# PART : MATHEMATICS

**1.** If $x^2 f(x)-x = 4\int_0^x tf(t)dt$ and f(1) = 2/3 then find value of 18f(3)

**Ans.** **(160)**

**Sol.** Diff. W.r.t to x

$2xf(x) + x^2 f'(x)-1 = 4xf(x).\ 1-0$

$\frac{dy}{dx}-\frac{2y}{x}=\frac{1}{x^2}$

$I.F = e^{\int -\frac{2}{x}dx} = e^{-2\ell nx} = \frac{1}{x^2}$

So $\quad y.\frac{1}{x^2} = \int \frac{1}{x^4}dx$

$\frac{y}{x^2} = -\frac{1}{3x^3}+c$

$y = -\frac{1}{3x}+cx^2 \qquad \therefore f(1) = \frac{2}{3} \Rightarrow \frac{2}{3} = -\frac{1}{3}+c.1 \Rightarrow c = 1$

$f(x) = x^2-\frac{1}{3x} \Rightarrow f(3) = 9-\frac{1}{9} = \frac{80}{9}$

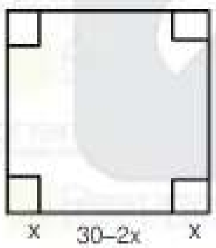
$18f(3) = 160$

**2.** From a square sheet of side 30 cm 4 square corners are cut out and then make a open box of maximum volume then total surface area of open box

(1) 400 cm² (2) 500 cm² (3) 800 cm² (4) 2000 cm²

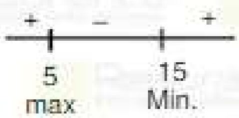
**Ans.** **(3)**

**Sol.**




$V = (30-2x)^2 x$

$\frac{dv}{dx} = (2x-30)(6x-30)$




Now $\quad$ T.S.A $= (30-2x)^2 + 4x(30-2x) = 400 + 400 = 800cm^2$

**3.** If slope of f(x) is $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$ and f(1) = 2 then f(8) =

(1) $2\sqrt{22}$ (2) $4\sqrt{22}$ (3) $\sqrt{44}$ (4) $\sqrt{22}$

**Ans.** (1)

**Sol.** $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$

Let $y = \upsilon x \therefore \frac{dy}{dx} = \upsilon + x\frac{d\upsilon}{dx}$ $\therefore \upsilon + x\frac{d\upsilon}{dx} = \frac{x^2+\upsilon^2x^2}{2x.\upsilon x}$

$x\frac{d\upsilon}{dx} = \frac{1+\upsilon^2}{2\upsilon} - \upsilon = \frac{1-\upsilon^2}{2\upsilon} \Rightarrow \frac{2\upsilon}{1-\upsilon^2}d\upsilon = \frac{dx}{x} \Rightarrow \int\frac{2\upsilon\, d\upsilon}{\upsilon^2-1} + \int\frac{dx}{x} = 0$

$\Rightarrow \ell n(\upsilon^2-1) + \ell nx = \ell nc \Rightarrow x(\upsilon^2-1) = c \Rightarrow x\frac{y^2-x^2}{x^2} = c \Rightarrow y^2 - x^2 = cx$

Now f(1) = 2 $\therefore$ 4–1 = c = 3

$\therefore y^2-x^2 = 3x$

$\therefore$ x = 8 then $y^2 - 64 = 24$

$y^2 = 88$

$y = f(8) = \sqrt{88}$

**4.** If coefficient of $x^7$ in $\left(ax - \frac{1}{bx^2}\right)^{13}$ is equal to coefficient of $x^{-5}$ in $\left(ax + \frac{1}{bx^2}\right)^{13}$ then find value of $a^4b^4$.

**Ans.** (22)

**Sol.** $\left(ax - \frac{1}{bx^3}\right)^{13}$ $T_{r+1} = {}^{13}C_r(ax)^{13-r}\left(-\frac{1}{bx^2}\right)^r$

$T_{r+1} = {}^{13}C_r.\, a^{13-r}\left(-\frac{1}{b}\right)^r x^{13-3r}$

coeff. of $x^7 = {}^{13}C_2.a^{11}\left(-\frac{1}{b}\right)^2$ | $13-3r=7$, $3r=6$, $r=2$

$\left(ax + \frac{1}{bx^2}\right)^{13}$ $T_{r+1} = {}^{13}C_r(ax)^{13-r}\left(\frac{1}{bx^2}\right)^r$

