

11/04/2023

Morning



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Memory Based Answers & Solutions

Time : 3 hrs.

for

M.M. : 300

JEE (Main)-2023 (Online) Phase-2

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

- (1) The test is of **3 hours** duration.
- (2) The Test Booklet consists of 90 questions. The maximum marks are 300.
- (3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.
 - (i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer.
 - (ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10**. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

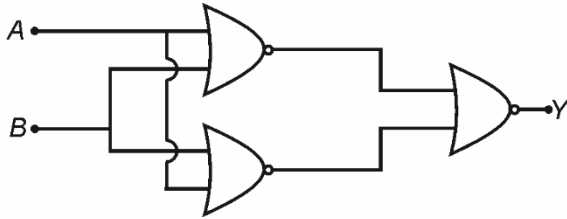
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Identify the logic operation of following circuit.



- (1) AND
- (2) OR
- (3) NOR
- (4) NAND

Answer (2)

Sol.

A	B	P	Q	Y	
1	0	0	0	1	OR gate
0	1	0	0	1	
1	1	0	0	1	
0	0	1	1	0	

2. Force acting on a particle moving along x-axis is given by $\vec{F} = (2 + 3x)\hat{i}$. The work done by this force from $x = 0$ to $x = 4$ m is

- (1) 16 J
- (2) 32 J
- (3) 4 J
- (4) 8 J

Answer (2)

Sol. $W = \int \vec{F} \cdot d\vec{r} = \int_0^4 (2 + 3x)\hat{i} \cdot dx\hat{i} = 2x + \frac{3}{2}x^2 \Big|_0^4 = 32 \text{ J}$

3. If half life of a radioactive nuclide A is equal to average life of another radioactive nuclide B. Find the ratio of decay constant of A to that of B.

- (1) $\ln 2 : 1$
- (2) $1 : \ln 2$
- (3) $2 : \ln 2$
- (4) $\ln 2 : 2$

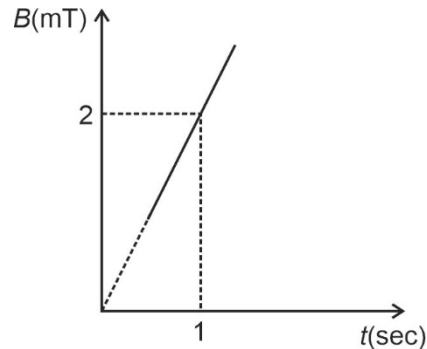
Answer (1)

Sol. $(t_{1/2})_A = (t_{\text{mean}})_B$

$$\frac{\ln(2)}{\lambda_A} = \frac{1}{\lambda_B}$$

$$\Rightarrow \frac{\lambda_A}{\lambda_B} = \ln 2$$

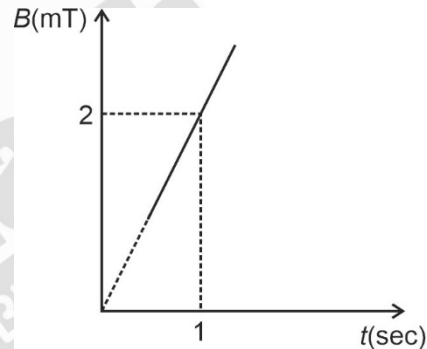
4. Variation of magnetic field through a coil of area 4 m^2 is shown in figure. What is the emf induced in the coil (in mV)?



- (1) 8
- (2) 16
- (3) 4
- (4) 2

Answer (1)

Sol. From given figure, $\frac{dB}{dt} = 2 \text{ mT/sec}$



$$\therefore \epsilon_{\text{ind}} = \left| A \frac{dB}{dt} \right| = 4 \times 2 \text{ mV} = 8 \text{ mV}$$

5. The characteristics of two coils is given below

	Coil-A	Coil-B
Radius	$r_A = 10 \text{ cm}$	$r_B = 20 \text{ cm}$
Number of turns	N_A	N_B
Current	I_A	I_B

If magnetic moment of both coil A and B are equal, then choose the correct relation

- (1) $2N_A I_A = N_B I_B$
- (2) $N_A I_A = N_B I_B$
- (3) $N_A I_A = 4N_B I_B$
- (4) $N_A I_A = 2N_B I_B$

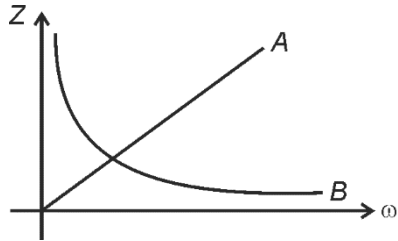
Answer (3)

Sol. $\mu = NIA \Rightarrow N_A I_A r_A^2 = N_B I_B r_B^2$

$$\Rightarrow N_A I_A = N_B I_B \times 4$$

$$N_A I_A = 4N_B I_B$$

6. The variation of impedance (Z) with angular frequency (ω) for two electrical elements is shown in the graph given. If X_L , X_C and R are inductive reactance, capacitive reactance and resistance respectively, then

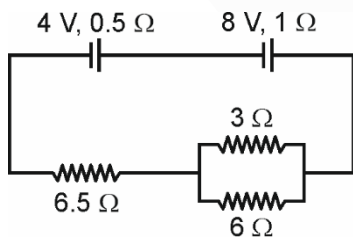


- (1) A is resistor, B is inductor
- (2) A is inductor, B is capacitor
- (3) A is inductor, B is resistor
- (4) A is capacitor, B is inductor

Answer (2)

