44 YEARS OF EXCELLENCE

NARAYANA EDUCATIONAL INSTITUTIONS

NARAYANA GRABS THE LION'S SHARE IN JEE-ADV.2022

5 RANKS in OPEN CATEGORY ONLY FROM NARAYANA IN TOP 10 AIR

JEE MAIN (APRIL) 2023 (10-04-2023-FN)

Memory Based Question Paper

MATHEMATICS

MATHEMATICS

1. If $|n^2 - 10n + 19| < 6$, then find number of integral value of n

Ans. 6

Sol. $-6 < n^2 - 10n + 19 < 6$

$\Rightarrow n^2 - 10n + 25 > 0$ and $n^2 - 10n + 13 < 0$

$(n-5)^2 > 0 \quad n \in [5 - 2\sqrt{3}, 5 + 2\sqrt{3}]$

$n \in R - \{5\}$

$\therefore n \in [1.3, 8.3]$

$\Rightarrow n = 2, 3, 4, 6, 7, 8$

2. How many mixed doubles tennis matches can be organised among 8 couples, such that no couple plays in the same match ? [Data may be different]

Ans. **(840)**

Sol. ${}^8C_2 \times {}^6C_2 \times 2$

$= \frac{8\times7}{2}\cdot\frac{6\times5}{2}\cdot 2$

$= 28 \times 15 \times 2$

$= 840$

3. Find the coefficient of x^7 in the expansion of $(1 - x + x^3)^{10}$.

(1) 780 (2) 1140 (3) 900 (4) 520

Ans. **(1)**

Sol. General term $= \frac{10!}{r_1!.r_2!.r_3!}(-1)^{r_2}.x^{r_2+3r_3}$ where $r_1 + r_2 + r_3 = 10$ and $r_2 + 3r_3 = 7$

Now

r_1	r_2	r_3
3	7	0
5	4	1
7	1	2

Reqd. coefficient $= \frac{10!}{3!.7!}(-1)^7 + \frac{10!}{5!.4!}(-1)^4 + \frac{10!}{7!.2!}(-1)^1$

$= -120 + 1260 - 360$

$= 780$

4. If $|A| = 2$ where A is a 3×3 matrix, then $|3A \text{ adj } (3|A|\, A^2)|$.

(1) 6^9 (2) $6^9 \cdot 2^5$ (3) $4\cdot6^9$ (4) $3^9\cdot2^{10}$

Ans. **(3)**

Sol. $|3a \text{ adj}(3|A|A^2)| = |3A|\cdot|\text{adj}(6A^2)|$

$= 3^3\,|A|\cdot|6A^2|^2$

$= 3^3 \times 2 \times (6^3|A^2|)^2 = 6^6\times3^3\times2.|A|^4$

$= 6^6\cdot3^3\cdot2^5$

$= 4\cdot6^9$

5. Let $\int e^{\sin^2 x}(\sin 2x.\cos x - \sin x)\,dx = I(x)$. If $I(0) = 1$, then find $I\left(\frac{\pi}{2}\right)$,

Ans. **(0)**

Sol. $I(x) = \int \frac{e^{\sin^2 x}.\sin 2x}{II}.\frac{\cos x}{I}\,dx - \int e^{\sin^2 x}.\sin x\,dx$

using IBP

$= \cos x.\ e^{\sin^2 x} - \int(-\sin x).e^{\sin^2 x}dx - \int e^{\sin^2 x}.\sin x\,dx$

$\Rightarrow I(x) = e^{\sin^2 x}.\cos x + c$

put x = 0, c = 0

$\therefore I\left(\frac{\pi}{2}\right) = e^1.\cos\frac{\pi}{2} = 0$

6. In the series 3 + 8 + 13 + ... + 373. Find sum of numbers which are not divisible by 3.

Ans. **(9525)**

Sol. Required sum = (3 + 8 + 13 + 18 + ... +373) – (3 + 18 + 33 + ... + 363)