$T_{r+1} = {}^{13}C_r.a^{13-r}\left(\frac{1}{b}\right)^r x^{13-3r}$

coeff. of $x^{-5} = {}^{13}C_6.a^7\left(\frac{1}{b}\right)^6$ | $13-3r=-5$, $3r=18$, $r=6$

Now ${}^{13}C_6a^7\left(\frac{1}{b}\right)^6 = {}^{13}C_2a^{11}\left(-\frac{1}{b}\right)^2$

$a^4b^4 = \frac{{}^{13}C_6}{{}^{13}C_2} = \frac{\lfloor 13}{\lfloor 6\lfloor 7} \times \frac{\lfloor 2\lfloor 11}{\lfloor 13}$

$a^4b^4 = 22$

**5.** If $|A|_{3\times3} = 2$ then value of $|3adj(|3A|A^2)|$ =

(1) $3^{11}6^{15}$ (2) $3^{11}6^{10}$ (3) $3^{9}6^{15}$ (4) $3^{9}6^{10}$

**Ans.** (2)

**Sol.** Given $|A_{3\times3}| = 2$

Now $|3adj(|3A|A^2)|$

$= 3^3\left|adj\left(3|A|A^2\right)\right| = 3^3\left|3|A|A^2\right|^2$

$= 3^3\left(|3A|^3|A^2|\right)^2 = 3^3.\left(3^9|A|^3.|A|^2\right)^2$

$= 3^3.3^{18}|A|^{10} = 3^{21}2^{10} = 3^{11}\times 6^{10}$

**6.** Find value of $96\cos\left(\frac{4\pi}{33}\right)\cos\left(\frac{2\pi}{33}\right)\cos\left(\frac{8\pi}{33}\right)\cos\left(\frac{16\pi}{33}\right)\cos\left(\frac{\pi}{33}\right)$

**Ans.** (3)

**Sol.** $96\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}\cos\frac{\pi}{33}$

Let $\frac{\pi}{33} = A$

$96\cos A\cos 2A\cos 2^2A\cos 2^3A\cos 2^4A$

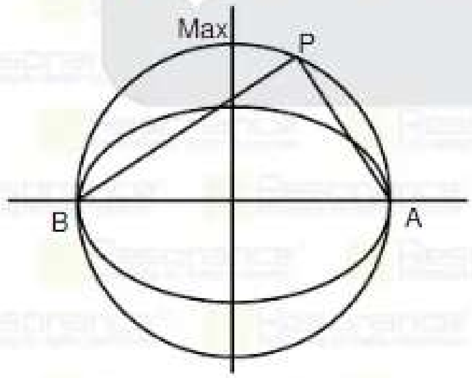
$= 96\,\frac{\sin 2^5A}{2^5\sin A} = \frac{96.\sin\frac{32\pi}{33}}{32\sin\frac{\pi}{33}} = \frac{96.\sin\left(\pi-\frac{\pi}{33}\right)}{32\sin\frac{\pi}{33}} = \frac{96.\sin\frac{\pi}{33}}{32\sin\frac{\pi}{33}} = 3$ Ans.

**7.** Let ellipse $x^2 + 9y^2 = 9$ cuts x axis at points A & B. If P is a point on the circle drawn on AB as diameter then max. Area of $\Delta$ PAB is_______. [

**Ans.** (9)

**Sol.** A (-3, 0) & B (3, 0)

For maximum area P must lie on y– axis so Area of DPAB = $\frac{1}{2}\times6\times3 = 9$ sq units



**8.** Find the total number of values $n \in z$, given that $|n^2 - 10n + 19| < 6$

**Ans.** **(6)**

**Sol.** $|n^2 - 10n + 19| < 6$

$-6 < n^2 - 10n + 19 < 6$

$+6 \qquad +6 \qquad +6$

$0 < (n-5)^2 < 12$

$0 < n - 5 < \sqrt{12}$ $\qquad$ $-\sqrt{12} < n - 5 < 0$

$5 < n < 8.5$ $\qquad$ $1.5 < n < 5$

$\{6, 7, 8\}$ $\qquad$ $n \in \{2, 3, 4\}$

number of values of n = 6

**9.** Negation of $(p \vee q) \wedge (p \wedge \sim r)$

(1) $p \vee r$ (2) $\sim p \vee r$ (3) $p \vee \sim r$ (4) $p \wedge (\sim r)$

**Ans.** **(2)**

**Sol.** $(p \vee q) \wedge (p \wedge \sim r)$

$= (p \vee q) \wedge p) \wedge r$

$= p \wedge (\sim r)$

Negation = $(\sim p \vee r)$

**10.** There is a set of numbers {1, 2, 3, 4, 5, 6, 7} then find how many 7 digits numbers are formed such that there numbers {1, 2, 3} are not together as well as {3, 5, 6, 7} are also not together.

