Mathematics – Key (@2023)

1) $3\overline{)12, 15, 21}$
$4, 5, 7$

L.C.M $= 3 \times 4 \times 5 \times 7$
$= 420$

(2) $12 = 2^2 \times 3$
$15 = 3 \times 5$
$21 = 3 \times 7$
LCM $= 2^2 \times 3 \times 5 \times 7$
$= 4 \times 3 \times 5 \times 7$
$= 420$

2) $A = \{1, 2, 3, 4, 5\}$

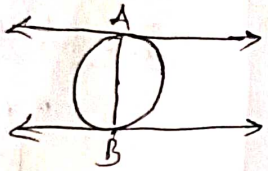
3) A

4) A

5) 2

6) If a line drawn parallel to one side of a triangle will divide the other two sides in the same ratio.

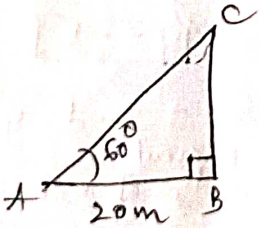
7) 2



8) Side (s) $= 4$ cm
Vol. of cube $= s^3$
$= 4^3$
$= 4 \times 4 \times 4$
$= 64$ cm^3

9) B

10)



11) $P(\text{not } E) = P(\bar{E})$

$\boxed{P(\bar{E}) = 1 - P(E)}$

$= 1 - 0.05$
$= 0.95$

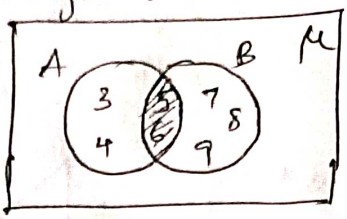
12) $\boxed{\text{Mean} = \frac{\text{Sum of all observations}}{\text{Total no. of observations}}}$

$= \frac{2+3+7+6+6+3+8}{7}$

$= \frac{35}{7} = 5$

(13) $A \cap B = \{3, 4, 5, 6\} \cap \{5, 6, 7, 8, 9\}$
$= \{5, 6\}$

Venn diagram



$\square = A \cap B$

14) cost of a pencil $= ₹ x$
cost of a pen $= ₹ y$
Cost of 6 pencils & 4 pens together $= ₹ 50$
$6x + 4y = 50 \longrightarrow (1)$
cost of 5 pencils & 6 pens together $= ₹ 46$
$5x + 6y = 46 \longrightarrow (2)$

15) $(x-2)^2 + 1 = 2x - 3$
$x^2 + 4 - 4x + 1 = 2x - 3$
$x^2 + 5 - 4x - 2x + 3 = 0$
$x^2 - 6x + 8 = 0$
∴ The given is a Q.E

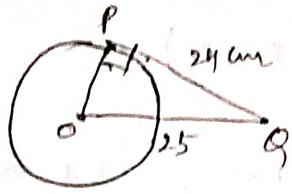
16) $\boxed{n\text{th term } (a_n) = a_1 + (n-1)d}$

here, a_1 = first term
d = common difference
n = nth value/terms

17) A(7, 8) B(−2, 3)
$x_1\ y_1$ $x_2\ y_2$

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(-2-7)^2 + (3-8)^2}$
$= \sqrt{(-9)^2 + (-5)^2}$
$= \sqrt{81 + 25}$
$= \sqrt{106}$ units.

18)



$\text{Radius}(OP) = \sqrt{OQ^2 - PQ^2}$

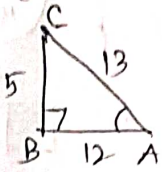
$= \sqrt{25^2 - 24^2}$

$= \sqrt{625 - 576}$

$= \sqrt{49}$

$= 7 \text{ cm}$

19) $\cos A = \frac{12}{13}$



$\cos A = \frac{\text{adj. Side}}{\text{Hyp}}$

$BC^2 = AC^2 - AB^2$ (Pythagoras theorem)

$= 13^2 - 12^2$

$= 169 - 144$

$BC^2 = 25$

$\boxed{BC = 5}$

$\sin A = \frac{\text{opp. Side}}{\text{Hyp}} = \frac{BC}{AC} = \frac{5}{13}$

$\tan A = \frac{\text{opp. Side}}{\text{adj. Side}} = \frac{BC}{AB} = \frac{5}{12}$

20) A die is thrown once,

Total no. of all possible outcomes $= 6$

Sample space $= \{1, 2, 3, 4, 5, 6\}$

i) Let the event be 'E' of getting a prime number

No. of favourable outcomes to E $= 3 \{2, 3, 5\}$

$P(E) = \boxed{P(\text{prime numbers}) = \frac{\text{No. of favourable outcomes}}{\text{Total no. of all possible outcomes}}}$