Sol. $X_L \propto \omega$, $X_C \propto \frac{1}{\omega}$, R is independent of ω

7. Find the current flowing in 3Ω resistor in the given circuit.



- (1) 0.4 A
- (2) 0.2 A
- (3) 0.8 A
- (4) 0.6 A

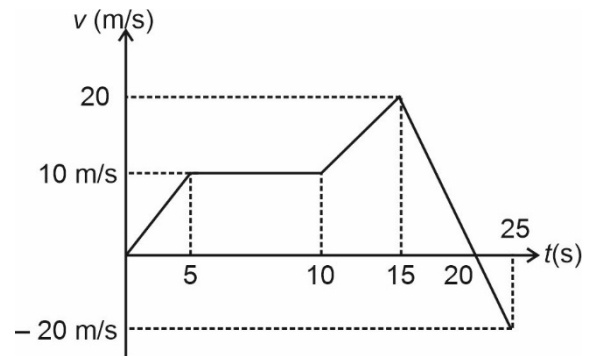
Answer (3)

Sol. \therefore Current (i) through equivalent battery

$$= \frac{12}{10} = 1.2 \text{ A}$$

$$\therefore i_{3\Omega} = \frac{6}{9} \left(\frac{12}{10} \right) = 0.8 \text{ A}$$

8. Velocity of particle moving along a straight line is shown in figure. The distance and displacement travelled by the body is



- (1) 150 m and 250 m
- (2) 250 m and 250 m
- (3) 150 m and 150 m
- (4) 50 m and 150 m

Answer (1)

Sol. Displacement

$$= \frac{1}{2} \times 10 \times 5 + 5 \times 10 + 50 + \frac{1}{2} \times 10 \times 5 + 10 \times 5 - 50$$

$$= 25 + 50 + 50 + 25$$

$$= 150 \text{ m}$$

Distance = 250 m

9. If light is passing through a medium of critical angle 45° , then the wave speed will be

- (1) $\frac{3}{\sqrt{2}} \times 10^8 \text{ m/s}$
- (2) $3\sqrt{2} \times 10^8 \text{ m/s}$
- (3) $\frac{3}{2} \times 10^8 \text{ m/s}$
- (4) $3 \times 10^8 \text{ m/s}$

Answer (1)

Sol. Refractive index of medium, $\mu = \frac{1}{\sin \theta_c} \Rightarrow \mu = \sqrt{2}$

$$\therefore \text{Light speed, } v = \frac{c}{\mu} = \frac{3}{\sqrt{2}} \times 10^8 \text{ m/s}$$

10. In moving coil galvanometer if number of turns increases by 25%, then change in voltage sensitivity is

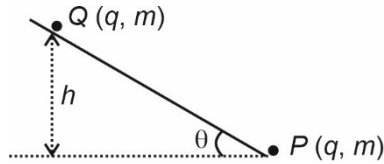
- (1) Zero
- (2) 1%
- (3) 25%
- (4) 50%

Answer (1)

Sol. Voltage sensitivity = $\left(\frac{NAB}{KR} \right) \times \left(\frac{N}{R} \right)$

$\frac{N}{R}$ remains same.

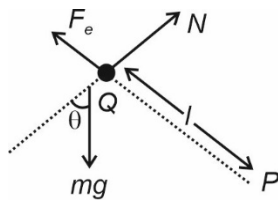
11. A fixed charge P and another free charge Q having same mass and charge are shown in the diagram find the maximum height (h) attained by charge Q in equilibrium state on smooth inclined plane if $q = 2\mu\text{C}$, $\theta = 30^\circ$, $m = 20\text{ g}$



- (1) 0.1 m (2) 0.3 m
 (3) 0.4 m (4) 0.5 m

Answer (2)

Sol.



$$\therefore F_e = mg \sin \theta$$

$$\frac{Kq^2}{l^2} = mg \sin \theta$$

$$l = \sqrt{\frac{Kq^2}{mg \sin \theta}} = 0.6\text{ m}$$

$$\text{Also, } h = l \sin \theta = 0.3\text{ m}$$

12. If a planet 'A' has density ρ and radius r , planet 'B' has density $\frac{\rho}{3}$ and radius $4r$. Then, find ratio of their acceleration due to gravity at their surface.

- (1) 3 : 4 (2) 4 : 3
 (3) 1 : 3 (4) 2 : 3

Answer (1)

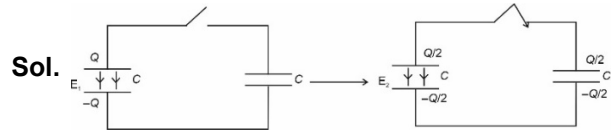
Sol. $\therefore g \propto \rho R$

$$\therefore \frac{g_A}{g_B} = \frac{\rho r}{\frac{\rho}{3} \times 4r} = \frac{3}{4}$$

13. A $2\ \mu\text{F}$ capacitor is charged with potential V and energy stored in capacitor is E_1 . Now the capacitor is disconnected with battery and connected with another identical capacitor in parallel. Now the energy stored in capacitor is E_2 . Find $\frac{E_1}{E_2}$

- (1) 2 (2) 4
 (3) 5 (4) 6

Answer (4)



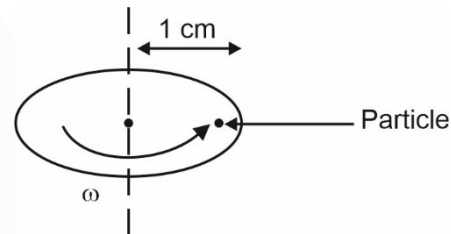
Sol.