**Ans.** **(4032)**

**Sol.** $n(\overline{A} \cap \overline{B}) = n(\cup) - n(A \cup B)$

$7! - (5!3! + 4!4! - 2!3!4!) = 4032$

**11.** If $\frac{2z + 2i}{2z - i}$ is purely Imaginary where $z = x + iy$, $y < 0$ & $x^2 + y = 0$ then find value of $x^2 + y^2 + 1$

(1) $\frac{7}{4}$ (2) $\frac{3}{4}$ (3) $\frac{5}{4}$ (4) $\frac{9}{4}$

**Ans.** **(1)**

**Sol.** $\frac{2z + 2i}{2z - i} + \frac{2\bar{z} - 2i}{2\bar{z} + i} = 0$

$(z + i)\ (2\bar{z} + i) + (\bar{z} - i)(2z - i) = 0$

$2z\bar{z} + zi + 2i\bar{z} - 1 + 2z\bar{z} - \bar{z}i - 2iz - 1 = 0$

$4z\bar{z} - iz + i\bar{z} - 2 = 0$

$4(x^2 + y^2) - i(x + iy) + i(x - iy) - 2 = 0$

$4x^2 + 4y^2 + 2y - 2 = 0$

$2x^2 + 2y^2 + y - 1 = 0$

$-2y + 2y^2 + y - 1 = 0$

$2y^2 - y - 1 = 0$

$y = 1, -\frac{1}{2}$

now $x^2 + y^2 + 1 = \frac{1}{2} + \frac{1}{4} + 1 = \frac{7}{4}$

**12.** Let point of intersection of line $\frac{x-3}{2}=\frac{y-5}{3}=\frac{z-1}{5}$ & plane $x+y+z=12$ is P then find distance of P from $2x+5y+7z=32$

(1) $\frac{111}{5\sqrt{78}}$ (2) $\frac{222}{5\sqrt{78}}$ (3) $\frac{333}{5\sqrt{72}}$ (4) $\frac{444}{5\sqrt{78}}$

**Ans.** (1)

**Sol.** Let point on line is $P(2\lambda+3, 3\lambda+5, 5\lambda+1)$

Now it also lies on plane

$x+y+z=12$

$2\lambda+3+3\lambda+5+5\lambda+1=12$

$10\lambda=3$

$\lambda=\frac{3}{10}$

So $\left(\frac{36}{10},\frac{59}{10},\frac{25}{10}\right)$

so distance from plane

$2x+5y+7z=32$ is

$\left|\frac{\frac{72}{10}+\frac{295}{10}+\frac{175}{10}-32}{\sqrt{4+25+49}}\right| \Rightarrow \frac{222}{10\sqrt{78}}=\frac{111}{5\sqrt{78}}=$

**13.** Two dice are rolled and sum of numbers of two dice is N then probability that $2^N < N!$ is $\frac{m}{n}$, where m and n are co-prime then $11m-3n$ is

**Ans.** (85)

**Sol.** Sum $2\to1$

Sum $3\to2$

Sum $4\to3$

Sum $5\to4$

Sum $6\to5$

Sum $7\to6$

Sum $8\to5$

Sum $9\to4$

Sum $10\to3$

Sum $11\to2$

Sum $12\to1$

So probability $=\frac{33}{36}=\frac{11}{12}=\frac{m}{n}$

Now $11m-3n=121-36=85$

**14.** Find the sum of all theoe terms of 3, 8,13, ...... 373 which are not divisible by 3.

(1) 9515 (2) 9525 (3) 9535 (4) 9545

**Ans.** **(2)**

**Sol.** 3, 8, 13, ..... 373

$t_n = 3 + (n-1)5 = 373$

$(n-1) = \frac{370}{5} = 74$

$n = 75$

$\text{sum} = \frac{75}{2}(3+373) = 14100$

Terms divisible by 3 : 3, 18,33 .......

$t_n = 3 + (n-1)\,15 \le 373$

$\Rightarrow n - 1 \le \frac{370}{15}$

$\Rightarrow n \le 25.66$

$n = 25$

$\text{Sum} = \frac{25}{2}[2.3 + (25-1)15]$

$= \frac{25}{2}(6 + 24 \times 15)$

$= 4575$

$\therefore$ Request sum = 14100 – 4575

= 9525 Ans.