$= \frac{3}{6} = \frac{1}{2}$

odd numbers $= \{1, 3, 5\}$

ii) $P(\text{odd numbers}) = \frac{3}{6} = \frac{1}{2}$

21) $2 \log 5 + \frac{1}{2} \log 9 - \log 3 = \log x$

$\log 5^2 + \log 9^{\frac{1}{2}} - \log 3 = \log x$

$\log 25 + \log (3^2)^{\frac{1}{2}} - \log 3 = \log x$

$\log 25 + \log 3 - \log 3 = \log x$

$\log x = \log 25$

$\therefore \boxed{x = 25}$

22) Let $p(x) = x^4 - 81$

$p(-3) = (-3)^4 - 81$

$= 81 - 81$

$= 0$

$p(3) = 3^4 - 81$

$= 81 - 81$

$= 0$

$\therefore$ -3 and 3 are zeroes of $p(x)$.

23) Let $3x + 2y = -1 \longrightarrow (1)$

$2x + 3y = -9 \longrightarrow (2)$

To eliminate the variable 'y' we have to equate their coefficient in both eqns

$(1) \times 3 \Rightarrow 9x + 6y = -3$

$(2) \times 2 \Rightarrow 4x + 6y = -18$

$5x = 15$

$\boxed{x = 3}$

Sub 'x' value in equation (1), we get

$3x + 2y = -1$

$3 \times 3 + 2y = -1$

$9 + 2y = -1$

$2y = -10$

$\boxed{y = -5}$

$\therefore x = 3$ and $y = -5$

24) Let present Age of Rohan $= x$ years

Present age of Rohan's mother $= (x + 26)$ yrs

Product of their ages after 3 years $= 360$ yrs

$(x + 3) \times (x + 29) = 360$

$x^2 + 32x + 87 = 360$

$x^2 + 32x + 87 - 360 = 0$

$\boxed{x^2 + 32x - 273 = 0}$

$+39 \quad -7$

$x^2 + 39x - 7x - 273 = 0$

$x(x + 39) - 7(x + 39) = 0$

$(x + 39)(x - 7) = 0$

273

3×91

$3 \times 7 \times 13$

-273

$x+39=0$ or $x-7=0$

$x=-39$ ✗ $\boxed{x=7}$

∴ Present age of Rohan = 7 years

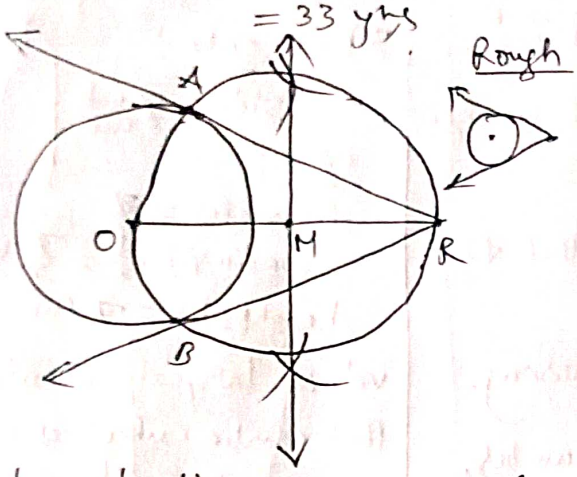
Present age of Rohan's Mother

$= (x+26)$ yrs

$= 7+26$

$= 33$ yrs

25)



Steps of construction: 1) Draw a circle with any radius of centre 'O'.

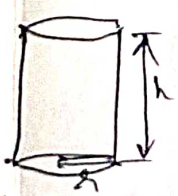
2) Mark a point R outside of the circle and join OR.

3) Draw a perpendicular bisector of OR which meets OR at M.

4) Taking OM or MR as radius and with centre M draw another circle which intersects the previously drawn circle at A and B.

5) Join RA and RB are the required tangents to the circle.