$$E_1 = \frac{Q^2}{2C}$$

$$E_2 = \frac{(Q/2)^2}{2C}$$

$$\therefore \frac{E_1}{E_2} = 4$$

14. A particle is kept at rest at 1 cm from axis on the disc rotating with angular velocity ω . If angular velocity is reduced to half of its initial value, then find the distance from axis, where particle again remains at rest



- (1) 4 cm
 (2) 6 cm
 (3) 8 cm
 (4) 12 cm

Answer (1)

Sol. $\mu mg = m \omega^2 x_1$

$$\mu mg = m \frac{\omega^2}{4} x_2$$

$$\Rightarrow \frac{4x_1}{x_2} = 1$$

$$\Rightarrow x_2 = 4x_1$$

$$\Rightarrow x_2 = 4\text{ cm}$$

15. Stopping potential for a metal when illuminated with light of wavelength λ is V_0 and that for wavelength

2λ is $\frac{V_0}{4}$. The threshold wavelength of metal is

- (1) λ
 (2) 2λ
 (3) 3λ
 (4) 4λ

Answer (3)

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Equation of progressive wave is

$$y = A \sin(160t - 0.5x).$$

Let the speed of wave is $10x$, find x .

Answer (32)

Sol. From given equation,

$$\omega = 160 \text{ and } k = 0.5$$

\therefore Speed of wave,

$$v = \frac{\omega}{k} = \frac{160}{0.5} = 320 \text{ m/s}$$

22. A machine gun is firing 10 g bullets with speed 250 m/s. To keep machine gun in position 125 N force is required. Find no. of bullets fired per second.

Answer (50)

$$\text{Sol. } F = n_{1\text{sec}} \cdot mv \Rightarrow n_{1\text{sec}} = \frac{125}{10 \times 10^{-3} \times 250} = 50$$

23. A particle is projected at an angle of 30° with horizontal. Height of particle at 3 s and 5 s are same. Find the speed of projection in m/s. ($g = 10 \text{ m/s}^2$)

Answer (80)

Sol. $T = 8 \text{ sec}$

$$\frac{T}{2} = 4 \text{ sec}$$

$$\frac{u \sin \theta}{g} = 4$$

$$u = \frac{40}{\sin 30} = 80 \text{ m/s}$$

24. An antenna is required for LOS communication upto a distance of 4 km. The height (in m) of the antenna is (Radius of earth is 6400 km)

Answer (01.25)

Sol. $d = \sqrt{2Rh}$

$$4 = \sqrt{2 \times 6400 \times h}$$

$$h = 1.25 \text{ m}$$

25. A material is placed in a toroid. Find the percentage change in magnetic field of toroid if susceptibility of material is $\chi = 2 \times 10^{-2}$

Answer (2)

Sol. $\frac{\Delta B}{B_0} = (\chi)$

$$\frac{\Delta B}{B_0} \times 100 = 100\chi$$

$$= 2 \times 10^{-2} \times 100 = 2\%$$

26.

27.

28.

29.

30.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. In a container at a constant temperature, arrange the RMS velocity of following

Ne, Cl₂, UF₆

- (1) Ne > Cl₂ > UF₆ (2) Cl₂ > UF₆ > Ne
 (3) UF₆ > Ne > Cl₂ (4) UF₆ > Cl₂ > Ne

Answer (1)

Sol. $U_{rms} = \sqrt{\frac{3RT}{M_w}}$

$M_w \uparrow U_{rms} \downarrow$

2. Which of the following is correct order of first ionisation energy for

Li, Be, C, B, N, O, F

- (1) B > N > O > Li > Be > F > C
 (2) N > F > O > C > B > Be > Li
 (3) F > O > N > C > B > Be > Li
 (4) F > N > O > C > Be > B > Li

Answer (4)

Sol. Option (4) is correct order

F > N > O > C > Be > B > Li

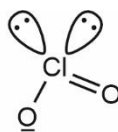
3. Match the columns

	Column-I		Column-II
(A)	ClO ₂ ⁻	(1)	Linear
(B)	N ₃ ⁻	(2)	Tetrahedral
(C)	NH ₄ ⁺	(3)	Bent
(D)	SF ₄	(4)	See-Saw

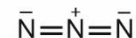
- (1) A → 1; B → 2; C → 3; D → 4
 (2) A → 3; B → 1; C → 2; D → 4
 (3) A → 4; B → 2; C → 1; D → 3
 (4) A → 3; B → 2; C → 1; D → 4

Answer (2)

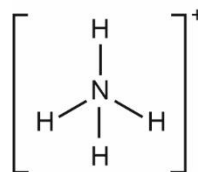
Sol.



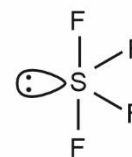
Bent



Linear

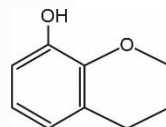


Tetrahedral

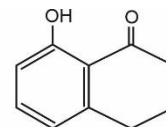


See-Saw

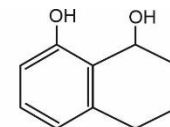
4. Increasing order of electrophilic aromatic substitution reaction



(A)



(B)



(C)

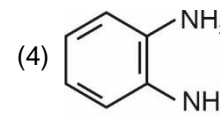
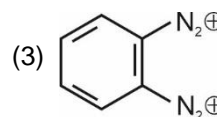
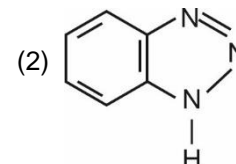
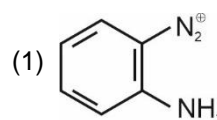
- (1) A < B < C (2) B < C < A
 (3) C < B < A (4) B < A < C

Answer (2)

Sol. The reactivity of an aromatic compound towards electrophilic aromatic substitution (EAS) is decided by the kind of substituents bonded to it. Any substituent that increases the electron density of benzene makes it more reactive towards EAS. In compound(A), the O-atom directly bonded to benzene ring increases the electron density by +R. In compound(B), the carbonyl group decreases the electron density by -R. In compound(C), the electron density of benzene increases by +I effect.

