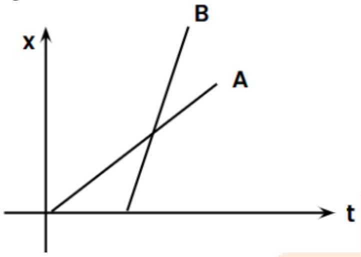
JEE-Mains-10-04-2023 [Memory Based]
[Morning Shift]

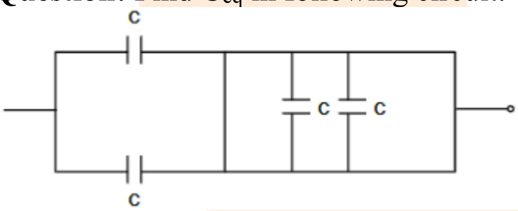
Physics

Question: Which all are correct?

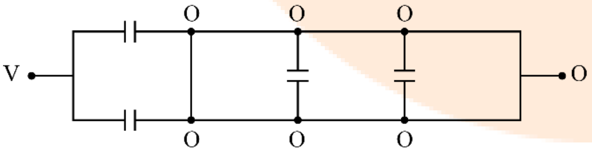


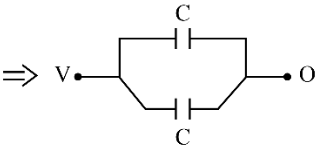
Options:
(a) A's home is closer
(b) B's home is closer
(c) A takes less time
(d) B travels fast
(e) A travels fast
Answer: (d)
Solution: Slope is more for B, so B travels faster.

Question: Find C_{eq} in following circuit.



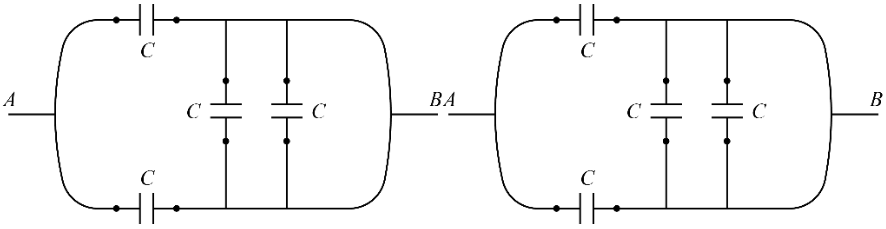
Answer: 2.00
Solution:





$C_{eq} = C + C = 2C$

Question: Find the equivalent capacitance across points A and B in the given electrical circuit.



Options:
(a) C/2
(b) 2C
(c) 5C/3
(d) 3C/4
Answer: (b)
Solution:

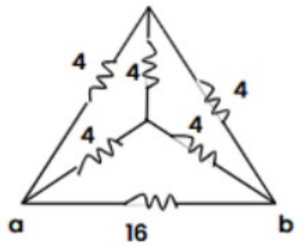
Question: A particle of mass in moving with a velocity v collides with a particle of mass 2m at rest and sticks to it. Velocity of combined mass is equal to
Options:
(a) v
(b) v/2
(c) v/3
(d) v/4
Answer: (c)
Solution:
$mv = 3mu$

$v = \frac{u}{3}$

Question: An object weighs 200 N at the surface of earth. Find the weight at a depth of R/2, where R is radius of earth.
Options:
(a) 100 N
(b) 300 N
(c) 50 N
(d) 150 N
Answer: (a)
Solution:
$$g' = g\left(1 - \frac{d}{R}\right)$$
$$\Rightarrow g' = g\left(1 - \frac{R}{2R}\right)$$
$$\Rightarrow \frac{g}{2}$$

Question: The equation of progressive wave is y = 5 sin(6t + 0.03x). Find the speed of wave.
Answer: 200.00
Solution: $u = \frac{\omega}{k} = \frac{6}{0.03} = 200$ m/s

Question: Find R_{eq} in following circuit



Options:
(a) 5/16 Ω
(b) 16/5 Ω
(c) 5/7 Ω
(d) 7/5 Ω
Answer: (b)
Solution:
$$\frac{1}{R_{eq}}=\frac{1}{8}+\frac{1}{8}+\frac{1}{16}=\frac{2+3+1}{16}\Rightarrow\frac{16}{5}=3.2$$

Question: A gas is having wavelength λ at temp 300 k if the temp is changed to 600 k what is the new de broglie wavelength?
Options:
(a) λ
(b) $\sqrt{2}\lambda$
(c) $1/\sqrt{2}\lambda$
(d) 2λ
Answer: (c)
Solution:
$$v_{rms}\propto\sqrt{T} \text{ and } \lambda=\frac{h}{mv_{rms}} \text{ or } \lambda\propto\frac{1}{\sqrt{T}}$$
So, $\frac{\lambda_1}{\lambda_2}=\sqrt{\frac{600}{300}}\Rightarrow\lambda_2=\frac{\lambda_1}{\sqrt{2}}$