26) Oil drum (cylinder):



Diameter $(d) = 2$ m

Radius $(r) = \frac{d}{2} = \frac{2}{2} = 1$ m

Height $(h) = 7$ m

T.S.A of cylinder $= 2\pi r(h+r)$

$= 2\times\frac{22}{7}\times 1\times(7+1)$

$= \frac{44}{7}\times 8 = \frac{352}{7}$ m^2

Cost of painting 1 m^2 area = ₹ 3

Cost of painting $\frac{352}{7}$ m^2 area = ₹ $3\times\frac{352}{7}$

$= ₹\ \frac{1056}{7}$

$= ₹\ 150.85$

Total cost to be paid for 10 drums

$= ₹\ 150.85 \times 10$

$= ₹\ 1508.5$

27) LHS $= \dfrac{1-\tan^2 A}{\cot^2 A - 1}$

$= \dfrac{1-\frac{\sin^2 A}{\cos^2 A}}{\frac{\cos^2 A}{\sin^2 A}-1}$

$= \dfrac{\frac{\cos^2 A-\sin^2 A}{\cos^2 A}}{\frac{\cos^2 A-\sin^2 A}{\sin^2 A}}$

$= \dfrac{\cos^2 A-\sin^2 A}{\cos^2 A}\times\dfrac{\sin^2 A}{\cos^2 A-\sin^2 A}$

$= \dfrac{\sin^2 A}{\cos^2 A} = \tan^2 A$ = RHS

28)

Family Size	No. of families
1-3	7 (f_0)
3-5	8 (f_1) → modal class
5-7	2 (f_2)
7-9	2
9-11	1

From the table,

$l=3,\ f_1=8,\ f_0=7,\ f_2=2,\ h=2$

$$\text{Mode} = l+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$$

$= 3+\left(\frac{8-7}{2\times 8-7-2}\right)\times 2$

$= 3+\frac{1}{16-9}\times 2$

$= 3+\frac{2}{7}$

$= 3+0.28$

$= 3.28$ (approx.)

29) Let us assume $6+\sqrt{2}$ is a rational no.

$\therefore 6+\sqrt{2} = \frac{a}{b}$ (a, b are coprime)

$\sqrt{2} = \frac{a}{b} - 6$

$\sqrt{2} = \frac{a-6b}{b}$

Since a and b are integers $\frac{a-6b}{b}$ is a rational numbers. So, $\sqrt{2}$ is also a rational numbers.

But this contradicts the fact that $\sqrt{2}$ is irrational.

Hence our assumption is wrong.

$\therefore 6+\sqrt{2}$ is irrational numbers.

b) i) $a_n = 3+4n$

$a_1 = 3+4\times1 = 3+4 = 7$

$a_2 = 3+4\times2 = 3+8 = 11$,

$a_3 = 3+4\times3 = 3+12 = 15$

7, 11, 15 .....

C.d (d) $= a_2 - a_1 = 11-7 = 4$

$= a_3 - a_2 = 15-11 = 4$

$\therefore$ 7, 11, 15, --- form an AP

$\boxed{S_n = \frac{n}{2}[2a+(n-1)d]}$

$S_{15} = \frac{15}{2}[2\times7+(15-1)\times4]$

$= \frac{15}{2}[14+56]$

$= \frac{15}{2}\times70 = 525$

ii) $a_n = 1-5n$

$a_1 = 1-5\times1 = 1-5 = 4$

$a_2 = 1-5\times2 = 1-10 = -1$

$a_3 = 1-5\times3 = 1-15 = -6$

C.d (d) $= a_2 - a_1 = -1-4 = -5$

$= a_3 - a_2 = -6-(-1)$

$= -6+1 = -5$

$\therefore$ 4, -1, -6, --- form an AP

$S_{15} = \frac{15}{2}[2\times4+(15-1)-5]$

$= \frac{15}{2}[8+14\times-5]$

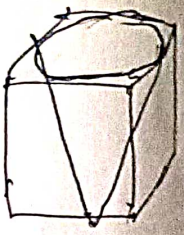
$= \frac{15}{2}[8+(-70)]$

$= \frac{15}{2}\times -31$

$= -465$

30) a) cube:

Side = 7 units cm



cone:

diameter = 7

radius (r) $= \frac{d}{2} = \frac{7}{2}$ units cm

height (h) = 7 cm

vol. of largest circular cone that can be cut out of a cube

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\times\pi\times\left(\frac{7}{2}\right)^2\times a$

