\sqrt{2}}$$

$$y=\frac{\sqrt{2}}{1+\sqrt{2}}$$

$$\Rightarrow D\left(\frac{\sqrt{2}}{1+\sqrt{2}},\frac{\sqrt{2}}{1+\sqrt{2}}\right)$$

$$\Rightarrow x^2=4ax$$

$$x=4a$$

$$\frac{\sqrt{2}}{1+\sqrt{2}}=4a$$

$$\Rightarrow \text{ Focus } S\left(\frac{\sqrt{2}}{4\left(1+\sqrt{2}\right)},0\right)$$

$$\Rightarrow \frac{\sqrt{2}}{4\left(1+\sqrt{2}\right)} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$= \frac{\sqrt{2}\left(\sqrt{2}-1\right)}{4}$$

$$= \frac{2-\sqrt{2}}{4} = \frac{1}{2} - \frac{\sqrt{2}}{4}$$

$$\Rightarrow \alpha = \frac{1}{2},\ \beta = -\frac{1}{4}$$

$$\frac{\beta}{\alpha} = \frac{2}{4 \times 4} = \frac{1}{8}$$