Question: If P= $a^2b^3/(c\sqrt{d})$ if % change in a, b, c, d is 1%, 2%, 3%, and 4% find % change in P.
Options:
(a) 10%
(b) 13%
(c) 15%
(d) 18%
Answer: (b)
Solution:
$$P=\frac{a^2b^3}{c\sqrt{d}}$$
So,

$$100\times\frac{\Delta P}{P}=2\frac{\Delta a}{a}\times100+3\frac{\Delta b}{b}\times100+\frac{\Delta c}{c}\times100+\frac{1}{2}\frac{\Delta d}{d}\times100$$

$$=(2\times1\%)+(3\times2\%)+(3\%)+\left(\frac{1}{2}\times4\%\right)$$

$$=2+6+3+2=13\%$$

Question: S1 : In a reservoir of water at the same level pressure remains the same
S2: When pressure is applied on closed vessel it is equally transmitted throughout the water.
Options:
(a) S1 true, S2 False
(b) S1 False, S2 False
(c) S1 true, S2 True
(d) S1 False, S2 True
Answer: (c)
Solution:

Question: S1 : Maximum power is dissipated in a circuit with inductor capacitor and resistor in the condition of resonance
S2 : Maximum power is dissipated when the phase difference is 0
Options:
(a) S1 true, S2 False
(b) S1 False, S2 False
(c) S1 true, S2 True
(d) S1 False, S2 True
Answer: (c)
Solution:

Question: If Body is thrown with velocity u with angle of 15° with horizontal has range 50 m. Find the new range if same body is projected at an angle of 45° with velocity u.
Answer: 100.00
Solution:

$$R_1=\frac{u^2\sin(2\theta)}{g}$$

so $\dfrac{R_1}{R_2}=\dfrac{\sin(2\times15)}{\sin(2\times45)}$

$$\Rightarrow R_2=\frac{\sin90^\circ}{\sin30^\circ}\times R_1$$

$$=\frac{1}{1/2}\times50=100$$

Question: Assertion - number of turns is doubled then the current sensitivity gets doubled.
Reason - when number of turns is increased voltage sensitivity also increases with current sensitivity.
Options:
(a) A - true, R - False
(b) A - False, R - False
(c) A - true, R - True

(d) A - False, R - True
Answer: (a)
Solution:

Question: A monatomic gas in process A is compressed isothermally to 1/8th of its original volume and in another process B it is compressed adiabatically to 1/8th of its original volume. Find the ratio of final pressure in process B to that of process A.
Answer: 4.00
Solution:
Isothermal

$$PV = P_2\left(\frac{V}{8}\right)$$

$$P_2 = 8P$$

Adiabatic

$$PV^{5/3} = P_3\left[\frac{V}{8}\right]^{5/3}$$

$$P_3 = 2^5 P = 32P$$

$$\text{so } \frac{P_3}{P_2} = \frac{32p}{8p} = 4$$

Question: The equation of progressive wave is y = 5 sin(6t + 0.03x). Find the speed of wave.
Options:
(a) 50 m/s
(b) 100 m/s
(c) 150 m/s
(d) 200 m/s
Answer: (d)

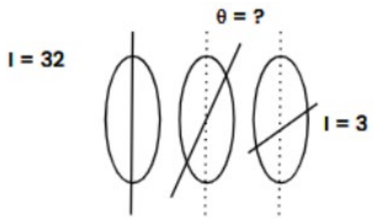
Solution: Speed = $\frac{\omega}{k} = \frac{6}{0.03} = \frac{600}{3} = 200$

Question: If 10 resistances of 10 Ω are connected to get maximum resistance and minimum resistance find R_{max}/R_{min}
Options:
(a) 50
(b) 100
(c) 150
(d) 200
Answer: (b)
Solution:
$R_{series} = R_{max} = 10R$

$R_{parallel} = R_{min} = \frac{R}{10}$

$$\text{So } \frac{R_{\max}}{R_{\min}} = \frac{10R}{R/10} = 100$$

Question: Unpolarized light has intensity I = 32. If 1st & 3rd Polaroids are perpendicular. Find angle between 1st & 2nd polaroids.