$= \frac{75}{2}(3 + 373) - \frac{25}{2}(3 + 363)$

$= 75 \times 188 - 25 \times 183$

$= 9525$

7. If y = f(x) satisfies, $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$ & $f(2) = 0$, then f(3) =

(1) $\sqrt{3}$ (2) $2\sqrt{3}$ (3) $\sqrt{2}$ (4) 3

Ans. **(1)**

Sol. $\frac{dy}{dx} = \frac{1 + \left(\frac{y}{x}\right)^2}{2\left(\frac{y}{x}\right)}$

Let y = tx

$\Rightarrow t + x\frac{dt}{dx} = \frac{1+t^2}{2t}$

$\Rightarrow x\frac{dt}{dx} = \frac{1-t^2}{2t}$

$\Rightarrow \int \frac{2t}{1-t^2}dt = \int \frac{dx}{x}$

$\Rightarrow -\ell n|1 - t^2| = \ell nx + \ell nc$

$\Rightarrow (1 - t^2)(cx) = 1$

$$\Rightarrow \left(1-\frac{y^2}{x^2}\right)cx = 1$$

$$y(2) = 0 \Rightarrow c = \frac{1}{2}$$

$$\left(1-\frac{y^2}{x^2}\right)\cdot\frac{1}{2}x = 1$$

t $x = 3$

$$\left(1-\frac{y^2}{9}\right)\times\frac{3}{2} = 1$$

$9 - 4^2 = 6$

$y^2 = 3$

$y = \pm\sqrt{3}$

8. If z is a complex number such that $\frac{z-2i}{z+2i}$ is purely imaginary, then $|z| =$

Ans. **(2)**

Sol. $\alpha + \bar{\alpha} = 0$

$$\frac{z-2i}{z+2i} + \frac{\bar{z}+2i}{\bar{z}-2i} = 0$$

$\Rightarrow |z| = 2$

9. Consider $f(x) = \begin{cases} x[x] & ; \quad x \in (-2, 0) \\ (x-1)[x] & ; \quad x \in [0, 2) \end{cases}$

If m is number of points of discontinuity and n is number of points of non-differentiable of f(x), then m + n =

Ans. **(4)**

Sol. $f(x) = \begin{cases} -2x & ; \quad x \in (-2, -1) \\ -x & ; \quad x \in [-1, 0) \\ 0 & ; \quad x \in [0, 1) \\ x-1 & ; \quad x \in [1, 2) \end{cases}$

$\therefore$ m = 1 & n = 3

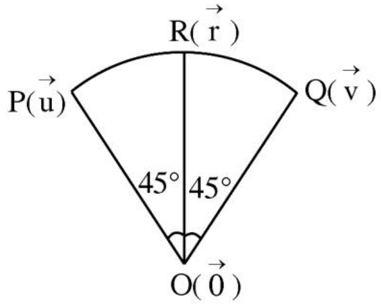
$\therefore$ m + n = 4

10. If arc PQ of a circle subtends 90° at point O, and $\overrightarrow{OP} = \vec{u}$ and $\overrightarrow{OQ} = \vec{v}$, then $\overrightarrow{OR} = a\vec{u} + b\vec{v}$, where R is mid-point of arc PQ. Then ab =

(1) 1 (2) 0.8 (3) $\frac{1}{2}$ (4) $\frac{4}{7}$

Ans. **(3)**

Sol.



$\vec{r} = a\vec{u} + b\vec{v}$

dot with $\vec{u} \Rightarrow \vec{u}.\vec{r} = a|\vec{u}|^2 + 0$

$\Rightarrow \frac{1}{\sqrt{2}} = a$

dot with $\vec{v} \Rightarrow \vec{r}.\vec{v} = b|\vec{v}|^2$

$\Rightarrow b = \frac{1}{\sqrt{2}}$

$\therefore ab = \frac{1}{2}$

11. If a differentiable function f(x) satisfies $x^2 f(x) - x = 4.\int_0^x t\,f(t)dt$ (for all $x > 0$), $f(1) = -1$, then $f(2) =$