$= \frac{1}{3}\times\frac{22}{7}\times\frac{7^2}{4_2}\times$

$= \frac{11\times49}{6}$

$= \frac{539}{6} = 89.8$ cm^3

b) $A \cup B = \{1,2,3,4,5\} \cup \{3,4,5,6,7\}$

$= \{1,2,3,4,5,6,7\}$

$B \cup C = \{3,4,5,6,7\} \cup \{1,3,5,7\}$

$= \{1,3,4,5,6,7\}$

$A \cup D = \{1,2,3,4,5\} \cup \{2,4,6,8\}$

$= \{1,2,3,4,5,6,8\}$

$B - D = \{3,4,5,6,7\} - \{2,4,6,8\}$

$= \{3,5,7\}$

$A \cap B = \{1,2,3,4,5\} \cap \{3,4,5,6,7\}$

$= \{3,4,5\}$

$B \cap D = \{3,4,5,6,7\} \cap \{2,4,6,8\}$

$= \{4,6\}$

$C \cap D = \{1,3,5,7\} \cap \{2,4,6,8\}$

$= \{\} = \phi$

$A - D = \{1,2,3,4,5\} - \{2,4,6,8\}$

$= \{1,3,5\}$

31) a)

weight	No. of Students	Cumulative frequency
40-45	2	2
45-50	3	5
50-55	8	13 (c.f)
55-60	6 (f)	19 → Median class
60-65	6	25
65-70	3	28
70-75	2	30

$N = 30$

$\frac{N}{2} = \frac{30}{2} = 15$

55-60 is the median class

$l = 55,\ \frac{n}{2} = 15,\ c.f = 13,\ f = 6,\ h = 5$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - c.f}{f}\right) \times h$$

$$= 55 + \frac{15-13}{6} \times 5$$

$$= 55 + \frac{2}{6} \times 5$$

$$= 55 + \frac{5}{3}$$

$$= 55 + 1.66\ldots$$

$$= 56.66\ldots$$

$$= 56.67$$

∴ Median weight = 56.67 Kgs.

b) $A(1,2)$ $B(-1,b)$ $C(-3,-4)$
$x_1\ y_1$ $x_2\ y_2$ $x_3\ y_3$

Given that, A, B, C are collinear.

Then $ar(\Delta ABC) = 0$

$$\frac{1}{2}\left|x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\right| = 0$$

$$\frac{1}{2}\left|1(b+4) + (-1)(-4-2) + (-3)(2-b)\right| = 0$$

$$b + 4 + (-1)\times(-b) + (-6) + 3b = 0$$

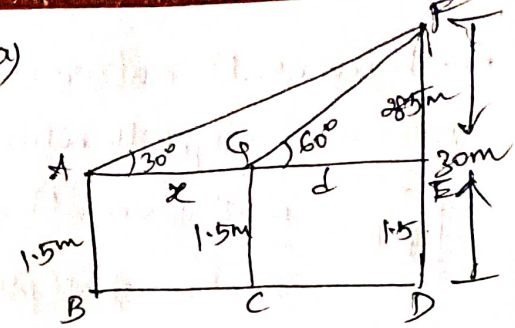
$$b + 4 + 6 - 6 + 3b = 0$$

$$4b + 4 = 0$$

$$4b = -4$$

$$\boxed{b = -1}$$

32) a)



Height of the boy = (AB) = 1.5 m

Height of the temple (DF) = 30 m

$\angle EAF = 30°$

$\angle EGF = 60°$

$EF = DF - DE = 30 - 1.5 = 28.5$ m

In ΔAEF, $\tan 30° = \frac{\text{opp. side}}{\text{adj side}} = \frac{EF}{AE}$

$$\frac{1}{\sqrt{3}} = \frac{28.5}{x+d}$$

$$x + d = 28.5 \times \sqrt{3} \longrightarrow (1)$$

In ΔGEF,

$$\tan 60° = \frac{EF}{GE}$$

$$\sqrt{3} = \frac{28.5}{d}$$

$$\sqrt{3}d = 28.5$$

$$d = \frac{28.5}{\sqrt{3}} \longrightarrow (2)$$

From (1) & (2),

$$x + d = 28.5\sqrt{3}$$

$$x + \frac{28.5}{\sqrt{3}} = 28.5\sqrt{3}$$

$$x = 28.5 \times \sqrt{3} - \frac{28.5}{\sqrt{3}}$$

$$x = \frac{28.5 \times 3 - 28.5}{\sqrt{3}}$$

$$x = \frac{57}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{57\sqrt{3}}{3}$$

$$= 19 \times 1.732$$

$$= 32.908$$

∴ Distance walked by the boy towards the temple (x) = 32.908 m

b) Total number of cards = 52

i) Let the event be E of getting a "King of Red colour"