Options:
(a) 30°
(b) 45°
(c) 60°
(d) 70°
Answer: (a)
Solution:

$$I_f = \frac{I_0}{2}\cos^2(\theta)\cos^2(90-\theta)$$

$$3 = \frac{32}{2}\text{sen}^2\,\theta\cdot\cos^2\theta$$

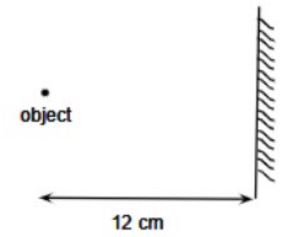
$$\frac{3}{4} = \left(2\sin\theta\cos\theta\right)^2$$

$$\frac{\sqrt{3}}{2} = \sin(2\theta)$$

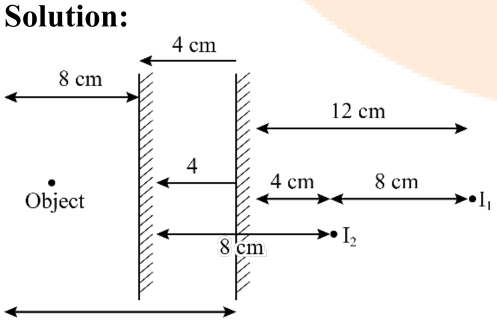
So $2\theta = 60°$

$\Rightarrow \theta = 30°$

Question: Mirror is moved by 4 cm towards object. Find shift in Image.



Answer: 8.00
Solution:



Question: In an AM wave, amplitude of modulating wave = 3 units and amplitude of carrier wave = 15 units. Find the ratio of maximum to minimum intensity $\frac{I_{max}}{I_{min}}$.

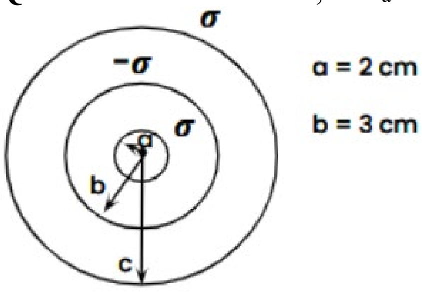
Options:

(a) 7 : 5
(b) 3 : 2
(c) 5 : 2
(d) 9 : 4
Answer: (d)
Solution:

$$\frac{I_{max}}{I_{min}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \left(\frac{15+3}{15-3}\right)^2$$

$$= \left(\frac{18}{12}\right)^2 = \frac{9}{4}$$

Question: Find radius C, if $V_a = V_c$



Answer: 5.00

Question: If angular momentum of electron in Bohr orbit is L find change in L if electron goes to 2nd orbit.
Options:
(a) L
(b) 2L
(c) 4L
(d) 5L
Answer: (a)

JEE-Mains-10-04-2023 [Memory Based] [Morning Shift]

Chemistry

Question: $Na_2O + H_2O \rightarrow 2X$
$Cl_2O_7 + H_2O \rightarrow 2Y$
Sum of Number of Oxygen atoms in X and Y.
Options:
(a) 10
(b) 5
(c) 6
(d) 8
Answer: (b)
Solution:

$Na_2O + H_2O \rightarrow 2NaOH$
$Cl_2O_7 + H_2O \rightarrow 2HClO_4$

Question: Find the sum of total number of lone pairs in IF_5 and IF_7.
Options:
(a) 1
(b) 2
(c) 3
(d) 4
Answer: (a)
Solution: IF_5 is sp^3d^2 with one lone pair and IF_7 is sp^3d^3 No lone pair

Question: Number of Diamagnetic & low spin species
Options:
(a) $[Co(NH_3)_6]^{3+}$
(b) $[CoCl_6]^{3-}$
(c) $[CoF_6]^{3-}$
(d) $[Fe(H_2O)_6]^{3+}]$
Answer: (a)
Solution: NH_3 act as SFL in Co^{+3} rest all are WFL.

Question: Find Number of moles and molecules of Oxygen at STP, given that volume of oxygen is 2.8375 L.
Options:
(a) 0.12 and 7.6×10^{22}
(b) 0.13 and 6.7×10^{22}
(c) 0.11 and 7.6×10^{23}
(d) 0.13 and 7.6×10^{21}

Answer: (a)

Solution: Moles $= \dfrac{2.8375}{22.4} = 0.1266$ and molecules $= 0.1266 \times 10^{23} = 7.6 \times 10^{22}$

Question: Which one does not stabilize 2° & 3° structure of proteins?
Options:
(a) Van Der Waals
(b) H-Bonds
(c) S–S bonds
(d) O–O bonds
Answer: (d)
Solution: 2° & 3° structure of proteins are Stabilized by the hydrogen bond, disulphuric linkages and Vanderwaals bond.

Question: $C + H_2O \xrightarrow{T_1} CO$
$C + H_2O \xrightarrow[\text{Catalyst}]{T_2} CO$
Options:
(a) $T_1 > T_2$
(b) $T_2 > T_1$
(c) $T_1 = T_2$
(d) $T_1 = 100$ K, $T_2 = 1270$ K
Answer: (a)
Solution: $C(s) + H_2O(g) \xrightarrow{1270\,K} CO(g) + H_2(g)$

$CO(g) + H_2O(g) \xrightarrow[\text{Catalyst}]{673\,K} CO_2(g) + H_2(g)$

Question: Stabilizer used for concentrating sulphuric ore:
Options:
(a) Fatty acid
(b) Pine oil
(c) Cresol
(d) Xanthates
Answer: (c)
Solution: Particles and froth stabilizers (e.g., cresols, aniline) stabilize the forth.