(1) 1 (2) $\frac{-1}{11}$ (3) $-\frac{17}{6}$ (4) $\frac{1}{4}$

Ans. **(3)**

Sol. Differentiate the given equation

$\Rightarrow 2x\,f(x) + x^2 f'(x) - 1 = 4x\,f(x)$

$\Rightarrow x^2\frac{dy}{dx} - 2xy = 1$

$\Rightarrow \frac{dy}{dx} + \left(-\frac{2}{x}\right)y = \frac{1}{x^2}$

$I.F = e^{\int -\frac{2}{x} \ell n} = e^{-2\ell n x} = \frac{1}{x^2}$

$\therefore y\left(\frac{1}{x^2}\right) = \int \frac{1}{x^4}dx$

$\Rightarrow \frac{y}{x^2} = \frac{-1}{3x^3} + c$

$\Rightarrow y = -\frac{1}{3x} + cx^2$

$\because f(1) = -1 = -\frac{1}{3} + c \Rightarrow c = -\frac{2}{3}$

$\therefore f(2) = \frac{-1}{6} - \frac{2}{3} \times 4 = -\frac{17}{6}$

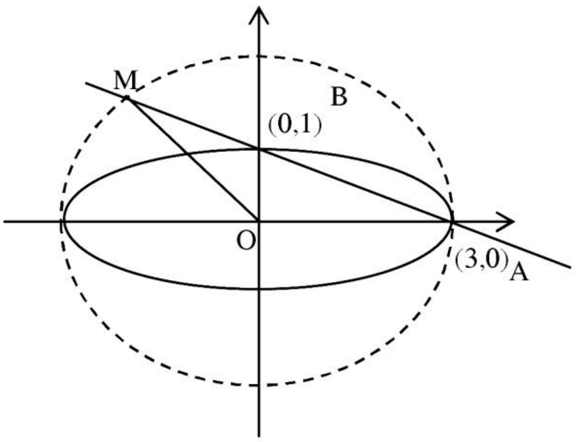
12. An ellipse $x^2 + 9y^2 = 9$ intersect positive x-axis and y-axis at A and B respectively. A circle with diameter equal to major axis is drawn, line AB intersect circle at R, (O being origin) Area of triangle ARO is $\frac{m}{n}$ (m and n are coprime) then the equation whose roots are m and n, is

(1) $2x^2 - 37x + 270 = 0$ (2) $x^2 - 34x + 189 = 0$

(3) $x^2 - 37x + 270 = 0$ (4) $2x^2 - 34x + 189 = 0$

Ans. **(3)**

Sol.



For line AB $x + 3y = 3$ and circle is $x^2 + y^2 = 9$

$$(3 - 3y)^2 + y^2 = 9$$

$\Rightarrow 10y^2 - 18y = 0$

$\Rightarrow y = 0, \frac{9}{5}$

$\therefore$ Area $= \frac{1}{2} \times 3 \times \frac{9}{5} = \frac{27}{10}$

Equation is $x^2 - 37x + 270 = 0$

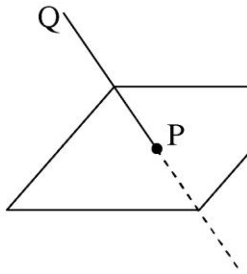
13. Intersection point of line $\frac{x-3}{-2} = \frac{y-5}{3} = \frac{z-7}{5}$ and the plane $x + y + z = 3$ is P then distance of P from plane $2x + 5y + 7z = 32$ is

(1) $\frac{44}{\sqrt{78}}$ (2) $\frac{41}{\sqrt{78}}$

(3) $\frac{45}{\sqrt{78}}$ (4) $\frac{47}{\sqrt{78}}$

Ans. **(1)**

Sol.



$$\frac{x-3}{-2}=\frac{y-5}{3}=\frac{z-7}{5}=\lambda$$

$P(-2\lambda + 3, 3\lambda + 5, 5\lambda + 7)$lies on $x + y + z = 3$

$-2\lambda + 3 + 3\lambda + 5 + 5\lambda + 7 = 3$

$6\lambda + 15 = 3$

$6\lambda = -12$

$\lambda = -2$

$P(7, -1, -3)$

Distance of P from plane $2x + 5y + 7z = 32$

$$= \left|\frac{14-5-21-32}{\sqrt{4+25+49}}\right|$$

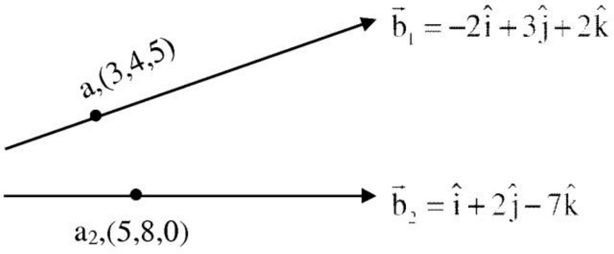
$$= \left|\frac{9-53}{\sqrt{29+49}}\right| = \left|\frac{44}{\sqrt{78}}\right|$$