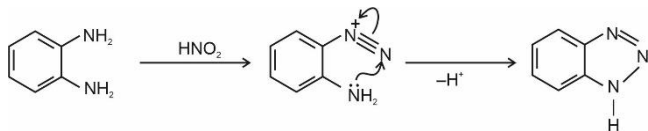
∴ The correct increasing order towards EAS is B < C < A

5. o-phenylenediamine $\xrightarrow{HNO_2}$

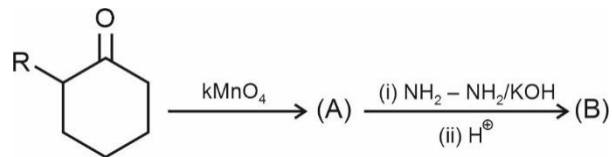


Answer (2)

Sol.



6. Consider the reaction sequence

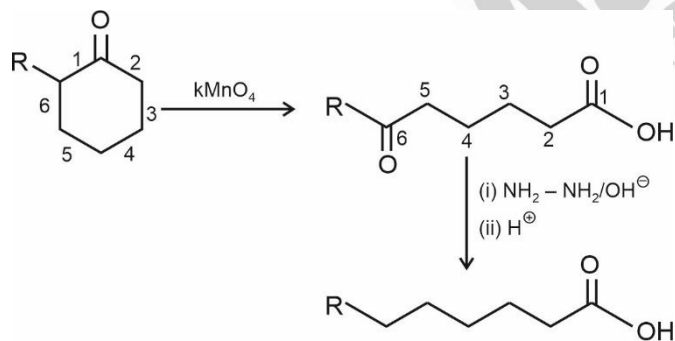


B is :

- (1)
- (2)
- (3)
- (4)

Answer (4)

Sol.



7. Which of the following does not contain ambidentate ligand

- (1) $C_2O_4^{2-}$, H_2O
- (2) $EDTA^{4-}$, NO_2^-
- (3) NO_2^- , SCN^-
- (4) SCN^- , CN^-

Answer (1)

Sol. $C_2O_4^{2-}$ is bidentate (chelate) ligand and H_2O is simple monodentate ligand.

8. Which of the following can be represented as meridional isomer

- (1) $[Pt(NH_3)_3Cl_3]^+$
- (2) $[Pt(en)_3]^{4+}$
- (3) $[Pt(en)_2Cl_2]^{2+}$
- (4) $[Pt(en)_2(NH_3)_2]^{4+}$

Answer (1)

Sol. $[Ma_3b_3]$ exist as fac. & mer isomer.

9. When ethene reacts with Ziegler-Natta and at 6–7 atm pressure it gives.

- (1) Low density polythene
- (2) Polyacrylonitrile
- (3) Polyamide
- (4) High density polythene

Answer (4)

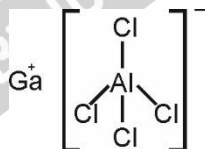
Sol. Addition polymerisation of ethene at pressure 6–7 atm in the presence of Ziegler-Natta catalyst gives high density polythene.

10. Identify the correct statement about the compound $GaAlCl_4$.

- (1) Chlorine atom is bonded to both Ga and Al
- (2) Ga is cationic part and less electronegative than Al
- (3) Chlorine atom forms co-ordinate bond with Ga
- (4) Chlorine atom is bonded to Al

Answer (4)

Sol. The structure of $GaAlCl_4$ is



Chlorine atoms are bonded to Al only. Gallium is the cationic part but more electronegative than Al.

11. Match the column

- | Column - I | Column - II |
|--|-----------------------------|
| (A) K^+ ions | (P) Thermonuclear reactions |
| (B) KCl | (Q) Sodium-potassium pump |
| (C) Mg | (R) Absorbent of CO_2 |
| (D) KOH | (S) Fertiliser |
| (1) $A \rightarrow Q; B \rightarrow S; C \rightarrow R, D \rightarrow P$ | |
| (2) $A \rightarrow P; B \rightarrow Q; C \rightarrow S; D \rightarrow R$ | |
| (3) $A \rightarrow Q; B \rightarrow S; C \rightarrow P, D \rightarrow R$ | |
| (4) $A \rightarrow P; B \rightarrow Q; C \rightarrow R; D \rightarrow S$ | |

Answer (3)

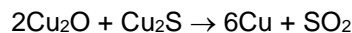
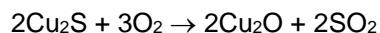
Sol. K^+ : Sodium-potassium pump

KCl : Fertiliser

Mg : Used in thermonuclear reactions

KOH : Absorber of CO_2

12. Which type of copper is formed by the following reactions?



- (1) Blister copper
- (2) Copper crisp
- (3) Reduced copper
- (4) Copper slag

Answer (1)

Sol. The solidified copper obtained has blistered appearance due to the evolution of SO_2 and so it is called blister copper.