Question: Number of species having bent shape?
N_3^-, I_3^-, NO_2^-, O_3, SO_2
Options:
(a) 2
(b) 3
(c) 4
(d) 5
Answer: (b)
Solution: NO_2^-, O_3 and SO_2 are bent shape

Question: Prolonged heating is avoided during preparation of ferrous ammonium sulphate is to prevent

Options:

(a) Oxidation

(b) Hydrolysis

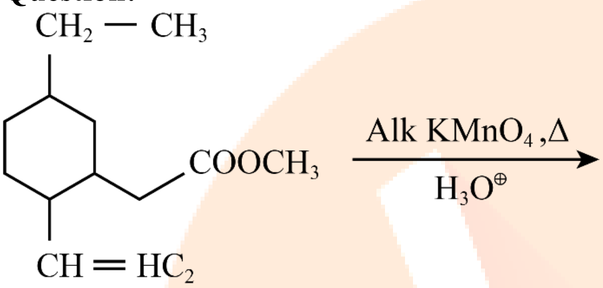
(c) Reduction

(d) Breaking

Answer: (a)

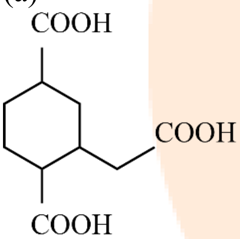
Solution: Avoid prolonged heating while preparing crystals of ferrous ammonium sulphate, as it may oxidise ferrous ions to ferric ions and change the stoichiometry of the crystals.

Question:

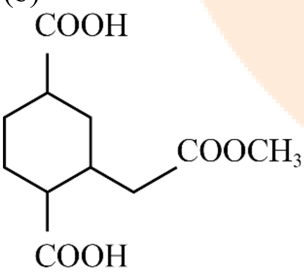


Options:

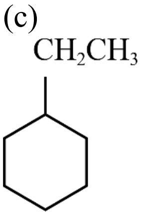
(a)



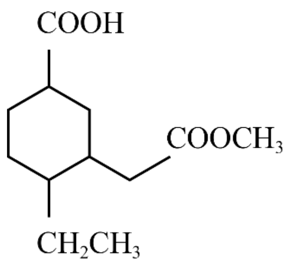
(b)



(c)



(d)



Answer: (a)
Solution: Alk $KMnO_4$ oxidises to carboxylic acid.

Question: Match the following.

Column I (Sources)	**Column II** (Waste Produces)
(A) Steel Industry	**(P)** Fly ash
(B) Thermal plant	**(Q)** Slag
(C) Paper mills	**(R)** Gypsum
(D) Fertilizers	**(S)** Biodegradable waste

Options:
(a) A – P; B – R; C – Q; D – S
(b) A – Q; B – P; C – R; D – S
(c) A – S; B – P; C – Q; D – R
(d) A – Q; B – P; C – S; D – R
Answer: (d)
Solution: Fact based.

Question: The compound which does not exist
Options:
(a) BeH_2
(b) NaO_2
(c) $PbEt_4$
(d) $(NH_4)_2BeF_4$
Answer: (b)
Solution: Superoxide's are formed by (K, Rb, Cs)

Question: Select the correct option:
$2CO(g) + O_2(g) \rightarrow 2CO_2(g)$; $\Delta H = -x$ KJ/mol
$C(graphite) + O_2(g) \rightarrow CO_2(g)$; $\Delta H = -y$ KJ/mol
Then ΔH for, $C(graphite) + \frac{1}{2} O_2(g) \rightarrow CO(g)$:
Options:
(a) $x-\frac{y}{2}$
(b) $\frac{x-2y}{2}$
(c) $\frac{x+2y}{2}$

(d) $\frac{x-y}{2}$

Answer: (b)

Solution:

$CO_2 \rightarrow \frac{1}{2}O_2 + CO$; $\Delta H = \frac{x}{2}$

$C(graphite) + O_2 \rightarrow CO_2$; $\Delta H = -y$

$C(graphite) + \frac{1}{2}O_2 \rightarrow CO$: $\Delta H = \frac{x}{2} - y$

$$\Delta H = \frac{x-2y}{2}$$

Question: Enthalpy of adsorption and enthalpy of formation micelle is respectively

Options:

(a) Positive, Positive

(b) Positive, Negative

(c) Negative, Positive

(d) Negative, Negative

Answer: (c)

Solution: Adsorption is exothermic and micelles is endo.

