MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

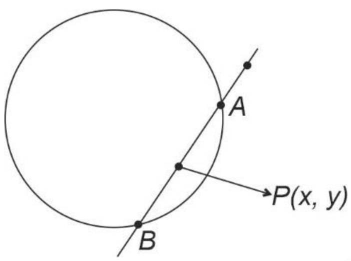
1. Let a circle $x^2 + y^2 = 16$ and line passing through (1, 2) cuts the curve at A and B then the locus of the mid-point of AB is

(1) $x^2 + y^2 + x + y = 0$ (2) $x^2 + y^2 - x + 2y = 0$

(3) $x^2 + y^2 - x - 2y = 0$ (4) $x^2 + y^2 + x + 2y = 0$

Answer (3)

Sol.



Let $P(x_1y_1)$ be the mid-point of AB

Then $T = S_1$

$$x_1^2 + y_1^2 - 16 = xx_1 + yy_1 - 16$$

$$\Rightarrow \quad xx_1 + yy_1 = x_1^2 + y_1^2 \quad \ldots\text{(i)}$$

$\because$ (i) passes through (1, 2)

$$\therefore \quad x_1 + 2y_1 = x_1^2 + y_1^2$$

$\therefore$ required locus

$x^2 + y^2 - x - 2y = 0$

2. Consider $f(x) = \sec^{-1}\left(\frac{2x}{5x+3}\right)$, domain of $f(x)$ is $[\alpha, \beta) \cup [\gamma, \delta]$, then the value of $|3\alpha + 10\beta + 5\gamma + 21\delta|$ is

(1) 22 (2) 23

(3) 21 (4) 19

Answer (3)

Sol. $\frac{2x}{5x+3} \ge 1$ OR $\frac{2x}{5x+3} \le -1$

$$\Rightarrow \quad \frac{2x}{5x+3} - 1 \ge 0$$

$$\Rightarrow \quad \frac{-3x-3}{5x+3} \ge 0$$

$$\Rightarrow \quad \frac{x+1}{5x+3} \le 0 \qquad \Rightarrow \quad x \in \left[-1, \frac{-3}{5}\right)$$

$$\frac{2x}{5x+3} + 1 \le 0$$

$$\Rightarrow \quad \frac{7x+3}{5x+3} \le 0 \qquad \Rightarrow \quad x \in \left(\frac{-3}{5}, \frac{-3}{7}\right]$$

$$\therefore \quad x \in \left[-1, \frac{-3}{5}\right) \cup \left(\frac{-3}{5}, \frac{-3}{7}\right]$$

$$\alpha = -1, \beta = \frac{-3}{5}, \gamma = \frac{-3}{5}, \delta = \frac{-3}{7}$$

$|3\alpha + 10\beta + 5\gamma + 21\delta|$

$= |-3 - 6 - 3 - 9| = 21$

3. 8 persons has to travel from A to B in 3 allotted cars. If a car can carry maximum 3 persons. Then find the number of ways they can travel.

(1) 1880 (2) 1800

(3) 1680 (4) 1600

Answer (3)

Sol. $\underset{3}{C_1}\ \underset{3}{C_2}\ \underset{2}{C_3} \rightarrow$

Total $\frac{8!}{3!\,3!\,2!\,2!}$ groups

So, they can travel in

$$\frac{8!}{3!\,3!\,2!\,2!} \times 31 \text{ ways}$$

= 1680

4. If $\frac{z+i}{4z+zi}$ is purely real $\Delta z = x + iy$ $(x, y \in R)$ then one of the possibility is

(1) $x \ne 0, y \ne -1$ (2) $x \ne 0, y = -1$

(3) $x = -1, y = 1$ (4) $x = 1, y \ne -1$

Answer (2)

Sol. $\text{Im}\left(\frac{x+i(y+1)}{2x+i(y+1)} \cdot \frac{2x-i(y+1)}{2x-i(y+1)}\right)$

$$= \frac{2x(y+1) - x(y+1)}{4x^2 + (y+1)^2}$$

$$= \frac{x(y+1)}{4x^2 + (y+1)^2} = 0$$

$\Rightarrow \quad x = 0$ or $y = -1$

5. If $\int\left(\left(\frac{x}{e}\right)^{2x}+\left(\frac{e}{x}\right)^{2x}\right)\ln x\,dx=\alpha\left(\frac{x}{e}\right)^{2x}+\beta\left(\frac{e}{x}\right)^{2x}+c$

where c is constant of integration, then

(1) $\alpha+\beta=0$ (2) $\alpha+\beta=1$

(3) $\alpha\beta=\frac{1}{2}$ (4) $\alpha\beta=\frac{1}{4}$

Answer (1)