13. To 25 ml of 1 M $AgNO_3$, 1.05 M KI is added dropwise. In the colloidal sol formed, fixed and diffused layer consists of respectively:

($AgNO_3$ is in excess)

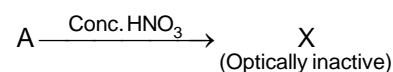
- (1) I^\ominus and NO_3^\ominus
- (2) Ag^\oplus and NO_3^\ominus
- (3) Ag^\oplus and K^\oplus
- (4) K^\oplus and Ag^\oplus

Answer (2)

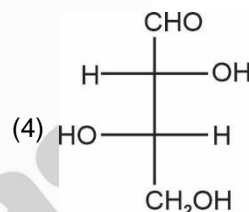
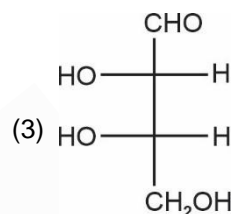
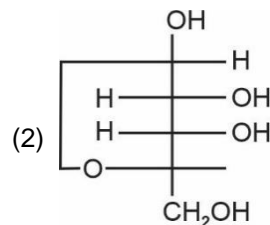
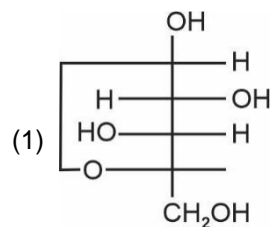
Sol. Fixed layer $\rightarrow Ag^\oplus$

Diffused layer $\rightarrow NO_3^\ominus$

14. L-isomer of a tetrose (A) gives Schiff's test having two chiral carbons.

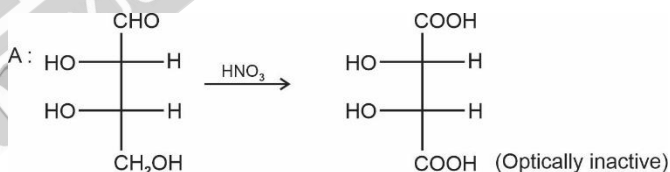


A is :



Answer (3)

Sol.



15. **Statement I (S I):** A water sample having BOD = 4 ppm is of good quality.

Statement II (S II): If the concentration of Zn and NO_3^\ominus each is 5 ppm, then water is of good quality.

- (1) Both S I and S II are correct
- (2) S I is incorrect and S II is correct
- (3) S I is correct and S II is incorrect
- (4) Neither S I nor S II is correct

Answer (1)

Sol. Clean water has BOD less than 5 ppm

The maximum concentration of Zn = 5 ppm

Concentration of NO_3^\ominus = 50 ppm

(as per international standards of drinking water)

16.
17.
18.
19.
20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Find spin only magnetic moment ratio for complexes $[\text{Cr}(\text{CN})_6]^{-3}$ and $[\text{Cr}(\text{H}_2\text{O})_6]^{+3}$

Answer (1)

Sol. Spin only magnetic moment for $[\text{Cr}(\text{CN})_6]^{-3}$ (d^3) = $\sqrt{15}$ B.M.

For $[\text{Cr}(\text{H}_2\text{O})_6]^{+3}$ (d^3) = $\sqrt{15}$ B.M.

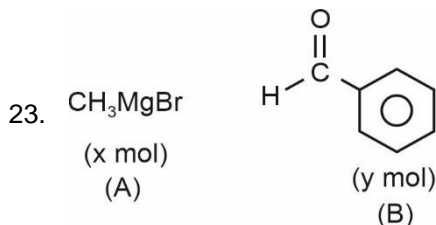
So ratio is 1 : 1

22. 25% of 250 g sugar solution and 40% of 500 g sugar solution are mixed then find out the mass percentage of sugar in solution

Answer (35.00)

Sol. Mass of Sugar = $(.25 \times 250) + (.40 \times 500)$
= 262.5 gm

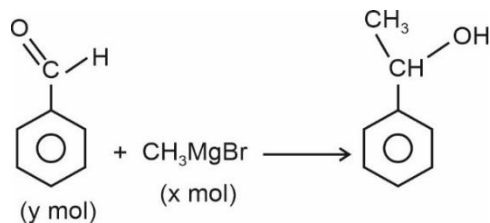
$$\% \left(\frac{w}{w} \right) = \frac{262.5}{750} \times 100 = 35\%$$



Value of $\frac{x}{y}$ when A & B completely react will be:

Answer (1)

Sol.



Hence value $\frac{x}{y} = 1$

24. Find the number of atoms per unit cell if edge length = 408 pm, density = 3 g/cm³, Molecular mass = 40 (Nearest integer)

Answer (3)

Sol. $3 = \frac{Z \times 40}{6 \times 10^{23} \times (4.08)^3 \times 10^{-24}}$

$$Z = \frac{3}{40} \times 6 \times 0.1 \times (4.08)^3$$

$$\approx 3.056$$

$$\approx 3$$

25. Given

Electrode	Potential
Pb^{+2}/Pb	M
Pb^{+4}/Pb	N
$\text{Pb}^{+2}/\text{Pb}^{+4}$?