Question: The pressure value of a gas is 930.2 mm Hg. The volume is then reduced to 40% of its initial value at a constant temperature. Then what is the final pressure (in mm Hg)

Options:

(a) 2325.5

(b) 2235.5

(c) 2532.5

(d) None

Answer: (a)

Solution: $P_1V_1 = P_2V_2$

$930.2 \times V_1 = 0.4\ V_1 \times P_2$

$P_2 = 2325.5$ mm Hg

Question: Read the following two statements.

Statement I: Potassium dichromate is used in volumetric analysis.

Statement II: $K_2Cr_2O_7$ is more soluble in water than $Na_2Cr_2O_7$

Options:

(a) Both statements I and II are correct

(b) Both statements I and II incorrect

(c) Statement I is correct and II is incorrect

(d) Statement I is incorrect and II is correct

Answer: (c)

Solution: Fact based

Question: The degree of dissociation of monobasic acid is 0.3. By what percent is the observed depression in freezing point greater than the calculated depression in freezing point?

Options:

(a) 30%

(b) 20%

(c) 10%

(d) 45%

Answer: (a)

Solution:

$$\alpha = \frac{i - 1}{n - 1}$$

n = 2

$\therefore \alpha = i - 1$ or $i = 1.3$

$\therefore$ 30%

JEE-Mains-10-04-2023 [Memory Based] [Morning Shift]

Mathematics

Question: $\left|n^2-10n+19\right|<6,\ n\in Z$. Find number of possible values of n _____.
Answer: 6.00
Solution:

$\left|n^2-10n+19\right|<6,\ n\in Z$

$\Rightarrow\left|(n-5)^2-6\right|<6$

$\Rightarrow 0<(n-5)^2<12$

$\Rightarrow (n-5)^2=1,4,9$

$\Rightarrow n-5=\pm1,\pm2,\pm3$

6 values of n.

Question: Find the coefficient of x^7 in $\left(1-x+2x^3\right)^{11}$.
Answer: 2310.00
Solution:

$\left(1-x+2x^3\right)^{11}$

${}^{11}C_0(1-x)^{11}+{}^{11}C_1(1-x)^{10}2x^3+{}^{11}C_2(1-x)^9\left(2x^3\right)^2+\ldots.$

${}^{11}C_0\times{}^{11}C_7(-x)^7+{}^{11}C_1\times{}^{10}C_4x^4 2x^3+{}^{11}C_2\times{}^{9}C_1(-x)4x^6$

$=2310$

Question: Coefficient of x^7 in $\left(ax-\frac{1}{bx^2}\right)^{13}$ is equal to coefficient of x^{-5} in $\left(ax+\frac{1}{bx^2}\right)^{13}$.

Find a^4b^4.
Answer: 22.00
Solution:

$$^{13}C_2(ax)^{11}\left(-\frac{1}{bx^2}\right)^2$$

$$^{13}C_6(ax)^7\left(\frac{1}{bx^2}\right)^6$$

$$^{13}C_2\frac{a^{11}}{b^2}={}^{13}C_6\frac{a^7}{b^6}$$

$$a^4b^4=\frac{^{13}C_6}{^{13}C_2}$$

Question: Dice is thrown 2 times. N is sum of numbers shown. $P\left(2^N<N!\right)=\frac{m}{n}$. Find $4m-3n$.

Answer: 8.00

Solution:

Sum of numbers

$2^2<2!$

$2^3<3!$

$2^4<4!$

$2^5<5!$

$\vdots$

$2^{12}<12!$

Then Probability of numbers

$$1-\left(\frac{1+2}{36}\right)$$

$$=\frac{11}{12}=\frac{m}{n}$$

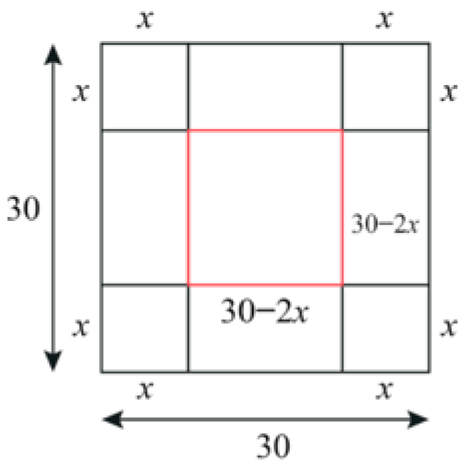
$$4m-3n=4\times11-3\times12$$

$$4m-3n=8$$

Question: An open box is made out of a square sheet of side 30 m by cutting out 4 squares from corners such that volume is maximum. Find the surface area of box.