Sol. Let $\left(\frac{x}{e}\right)^{2x}=t$

$2x(\ln x-1)=\ln t$

$\left[2(\ln x-1)+2x\left(\frac{1}{x}\right)\right]dx=\frac{1}{t}dt$

$\ln x=\frac{1}{2t}dt$

$I=\int\left(t+\frac{1}{t}\right)\times\frac{1}{2t}dt=\frac{1}{2}\int\left(1+\frac{1}{t^2}\right)dt$

$=\frac{1}{2}\left(t-\frac{1}{t}\right)+c$

$=\frac{1}{2}\left(\left(\frac{x}{e}\right)^{2x}-\left(\frac{e}{x}\right)^{2x}\right)+c$

$\Rightarrow\quad \alpha=\frac{1}{2},\beta=\frac{-1}{2}$

6. If a dice is thrown n-times and probability of getting 7 times odd is equal to 9 times even. Then $P(2\text{ even})$ is $\frac{K}{2^{15}}$ then K is

(1) 58 (2) 60

(3) 48 (4) 65

Answer (2)

Sol. P(getting odd 7 times) = P(getting even 9 times)

$^nC_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7}={}^nC_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9}$

$^nC_7={}^nC_9$

$n = 9 + 7 = 16$

$P(2\text{ times even})={}^{16}C_2\left(\frac{1}{2}\right)^{14}\left(\frac{1}{2}\right)^2$

$=\frac{^{16}C_2}{2^{16}}=\frac{16!}{2!\times14!}\times\frac{1}{2^{16}}$

$=\frac{15\times16}{2\times2^{16}}$

$=\frac{15\times4}{2^{15}}=\frac{K}{2^{15}}$

$\Rightarrow\quad K=60$

7. If $\int_0^{t^2}\left(f(x)+x^2\right)dx=\frac{4}{3}t^3$, then $f(x)$ is

(1) $x^2-2\sqrt{x}$ (2) $x^2+2\sqrt{x}$

(3) x^2 (4) $-x^2+2\sqrt{x}$

Answer (4)

Sol. $\left(f(t^2)+t^4\right)2t=4t^2$

$f(t^2)+t^4=2t$

$f(x^2)=-x^4+2x$

Let $x^2=u$

$f(u)=-u^2+2\sqrt{u}$

$f(x)=-x^2+2\sqrt{x}$

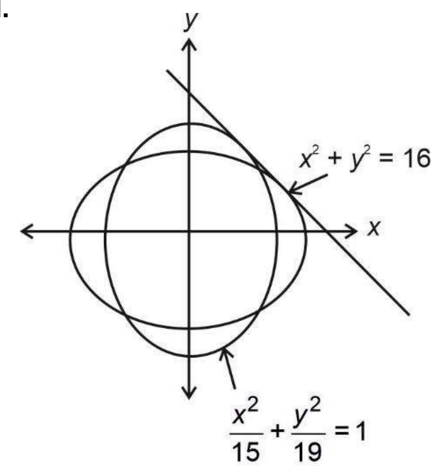
8. The equation of conic is $19x^2 + 15y^2 = 285$. A concentric circle with radius 4 units is given then angle of common tangent made by minor axis of ellipse is

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{6}$ (4) $\frac{\pi}{4}$

Answer (1)

Sol.