Value of $\text{Pb}^{+2}/\text{Pb}^{+4}$ is M-x N, then value of x is

Answer (2)

Sol. $E_{\text{Pb}^{+4}/\text{Pb}^{+2}}^{\circ} = \frac{4E_{\text{Pb}^{+4}/\text{Pb}}^{\circ} - 2E_{\text{Pb}^{+2}/\text{Pb}}^{\circ}}{2}$
= $\frac{4 \times N - 2M}{2}$
= $2N - M$

$$\therefore E_{\text{Pb}^{+2}/\text{Pb}^{+4}}^{\circ} = M - 2N$$

Hence x = 2

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

- If $x + y + z = 17$ and x, y, z are non-negative integers, then find the number of integral solutions is
 - 136
 - 171
 - 90
 - 130

Answer (2)

Sol. $x + y + z = 17$

$${}^{17+3-1}C_{3-1} = {}^{19}C_2$$

$$= 171$$

- Let $M = [a_{ij}]_{2 \times 2}$, $0 \leq i, j \leq 2$ where $a_{ij} \in \{0, 1, 2\}$ and A be the event such that M is invertible then $P(A)$ is

(1) $\frac{49}{81}$	(2) $\frac{16}{27}$
(3) $\frac{47}{81}$	(4) $\frac{46}{81}$

Answer (2)

Sol. If A is invertible then $|A| \neq 0$

Now, for $|A| = 0$

$$(1) \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ or } \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \rightarrow \text{Total matrix} = 3$$

- Two 1's and Two 0's \rightarrow Total matrix = 4
- Two 2's and Two 0's \rightarrow Total matrix = 4
- Two 1's and Two 2's \rightarrow Total matrix = 4
- One 1 and Three 0's \rightarrow Total matrix = 4
- One 2 and Three 0's \rightarrow Total matrix = 4

$$\therefore P(A) = 1 - \frac{23}{81} = \frac{48}{81} = \frac{16}{27}$$

- For a given function $f(x) = [x^2 - x] + \{x\}$
 - $f(x)$ is continuous at $x = 0, x = 1$
 - $f(x)$ is continuous and differentiable at $x = 0$ and $x = 1$
 - $f(x)$ is continuous but non-differentiable at $x = 0, 1$
 - $f(x)$ is continuous at $x = 1$ but discontinuous at $x = 0$

Answer (4)

Sol. $f(x) = \begin{cases} x+1 & -0.5 < x < 0 \\ 0 & x = 0 \\ -1+x & 0 < x < 1 \\ 0 & x = 1 \\ x-1 & 1 < x < 1.5 \end{cases}$

At $x = 0^+$ $f(x) = -1$

$x = 0^-$ $f(x) = 1$

$\therefore f(x)$ is discontinuous at $x = 0$

At $x = 1^+$ $f(x) = 0$

$x = 1^-$ $f(x) = 0$

$f(1) = 0$

$\therefore f(x)$ is continuous at $x = 1$

- Two complex numbers w_1 and w_2 , given by $w_1 = 3 + 5i$ and $w_2 = 5 + 4i$ are both rotated by 90° with respect to origin anti-clockwise and clockwise respectively to get the new complex numbers w_3 and w_4 . The principal argument of $w_3 - w_4$ is

(1) $-\pi - \tan^{-1} \frac{8}{9}$ (2) $-\pi - \tan^{-1} \frac{33}{5}$

(3) $\pi - \tan^{-1} \frac{8}{9}$ (4) $\pi - \tan^{-1} \frac{33}{5}$

Answer (3)

Sol : $w_3 = i w_1 = i(3 + 5i) = -5 + 3i$

$w_4 = -i w_2 = -i(5 + 4i) = 4 - 5i$

$w_3 - w_4 = -9 + 8i$

$\arg(w_3 - w_4) = \pi - \tan^{-1} \frac{8}{9}$

($\because w_3 - w_4 \in$ II Quadrant)

5. Let a and b are roots of $x^2 - 7x - 1 = 0$. The value of $\frac{a^{21} + b^{21} + a^{17} + b^{17}}{a^{19} + b^{19}}$ is

- (1) 29 (2) 49
 (3) 53 (4) 51

Answer (4)

Sol. $\frac{a^{17}(a^4 + 1) + b^{17}(b^4 + 1)}{a^{19} + b^{19}}$

$\alpha^2 - 1 = 7\alpha$

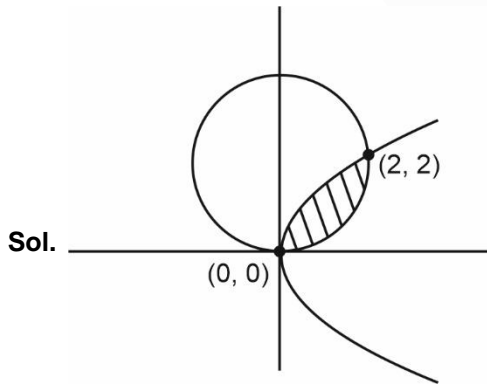
$\Rightarrow \alpha^4 + 1 = 51\alpha^2 \begin{cases} a \\ b \end{cases}$

$\therefore \frac{51a^{19} + 51b^{19}}{a^{19} + b^{19}} = 51$

6. Find the area bounded by $\begin{cases} x^2 + (y - 2)^2 \leq 4 \\ y^2 \leq 2x \end{cases}$

- (1) $\pi + \frac{4}{3}$ (2) $\pi - \frac{4}{3}$
 (3) $2\pi + \frac{8}{3}$ (4) $2\pi - \frac{8}{3}$

Answer (2)



On solving the curves, $(0, 0)$ and $(2, 2)$ will be the point of intersection

Required area = $\int_0^2 \left(\sqrt{4 - (y - 2)^2} - \frac{y^2}{2} \right) dy$

$= \left[\frac{1}{2}(y - 2)\sqrt{4 - (y - 2)^2} + \frac{4}{2} \sin^{-1} \left(\frac{y - 2}{2} \right) - \frac{y^3}{6} \right]_0^2$

$= -\frac{8}{6} - \left[\frac{1}{2}(-2) \times 0 - \pi \right] = \pi - \frac{4}{3}$