Answer: 800.00

Solution:



$V=(30-2x)^2\times x$

$\frac{dV}{dx}=0\Rightarrow x=5,15$

$x=5$ accepted

Surface area

$=(20)^2+4(5\times20)$
$=400+400$
$=800$

Question: $\sim[(p\vee q)\wedge(q\vee(\sim r))]$ is equivalent to

Answer: ()
Solution:

$\sim[(p\vee q)\wedge(q\vee(\sim r))]=-(p\vee q)\vee(\sim q\wedge r)$

$\equiv(\sim p\wedge\sim q)\vee(\sim q\wedge r)$
$\equiv(\sim p\vee r)\wedge(\sim q)$

Question: Couples play in mixed doubles, such that no husband and wife plays in the same match. If total 840 games are played then find the total number of people.
Answer: 16.00
Solution:

Number of couples = n

${}^{n}C_2\cdot{}^{n-2}C_2\times2=840$

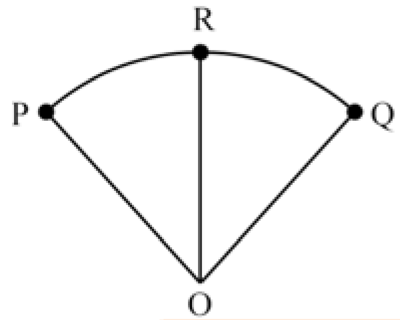
$\Rightarrow n(n-1)(n-2)(n-3)=840\times2$

$$\Rightarrow n(n-1)^2(n-2) = 21\times 40\times 2$$
$$= 7\times 3\times 8\times 5\times 2$$
$$= 8\times 7\times 6\times 5$$

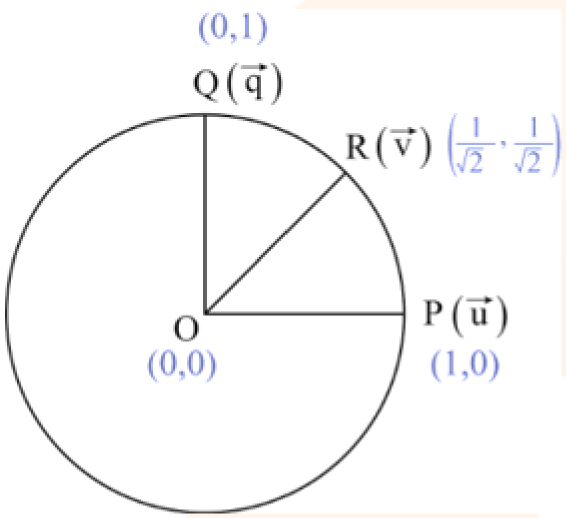
$n = 8$

Number of persons = 16

Question: Arc PQ subtends right angle at centre. R is the midpoint of arc PQ. $\overrightarrow{OP} = \vec{u}$, $\overrightarrow{OR} = \vec{v}$, $\overrightarrow{OQ} = \alpha\vec{u} + \beta\vec{v}$. Find quadratic equation whose roots are α, β^2.



Answer: $x^2 - x - 2 = 0$
Solution:



$$\hat{j} = \alpha\hat{i} + \beta\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)$$

$$\beta = \sqrt{2},\ \alpha + \frac{\beta}{\sqrt{2}} = 0$$

$$\Rightarrow \alpha = -1$$

Equation: $x^2 - (\alpha + \beta^2)x + \alpha\beta^2 = 0$

$x^2 - x - 2 = 0$

Question: $3, 8, 13, \ldots, 373$ are in AP. Find the sum of terms not divisible by 3.
Answer: 9525.00
Solution:

$T_n = a + (n-1)d$
$373 = 3 + (n-1)5$

$\Rightarrow n = 75$

Sum $= \frac{n}{2}(a + l)$

$= \frac{72}{2}(3 + 373) = 14100$

Sum = 14100

Number divisible by 3 are $3, 18, \ldots 363$

$363 = 3 + (k-1)15$

$\Rightarrow k = 25$

Sum $= \frac{25}{2}[3 + 363] = 4575$

$\therefore$ Required sum $= 14100 - 4575 = 9525$.

Question: If $f(x)$ is differentiable and $x^2 f(x) - x = 4\int_0^x t\, f(t)\, dt$ & $f(1) = \frac{2}{3}$ then $18 f(3) = ?$
Answer: 160.00
Solution:

$x^2 f(x) - x = 4\int_0^x t\, f(t)\, dt$ & $f(1) = \frac{2}{3}$

$\Rightarrow 2xf(x) + x^2 f'(x) - 1 = 4xf(x)$

$\Rightarrow x^2 f'(x) - 2xf(x) = 1$

$\Rightarrow \frac{x^2 f'(x) - 2xf(x)}{x^4} = \frac{1}{x^4}$

$$\Rightarrow \frac{d}{dx}\left(\frac{f(x)}{x^2}\right)=\frac{1}{x^4}$$

$$\Rightarrow \frac{f(x)}{x^2}=\frac{x^{-3}}{-3}+C$$

$$\Rightarrow f(x)=\frac{-1}{3x}+Cx^2$$

$$f(1)=\frac{-1}{3}+C\Rightarrow C=1$$

$$f(x)=x^2-\frac{1}{3x}$$

$$f(3)=9-\frac{1}{9}=\frac{80}{9}$$

$$18f(3)=160$$

Question: A parabola pass through (-1, 0), (0, 1) & (1, 0), has equation $y=P(x)$. If 'A' is the area between $(x+1)^2+(y-1)^2\le 1$ and $y\le P(x)$. Find $(12A+4)$.