$19x^2 + 15y^2 = 285$

$\Rightarrow \frac{x^2}{15}+\frac{y^2}{19}=1$

Let equation of tangent of ellipse be

$y = mx \pm \sqrt{15m^2+19}$ …(i)

If it is tangent to $x^2 + y^2 = 16$ then

$\left|\frac{\sqrt{15m^2+19}}{\sqrt{m^2+1}}\right| = 4$

$15m^2 + 19 = 16m^2 + 16$

$m^2 = 3$

$\boxed{m = \pm\sqrt{3}}$

$\therefore$ angle made by the tangent by minor axis (x- axis) will be $= \frac{\pi}{3}$

9. $y = f(x)$ is a quadratic function passing through (–1, 0) and tangent to it at (1, 1) is $y = x$. Find x intercept by normal at point $(\alpha, \alpha + 1)$, $(\alpha > 0)$

(1) 7
(2) –7
(3) 5
(4) –5

Answer (1)

Sol. Let $f(x) = ax^2 + bx + (b - a)$

$f'(1) = 2a + b = 1$ and $f(1) = 1 = 2b$

$b = \frac{1}{2}, a = \frac{1}{4}$

$\frac{-dx}{dy} = \frac{-1}{2ax+b} = \frac{-2}{x+1}$

and $\alpha+1 = \frac{1}{4}\alpha^2+\frac{1}{2}\alpha+\frac{1}{4}$

$4(\alpha + 1) = (\alpha + 1)^2$

$\Rightarrow \alpha + 1 = 4$ OR $\alpha = 3$

Slope of normal $= \frac{-2}{4} = \frac{-1}{2}$

Equation of normal

$y-4 = \frac{-1}{2}(x-3)$

$2y - 4 = -x + 3$

$2y + x = 7$

10.

11.

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. The sum of all 4 digit number using the digits 1, 2, 2, 3 is

Answer (26,664)

Sol. Sum of all unit place numbers

$= 3!\times 2+\frac{3!}{2!}\times 3+\frac{3!}{2!}\times 1$

$= 12 + 9 + 3 = 24$

$\therefore$ Sum of all 4 digit no.

$= 1 \times 24 + 10 \times 24 + 100 \times 24 + 1000 \times 24$

$= 24000 + 2400 + 240 + 24$

= 26,6764

22. 4, 11, 21, 34 ……. then find the value of $\frac{S_{29}-S_9}{60}$

Answer (223)

Sol. $S = 4+11+21+34+\ldots.. +T_n$

$$\frac{S = \quad 4+11+21+\ldots..+T_{n-1}+T_n}{T_n = 4+\underbrace{7+10+13+\ldots.+(T_n+T_{n-1})}_{n-1\text{ term}}}$$

$$T_n = 4 + \frac{(n-1)}{2}[14 + (n-2)3]$$

$$= 4 + \frac{(n-1)}{2}[8 + 3n]$$

$$T_n = 4 + \frac{1}{2}(3n^2 + 5n - 8)$$

$$\sum T_n = S_n = \frac{3}{2}\sum n^2 + \frac{5}{2}\sum n$$

$$= \frac{3}{2}\frac{n(n+1)(2n+1)}{6} + \frac{5}{2}\frac{n(n+1)}{2}$$

$$\frac{S_{29} - S_9}{60} = 223$$

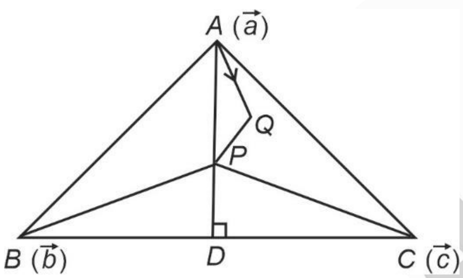
23. In ΔABC, P is circumcentre, ΔQ is orthocentre, then $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$ is

(1) $2\overrightarrow{PQ}$ (2) $\overrightarrow{PQ}$

(3) $3\overrightarrow{PQ}$ (4) $\frac{1}{2}\overrightarrow{PQ}$

Answer (2)

Sol.