7. The number of solutions of $\cos^4 \theta - 2\cos^2 \theta + 3\sin^6 \theta + 1 = 0$ in $[0, 2\pi]$ is

- (1) 1 (2) 2
 (3) 3 (4) 4

Answer (3)

Sol. $(1 - \cos^2 \theta)^2 + \sin^2 \theta = 0$

$\Rightarrow \sin^4 \theta + \sin^2 \theta = 0$

$\sin^2 \theta (\sin^2 \theta + 1) = 0$

$\Rightarrow \sin^2 \theta = 0$

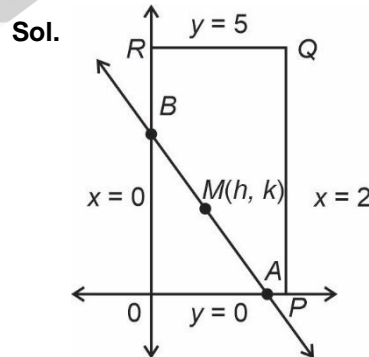
$\theta = 0, \pi, 2\pi$

\therefore Three solutions in $[0, 2\pi]$

8. A rectangle is drawn by lines $x = 0, x = 2, y = 0$ and $y = 5$. Points A and B lie on coordinate axes. If line AB divides the area of rectangle in $4 : 1$, then the locus of mid-point of AB is

- (1) Circle
 (2) Hyperbola
 (3) Ellipse
 (4) Straight line

Answer (2)



$A(2h, 0), B(0, 2k)$

Area of $\triangle OAB = 8$

$\frac{1}{2} \times 2h \times 2k = 8$

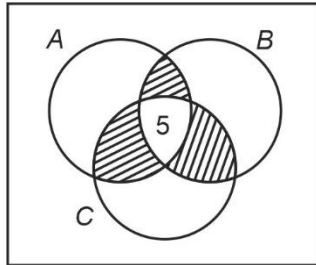
$hk = 4$

Locus is $xy = 4$

9. Let awards in event A is 48 and awards in event B is 25 and awards in event C is 18 and also $n(A \cup B \cup C) = 60$, $n(A \cap B \cap C) = 5$, then how many got exactly two awards is

- (1) 21 (2) 25
(3) 24 (4) 23

Answer (1)



Sol.

$$\begin{aligned} \therefore \text{Number of persons who get exactly two awards} &= n(A) + n(B) + n(C) - n(A \cup B \cup C) - 2n(A \cap B \cap C) \\ &= 48 + 25 + 18 - 60 - 10 \\ &= 21 \end{aligned}$$

10. Let p : I have fever
 q : I do not take medicine
 r : I take rest

If I have fever then I take medicine and I take rest is equivalent to

- (1) $(\sim p \vee \sim q) \wedge (\sim p \vee r)$
(2) $(\sim p \vee q) \wedge (\sim p \vee \sim r)$
(3) $(\sim p \vee q) \wedge (\sim p \vee r)$
(4) $(\sim p \vee q) \wedge (\sim p \vee \sim r)$

Answer (3)

Sol. Given statement is equivalent to

$$\begin{aligned} p \rightarrow (q \wedge r) &= \sim p \vee (q \wedge r) \\ &= (\sim p \vee q) \wedge (\sim p \vee r) \end{aligned}$$

11. Let A be a 2×2 matrix such that $A^T = \alpha A + 1$ and $|A^2 + 2A| = 4$, then a possible value of α is

- (1) 1
(2) -1
(3) 3
(4) 0

Answer (2)

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^T = \alpha A + 1$$

$$\Rightarrow \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} \alpha a + 1 & \alpha b \\ \alpha c & \alpha d + 1 \end{bmatrix}$$

$$a = \alpha c + 1 \Rightarrow a = \frac{1}{1 - \alpha} \dots(i)$$

$$b = \alpha c \dots(ii)$$

$$c = \alpha b \dots(iii)$$

$$(ii) \text{ and } (iii) \Rightarrow c = 0 \text{ or } \alpha = \pm 1 (\alpha \neq 1)$$

$$\therefore c = 0 \text{ or } \alpha = -1$$

$$\text{Also, } d = \alpha d + 1 \Rightarrow d = \frac{1}{1 - \alpha}$$

Case (I)

$$c = 0$$

$$\Rightarrow b = 0$$

$$|A^2 + 2A| = 4 \Rightarrow |A| \cdot |A + 2I| = 4$$

$$\left(\frac{1}{1 - \alpha}\right)^2 \left(\frac{3 - 2\alpha}{1 - \alpha}\right)^2 = 4$$

$$\frac{3 - 2\alpha}{(1 - \alpha)^2} = \pm 2$$

$$\Rightarrow \alpha = \frac{1 \pm \sqrt{3}}{2}$$

Case (II)

$$\alpha = -1$$

12. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $0 < |a_i| < 1, i = \{1, 2, 3\}$

Statement-A : $|a| \geq \max\{|a_1|, |a_2|, |a_3|\}$

Statement-B : $|a| < 3 \max\{|a_1|, |a_2|, |a_3|\}$

- (1) Both A and B are true
(2) Both A and B are false
(3) A is true, B is false
(4) A is false, B is true

Answer (1)

Sol. $|a|^2 = \sqrt{a_1^2 + a_2^2 + a_3^2} \geq \max\{|a_1|, |a_2|, |a_3|\}$

$$|a|^2 = \sqrt{a_1^2 + a_2^2 + a_3^2} < 3 \times \max\{|a_1|, |a_2|, |a_3|\}$$

13. Solution of differential equation is $(1 - x^2y^2)dx = xdy$

$+ ydx$ if $y(2) = 4$, then $\frac{5y(5)+1}{5y(5)-1} = k$ then k is

- (1) $\frac{49}{81}e^2$ (2) $\frac{81}{49}e^2$
 (3) e^2 (4) $\frac{9}{7}e^2$

Answer (2)