Answer: 3π

Solution:

$$x^2=-4a(y-1)$$

The point is (1, 0)

$$(1,0)\Rightarrow 1=-4a(-1)$$

$$a=\frac{1}{4}$$

$$x^2=-(y-1)$$

Area = $\frac{1}{2}(1)^2\times\frac{\pi}{2}-\frac{1}{2}\times 1\times 1$

$$(12A+4)=3\pi$$

Question: If $\frac{x^2+y^2}{2xy}=\frac{dy}{dx}$; $y(2)=0,\ x>0.$ Find $y(8)$.

Answer: 48.00

Solution:

$$\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$$

Let $y=vx$

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$v+x\frac{dv}{dx}=\frac{x^2+v^2x^2}{2\cdot x\cdot vx}$$

$$v+x\frac{dv}{dx}=\frac{1+v^2}{2v}$$

$$\Rightarrow x\frac{dv}{dx}=\frac{1+v^2}{2v}-v$$

$$\Rightarrow x\frac{dv}{dx}=\frac{1-v^2}{2v}$$

$$\Rightarrow \int\frac{2vdv}{1-v^2}=\int\frac{dx}{x}$$

$$\Rightarrow -\int\frac{-2vdv}{1-v^2}=\int\frac{dx}{x}$$

$$\Rightarrow -\ln\left|1-v^2\right|=\ln|x|+\ln C$$

$$y(2)=0$$

$$x=2;\ y=0\Rightarrow v=\frac{y}{x}=0$$

$$-\ln|1-0|=\ln(2)+\ln c$$
$$\ln c=-\ln 2$$

$$-\ln\left|1-v^2\right|=\ln|x|-\ln 2$$

$$x=8;\ v=\frac{y}{x}=\frac{y}{8}$$

$$-\ln\left|1-\frac{y^2}{64}\right|=\ln 8-\ln 2$$

$$-\ln\left|1-\frac{y^2}{64}\right|=\ln 4$$

$$1-\frac{y^2}{64}=\frac{1}{4}$$

$$\Rightarrow \frac{3}{4}=\frac{y^2}{64}$$

$$\Rightarrow y^2=48$$

Question: Find shortest distance between the lines:
$\frac{x-2}{1}=\frac{y}{2}=\frac{z-1}{0}$; $\frac{x-2}{1}=\frac{y-3}{-2}=\frac{z-1}{2}$.
Answer: 1.00
Solution:

$$\Delta=\frac{mag\begin{vmatrix}0 & -3 & 0\\ 1 & 2 & 0\\ 1 & -2 & 2\end{vmatrix}}{mag\begin{vmatrix}i & j & k\\ 1 & 2 & 0\\ 1 & -2 & 2\end{vmatrix}}=\frac{6}{\sqrt{4^2+2^2+4^2}}=\frac{6}{6}=1$$

Question: $I=\int e^{\sin^2 x}\left(\cos x\sin 2x-\sin x\right)dx$. $I(0)=$ $I\left(\frac{5}{3}\right)=?$

Answer: $e^{\sin^2 x}\left(\cos x\right)$
Solution:

$$I=\int e^{\sin^2 x}\left(\cos x\sin 2x-\sin x\right)dx$$

$$\int e^{g(x)}\left(f\times g'+f'\right)dx=e^g\times f$$

$$g(x)=\sin^2 x$$
$$f(x)=\cos x$$

$$e^{\sin^2 x}\times\cos x+c$$

Question: 1, 2, 3, 4, 5, 6, 7. In how many ways they can be arranged such that neither 1, 5, 3 nor 2, 4, 6, 7 strings should come.
Answer: 4898.00
Solution:

$$A=1\,5\,3$$

$$B=1\,2\,3\,4$$

$$n\left(A^c \cap B^c\right) = \text{Total} - n\left(A \cup B\right)$$
$$= 7! - \left(n\left(A\right) + n\left(B\right) - n\left(A \cap B\right)\right)$$
$$= 7! - \left(5! + 4! - 2!\right)$$
$$= 5! \times 41 - 22$$
$$= 4920 - 22$$
$$= 4898$$

Question: $\left|3\,adj\left(\det\left(3A\right)A^2\right)\right| = ?;\ \left|A_3\right| = 2,$