14. Find the shortest distance between lines $\frac{x-3}{-2}=\frac{y-4}{3}=\frac{z-5}{2}$ and $\frac{x-5}{1}=\frac{y-8}{2}=\frac{z}{-7}$ is

(1) $\frac{63}{\sqrt{818}}$ (2) $\frac{60}{\sqrt{718}}$ (3) $\frac{55}{\sqrt{618}}$ (4) $\frac{73}{\sqrt{518}}$

Ans. **(1)**

Sol.



Shortest distance = $\left|\frac{(\vec{a}_2-\vec{a}_1 \cdot (\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$

$\vec{a}_2-\vec{a}_1 = 2\hat{i}+4\hat{i}-5\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} i & j & k \\ -2 & 3 & 2 \\ 1 & 2 & -7 \end{vmatrix}$$

$= \hat{i}(-25) - \hat{j}(12) + \hat{k}(-7)$

$= -25\hat{i} - 12\hat{j} - 7\hat{k}$

Shortest distance = $\left|\dfrac{-50-48+35}{\sqrt{625+144+49}}\right|$

$= \left|\dfrac{63}{\sqrt{818}}\right|$

15. If mean of above observation is 30. Find variance

0-10	10-20	20-30	30-40	40-50
1	2	x	6	4

Ans. (115)

Sol.

x_i	f_i	$x_i f_i$
5	1	5
15	2	30
25	x	25x
35	6	210
45	4	180

$\sum f_i = 13 + x$

$\sum x_i f_i = 425 + 25x$

$\bar{x} = \dfrac{425+25x}{13+x} = 30$

$25x + 425 = 390 + 30x$

$35 = 5x$

$x = 7$

$\sum f_i = 20$

Variance $= \sigma^2 = \dfrac{1}{20}\ (\sum f_i x^2 - 30^2)$

$= \dfrac{1}{20}(1 \times 25 + 2 \times 225 + 7 \times 25^2 + 6 \times 35^2 + 4 \times 45^2) - 30^2$

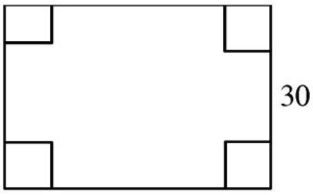
$= \dfrac{1}{20}(25 + 450 + 4375 + 7350 + 8100) - 900$

$= 1015 - 900 = 115$

16. A box made by cutting the four square from corner of a square sheet of side length 30 cm. If the volume of box is maximum then find the surface area of open box.

Ans. **(800)**

Sol.



Let side of small square = x

$v = x(30-2x)^2$

$\frac{dv}{dx} = (30-2x)^2 + x\,2(30-2x)(-2)$

$= (30-2x)(30-2x-4x)$

$= (30-2x)(30-6x)$

$x = 5$

Surface area $= 45(30-2x)x + (30-2x)^2$

$= 4 \times 5 \times 20 + (20)^2$

$= 400 + 400$

$= 800$

17. Let A & B are two points on a circle of radius 'λ'. If AB = λ and M is a point on line segment AB such that AM : MB = 2 : 3, then radius of locus of point 'M' is

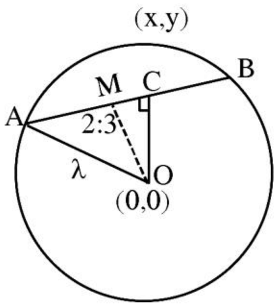
(1) $\frac{19}{5}\lambda$ (2) $\frac{\sqrt{19}}{5}\lambda$ (3) $\frac{\lambda}{5}$ (4) $\frac{\sqrt{19}}{10}\lambda$