Let P be origin then

$$\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \vec{a} + \vec{b} + \vec{c}$$

$$\overrightarrow{PD} = \frac{\vec{b} + \vec{c}}{2}$$

$$\overrightarrow{PQ} = 2\overrightarrow{PD}$$

$$= \vec{b} + \vec{c}$$

$$\overrightarrow{PQ} = \vec{a} + (\vec{b} + \vec{c})$$

$$= \vec{a} + \vec{b} + \vec{c}$$

24. Let $S = \left\{x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] : 9^{1-\tan^2 x} + 9^{\tan^2 x} = 10\right\}$ and $\beta = \sum_{x \in S}\left(\frac{x}{3}\right)$. Then $\frac{1}{7}(\beta - 14)^2$ is

Answer (28)

Sol. Let $9^{\tan^2 x} = t$

$$\frac{9}{t} + t = 10$$

$$t^2 - 10t + 9 = 0$$

$$\Rightarrow \quad t = 9 \quad \text{or} \quad 1$$

$$9^{\tan^2 x} = 9$$

$$\Rightarrow \quad \tan^2 x = 1$$

$$= \tan x = \pm 1$$

$$x = \pm\frac{\pi}{4}$$

or

$$\tan x = 0$$

$$\Rightarrow \quad x = 0$$

$$\beta = \frac{0}{3} + \frac{\pi}{12} - \frac{\pi}{12} = 0$$

$$\therefore \quad \frac{1}{7}(\beta - 14)^2 = \frac{14^2}{7} = 28$$

25. The coefficient of x and x^2 in $(1 + x)^p (1 - x)^q$ are 4 and – 5, then $2p + 3q$ is

Answer (63)

Sol. $(1+x)^p (1-x)^q = (1 + px + \frac{p(p-1)}{2}x^2 + ...)$

$$(1 - qx + q\frac{(q-1)}{2}x^2 + ...)$$

$\therefore$ Coefficient of $x = p - q = 4$ ...(i)

Coefficient of $x^2 = \frac{q(q-1)}{2} - pq + \frac{p(p-1)}{2}$

$= -5$

$\Rightarrow$ $(p - q)^2 - (p + q) = -10$

$\therefore$ $p + q = 26$ ...(ii)

By (i) and (ii) $p = 15$, $q = 11$

So, $2p + 3q = 30 + 33 = 63$

26. Let α be the remainder $(22)^{2022} + (2022)^{22}$ is divided by 3 and β be the remainder when the same is divided by 7 then $\alpha^2 + \beta^2$ is

Answer (05)

Sol. $(22)^{2022} + (2022)^{22}$

For α

$$(21+1)^{2022} + \underbrace{(2022)^{22}}_{\text{divisible by 3}}$$

$= (3k_1 + 1)$

For β

$(21+1)^{2022} + (2023-1)^{22}$

$= (7\lambda + 1) + (7\mu + 1)$

$= 7k_2 + 2$

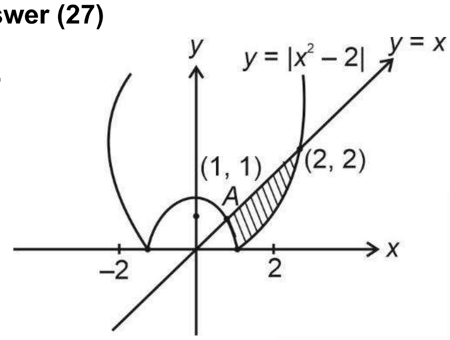
So, $\alpha = 1$, $\beta = 2$

$\alpha^2 + \beta^2 = 5$

27. If area bounded by region $\{x, y)|\ |x^2 - 2| \le y \le x\}$ is A, then $6A + 16\sqrt{2}$ is

Answer (27)

Sol.



∴ Required area

$$= \int_1^{\sqrt{2}} x - \{-(x^2-2)\}dx + \int_{\sqrt{2}}^{2} \{x - (x^2-2)\}dx$$

$$= \int_1^{\sqrt{2}} (x^2 + x - 2)dx + \int_{\sqrt{2}}^{2} (-x^2 + x + 2)dx$$

$$= \left(\frac{x^3}{3} + \frac{x^2}{2} - 2x\right)_1^{\sqrt{2}} + \left(\frac{-x^3}{3} + \frac{x^2}{2} + 2x\right)_{\sqrt{2}}^{2}$$

$$= \left(\frac{2\sqrt{2}}{3} + 1 - 2\sqrt{2}\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right)$$

$$+ \left(\frac{-8}{3} + 2 + 4\right) - \left(\frac{-2\sqrt{2}}{3} + 1 + 2\sqrt{2}\right)$$

∴ $6A + 16\sqrt{2} = 27 - 16\sqrt{2} + 16\sqrt{2}$

$= 27$

28.

29.

30.

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