Answer: $3^{21} \times 2^{10}$

Solution:

$$\left|3A\right| = 27\left|A\right|$$
$$= 54$$

$$\left|3adj\left(54A^2\right)\right|$$

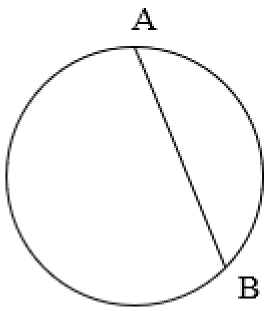
$$\left|3 \times 54^2\,adjA^2\right|$$

$$\left(3 \times 54^2\right)^3 \times \left|adj\ A\right|^2$$

$$3^3 \times 2^6 \times 27^6 \times 2^4$$

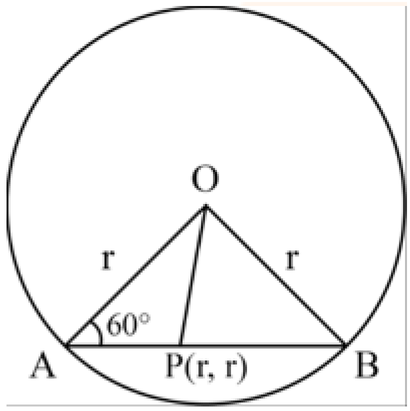
$$2^{10} \times 3^{21}$$

Question: Length of $\left|AB\right| =$ radius $= \lambda$. Find the locus of point which divides AB in the ratio $2:3$.



Answer: $x^2 + y^2 = \left(\frac{19}{25}\right)\lambda^2$

Solution:



$\angle OAP = 60°$

$$AP = \frac{2r}{5}$$

$$\cos 60° = \frac{OA^2 + AP^2 - OP^2}{2\,OA \cdot OP}$$

$$\frac{1}{2} = \frac{r^2 + \frac{4r^2}{25} - OP^2}{2r \cdot \frac{2r}{5}}$$

$$\Rightarrow \frac{2r^2}{5} = \frac{29r^2}{25} - OP^2$$

$$\Rightarrow OP^2 = \frac{29r^2}{25} - \frac{2r^2}{5}$$

$$\Rightarrow x^2 + y^2 = \frac{19r^2}{25} = \frac{19\lambda^2}{25}$$

Question: If the mean of the following data is 28, then find the variance.

Class	0-10	10-20	20-30	30-40	40-50
f_1	2	3	x	5	4

Answer: 121.00
Solution:

$$28 = \frac{10 + 45 + 25x + 140 + 225}{14 + x}$$

$x = 6$

Variance = $\frac{1}{20}\sum f_i x_i^2 - (28)^2 = 121$

Question: Let $f(x)=\int_0^x\left((a-1)\left(x^2+t+1\right)^2-(a+1)\left(t^4+t^2+1\right)\right)dt$. Then the total number of integral values of 'a' for which of $f'(x)=0$ has no real root.

Answer: 3.00

Solution:

$$f(x)=\int_0^x\left((a-1)\left(x^2+t+1\right)^2-(a+1)\left(t^4+t^2+1\right)\right)dt$$

$$f'(x)=0\Rightarrow(a-1)\left(x^2+x+1\right)^2-(a+1)\left(x^4+x^2+1\right)=0$$

$$\Rightarrow\left(x^2+x+1\right)\left[(a-1)\left(x^2+x+1\right)-(a+1)\left(x^2-x+1\right)\right]=0$$

$$\Rightarrow(a-1-a-1)x^2+(a-1+a+1)x+a-1-a-1=0$$

$$\Rightarrow-2x^2+2ax-2=0$$

$$\Rightarrow x^2-ax+1=0$$

$$D<0$$

$$a^2-4<0\Rightarrow a\in(-2,2)$$

$$\Rightarrow a=-1,0,1$$

3 values of a.

Question: $B(2,-4,2),P(-1,2,3),A(0,1,3),C(-4,2,0)$. Find projection of $\overrightarrow{OP}$ on $\overrightarrow{AB}\times\overrightarrow{AC}$

.

Answer: ()

Solution:

$$\overline{OP}\cdot\frac{\left(\overline{AB}\times\overline{AC}\right)}{\left|\overline{AB}\times\overline{AC}\right|}$$

$$\overline{AB}\times\overline{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -5 & -1\\ -4 & 1 & -3\end{vmatrix}$$

$$=i(16)-j(-10)+k(-18)$$

$$\left(-\hat{i}+2\hat{j}+3\hat{k}\right)\cdot\frac{\left(16\hat{i}-10\hat{j}-18\hat{k}\right)}{\sqrt{16^2+10^2+(18)^2}}$$

$$= \frac{-16+20-54}{\sqrt{256+100+324}}$$

$$= \frac{-50}{\sqrt{680}}$$