Ans. **(2)**

Sol. $AC = \frac{\lambda}{2}$

$AM = \frac{2\lambda}{5}$

$CM = \frac{\lambda}{2} - \frac{2\lambda}{5} = \frac{\lambda}{10}$



from ΔOCM

$OM^2 = CM^2 + OC^2$

$$\Rightarrow x^2 + y^2 = \frac{\lambda^2}{100} + \left(\lambda^2 - \frac{\lambda^2}{4}\right)$$

$$\Rightarrow x^2 + y^2 = \frac{\lambda^2}{100} + \frac{3\lambda^2}{4}$$

$$\Rightarrow x^2 + y^2 = \frac{76\lambda^2}{100} = \frac{19\lambda^2}{25}$$

18. If the coefficient of x^7 in $\left(ax - \frac{1}{bx^2}\right)^{13}$ and coefficient of x^{-5} in $\left(ax + \frac{1}{bx^2}\right)^{13}$ are equal then a^4b^4 is equal

(1) 20 (2) 22 (3) 27 (4) 32

Ans. **(2)**

Sol. $T_{r+1} = {}^{13}C_r(ax)^{13-r}.\left(-\frac{1}{bx^2}\right)^r$

$= {}^{13}C_r\, a^{13-r}\left(-\frac{1}{b}\right)^r .\, x^{13-r-2r}$

$13 - 3r = 7$ $\qquad$ $13 - 3r = -5$

$r = 2$ $\qquad$ $r = 6$

$${}^{13}C_2\, a^{11}\left(-\frac{1}{b}\right)^2 = {}^{13}C_6\, a^7.\left(-\frac{1}{b}\right)^6$$

$${}^{13}C_2\, a^{11}.\frac{1}{b^2} = {}^{13}C_6.a^7.\frac{1}{b^6}$$

$$a^4b^4 = \frac{{}^{13}C_6}{{}^{13}C_2}$$

$$= \frac{13!}{6!7!} \times \frac{11!2!}{13!}$$

$$= \frac{11\times10\times9\times8}{6!}\times2$$

$$= \frac{11\times10\times72\times2}{720} = 22$$

19. Find the number of permutation of 7 digit numbers formed by digits 1,2,3,4,5,6,7 without repetition such that string 153 and 2467 does not form.

Ans. **(4878)**

Sol. $7! - 5! \times 1 - 4! \times 1 + 2!$

$= 5040 - 120 - 22$

$= 4878$

20. In an GP all terms are positive integral terms. If sum of square of first three term is 33033. Find sum of these three terms.

Ans. **(231)**

Sol. $a^2 + a^2r^2 + a^2r^4 = 30333$

$\Rightarrow a^2 . (r^4 + r^2 + 1) = 3 \times 7 \times 11^2 \times 13$

$\Rightarrow a = 11 \ \& \ r^4 + r^2 + 1 = 273$

$\Rightarrow a = 11 \ \& \ r^4 + r^2 - 272 = 0$

$\Rightarrow a = 11 \ \& \ r^2 = 16, -17$

$\Rightarrow a = 11 \ \& \ r = 4$

Required sum $= a + ar + ar^2 = 11(1 + 4 + 16) = 11 \times 21 = 231$

21. Negation of $(\sim p \vee q) \wedge (q \vee \sim r)$ is

(1) $q \wedge (p \vee r)$ (2) $\sim p \wedge (q \vee r)$ (3) $\sim q \wedge (p \vee r)$ (4) $q \vee (\sim p \wedge \sim r)$

Ans. **(3)**

Sol. $S : q \vee (\sim p \wedge \sim r)$

$\Rightarrow \sim S : \sim q \wedge (p \vee r)$

22. If $f(x) = \dfrac{x \tan 1^o + \ell n123}{x \ell n1234 - \tan 1^o}$, then the minimum value of $f(f(x)) + f\left(f\left(\dfrac{4}{x}\right)\right)$ is $(x > 0)$

Ans. **(4)**

Sol. $f(f(x)) = x$

$\therefore f(f(x)) + f\left(f\left(\dfrac{4}{x}\right)\right) = x + \dfrac{4}{x} = 2\left(\dfrac{x}{2} + \dfrac{2}{x}\right) \geq 4$