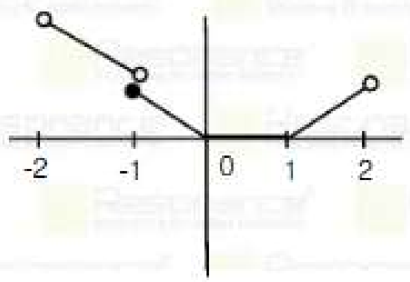
**15.** Let $f(x) = \begin{cases} x[x] & -2 < x < 0 \\ (x-1)[x] & 0 \le x < 2 \end{cases}$ and 'm' are the point of discontinuity and 'n' are the points of non-differentiability find 'm + n'.

**Ans.** **(4)**

**Sol.** $\begin{cases} -2x & : -2 < x < -1 \\ -x & : -1 \le x < 0 \\ 0 & : 0 \le x \le 1 \\ x-1 & : 1 \le x < 2 \end{cases}$

point of discontinuity → -1

point of non-differentiability → -1, 0, 1



**16.** Let $I = \int e^{\sin^2 x}$ (sin 2x.cosx– sinx) dx and I (0) =1 then find $I\left(\frac{\pi}{2}\right)$.

(1) 0 (2) 1 (3) 2 (4) 3

**Ans.** **(1)**

**Sol.** $\because I = \int e^{f(x)}(g'(x)+f'(x)g(x))dx = e^{f(x)}.g(x)$

$\therefore I = e^{\sin^2 x}.\cos x + c$

but $I(0) = 1 \Rightarrow c = 0$

$I = e^{\sin^2 x}\cos x$

$I\left(\frac{\pi}{2}\right) = 0$

**17.** In a G.P of positive Integral term $a_1^2 + a_2^2 + a_3^2$ =33033 then find $a_1 + a_2 + a_3$

**Ans.** **(231)**

**Sol.** $a_1^2 + a_2^2 + a_3^2 = 33033$

$a^2(1+r^2+r^4) = 33033 = 11^2 \times 3 \times 7 \times 13$

$a^2(1 + r^2(1+r^2)) = 11^2.273$

$a^2(1+r^2+r^4) = 11^2.(16(16+1)+1)$

$\therefore$ a = 11 and r = 4

$a_1 + a_2 + a_3 = a + ar + ar^2 = 11(1 + 4 + 16) = 231$

**18.**

| | | | | |
|---|---|---|---|---|
| 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
| 2 | 3 | x | 5 | 4 |

Given mean = 28 find variance

(1)150 (2) 151 (3) 149 (4) 152

**Ans.** **(2)**

**Sol.** Mean= $28 = \frac{5\times 2 + 15\times 3 + 25\times x + 35\times 5 + 45\times 4}{14 + x}$

392 + 28x = 410 + 25x

3x = 18

x = 6

Now variance

$= \frac{\sum f_i (x_i)^2}{\sum f_i} - (\bar{x})^2$

$= \frac{2(25) + 3(15)^2 + 6(25)^2 + 5(35)^2 + 4(45)^5}{20} - 28^2$

$= \frac{50 + 675 + 3750 + 6125 + 8100}{20} - 784$

$= \frac{18700}{20} - 784 = 935 - 784$

$= 151$

**19.** Shortest distance between line $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ and $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ is

(1) $\sqrt{29}$ (2) $3\sqrt{29}$ (3) $2\sqrt{29}$ (4) $4\sqrt{29}$

**Ans.** (3)

**Sol.** S.D. $=\left|\frac{(\vec{a}_2-\vec{a}_1).(\vec{b}\times\vec{d})}{|\vec{b}\times\vec{d}|}\right|$ where $\vec{b}\times\vec{d}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 7 & -6 & 1\\ 1 & -2 & 1\end{vmatrix}$

$=\left|\frac{(4\hat{i}+6\hat{j}+8\hat{k}).(-4\hat{i}-6\hat{j}-8\hat{k})}{\sqrt{16+36+64}}\right|$

$=\left|\frac{-16-36-64}{\sqrt{116}}\right|$

$=\sqrt{116}=2\sqrt{29}$

**20.** If the number of ways in which mixed double badminton can be played such that no couples (Husband and Wife) played into a same game is 840, find the number of couples.

**Ans.** (8)

**Sol** Let number of couples is n

${}^{n}C_2\ \ {}^{n-2}C_2 . 2 = 840$

${}^{n}C_2\ \ {}^{n-2}C_2 . = 420$

$\frac{n(n-1)}{2}.\frac{(n-2)(n-3)}{2}=420.$

$n(n-1)\ (n-2)\ (n-3) = 1680 = 8.7.6.5 = n = 8$