No. of favourable outcomes to E = 2

Total no. of all possible outcomes = 52

$$P(E) = \frac{\text{No. of favourable outcomes}}{\text{Total no. of all possible outcomes}}$$

$$= \frac{2}{52} = \frac{1}{26}$$

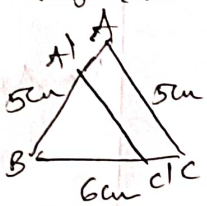
ii) $P(\text{face card}) = \frac{12}{52} = \frac{3}{13}$

iii) $P(\text{a jack of hearts}) = \frac{1}{52}$

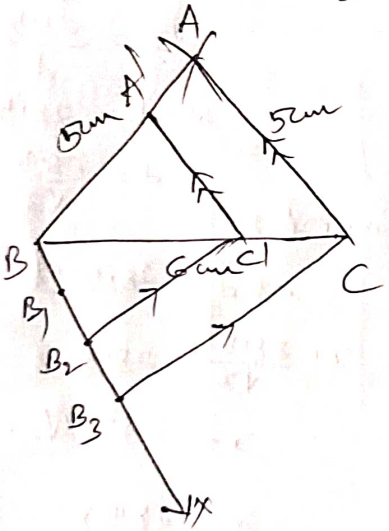
iv) $P(\text{a spade}) = \frac{13}{52} = \frac{1}{4}$

33) a)

Rough diagram



Main diagram



Steps of construction: 1) Construct $\triangle ABC$ with the given measurements.

2) Draw a ray $\overrightarrow{BX}$ making an acute angle with BC.

3) Mark B, B_1, B_2, B_3 on the ray $\overrightarrow{BX}$ such that $BB_1 = B_1B_2 = B_2B_3$

4) Join B_2C. Through B_3 draw a line parallel to B_2C which meets BC at C'

5) Through C' draw a line parallel to CA which meets BA at A'.

6) Hence $\triangle A'BC'$ is the required triangle whose sides are $\frac{3}{4}$ of $\triangle ABC$

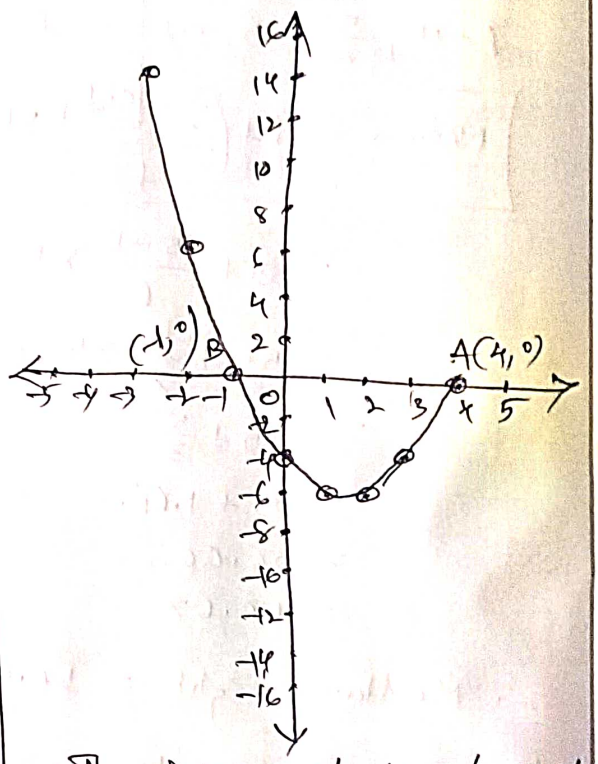
b) Let $y = x^2 - 3x - 4$

x	0	1	2	3	4	-1	-2	-3	-4
x^2	0	1	4	9	16	1	4	9	16
$-3x$	0	-3	-6	-9	-12	3	6	9	12
-4	-4	-4	-4	-4	-4	-4	-4	-4	-4
y	-4	-6	-6	-4	0	0	6	14	24

o.p's: (0,-4) (1,-6) (2,-6) (3,-4), (4,0) (-1,0) (-2,6) (-3,14) (-4,24)

Scale: On x-axis 1cm = 1 Unit

On y-axis 1cm = 2 units



The given quadratic polynomial represents a parabola which intersects the x-axis at A(4,0) and B(-1,0). So, the x-coordinates of these points are the zeroes of the quadratic polynomial

∴ Zeroes = {4, -1}

(-4) = -4 + 1

Verification: $x^2 - 3x - 4 = 0$

$x^2 - 4x + 1x - 4 = 0$

$x(x-4) + 1(x-4) = 0$

$(x-4)(x+1) = 0$

$x - 4 = 0$ & $x + 1 = 0$

$x = 4$ $\quad$ $x = -1$

∴ Zeroes = {4, -1}