Sol. $(1 - x^2y^2)dx = xdy + ydx$

$$(1 - x^2y^2)dx = d(xy)$$

$$\int dx = \int \frac{d(xy)}{1 - x^2y^2}$$

$$x = \frac{1}{2} \log \left| \frac{1+xy}{1-xy} \right| + c$$

$$\therefore y(2) = 4$$

$$2 = \frac{1}{2} \log \left| \frac{1+4 \times 2}{4 \times 2 - 1} \right| + c$$

$$4 = \log \left| \frac{9}{7} \right| + c$$

$$c = 4 - \log \frac{9}{7}$$

$$x = \frac{1}{2} \log \left| \frac{1+xy}{xy-1} \right| + 4 - \log \frac{9}{7}$$

For $y(5)$

$$5 = \frac{1}{2} \log \left| \frac{1+5y}{5y-1} \right| + 4 - \log \frac{9}{7}$$

$$2 \left(1 + \log \frac{9}{7} \right) = \log \left(\frac{1+5y}{5y-1} \right)$$

$$\frac{5y+1}{5y-1} = e^{2 \left(1 + \log \frac{9}{7} \right)} \Rightarrow \frac{81}{49} e^2$$

14.
15.
16.
17.
18.
19.
20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. 5 boys with allotted roll numbers and seat numbers are seated in such a way that no one sits on the allotted seat. The number of such seating arrangements is

Answer (44)

Sol. Number of seating arrangements

$$= 5! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} \right)$$

$$= 5! \left(\frac{120 - 120 + 60 - 20 + 5 - 1}{120} \right)$$

$$= 44$$

22. $(p \vee q) \wedge (p \vee r) \Rightarrow (q \vee r)$. Number of triplets (p, q, r) such that it is true, is _____.

Answer (7)

Sol. $(p \vee q) \wedge (p \vee r) \Rightarrow (q \vee r)$

$$\Rightarrow p \vee (q \wedge r) \Rightarrow (q \vee r)$$

This is always true if $p \vee (q \wedge r)$ is F.

$$\Rightarrow (p, q, r) \equiv (F, F, F), (F, F, T), (F, T, F)$$

This is true if $p \vee (q \wedge r)$ is T and $(q \vee r)$ is T

$$\Rightarrow (p, q, r) \equiv (T, T, F), (T, F, T), (T, T, T), (F, T, T)$$

\therefore 7 triplets are possible

23. The number of rational terms in the expansion of

$$\left(3^{3/4} + 5^{3/2} \right)^{60}$$

Answer (16)

$$\text{Sol. } T_{r+1} = {}^{60}C_r \left(3^{3/4} \right)^{60-r} \left(5^{3/2} \right)^r$$

$$= {}^{60}C_r (3)^{\frac{180-3r}{4}} 5^{3r/2}$$

Total rational terms = 16

24. Consider the plane $2x + y - 3z = 6$. If (α, β, γ) is the image of point $(2, 3, 5)$ in the given plane, then $\alpha + \beta + \gamma =$ _____

Answer (10)

Sol. $\frac{x-2}{2} = \frac{y-3}{1} = \frac{z-5}{-3} = -2 \frac{(-14)}{14}$

$\Rightarrow x = 6, y = 5, z = -1$

$\therefore \alpha = 6, \beta = 5, \gamma = -1$

$\alpha + \beta + \gamma = 6 + 5 - 1 = 10$

25. The mean of coefficients of x, x^2, \dots, x^7 in the binomial expansion of $(2 + x)^9$ is

Answer (2736)

Sol. Sum of coefficients $= {}^9C_1 \cdot 2^8 + {}^9C_2 \cdot 2^7 \dots + {}^9C_7 \cdot 2^2$
 $= (2 + 1)^9 - {}^9C_0 \cdot 2^9 - {}^9C_8 \cdot 2 - {}^9C_9$
 $= 3^9 - 2^9 - 19 = 19683 - 572 - 19 = 19152$

Mean $= \frac{\text{Sum}}{7} = \frac{19152}{7} = 2736$

26. Consider two sets A and B . Set A has 5 elements whose mean & variance are 5 and 8 respectively. Set B has also 5 elements whose mean & variance are 12 & 20 respectively. A new set C is formed by subtracting 3 from each element of set A and by adding 2 to each element of set B . The sum of mean & variance of the set C is _____.

Answer (58)

Sol. $\bar{x}_C = \text{mean of } c = \frac{(5-3) + (12+2)}{2} = 8$

$\sigma_{12}^2 = \text{variance of } c = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$

$d_1 = \bar{x}_{12} - \bar{x}_1$

$d_2 = \bar{x}_{12} - \bar{x}_2$

$n_1 = 5, \sigma_1^2 = 8, d_1 = 8 - 2 = 6$

$n_2 = 5, \sigma_2^2 = 20, d_2 = 8 - 12 = -4$

$\sigma_{12}^2 = \frac{5(8+36) + 5(20+36)}{10} = 50$

$\therefore \sigma_{12}^2 + \bar{x}_C = 50 + 8 = 58$

27. If $S = 109 + \frac{108}{5} + \frac{107}{5^2} + \dots + \frac{2}{5^{107}} + \frac{1}{5^{108}}$ then the value of $165 - (25)^{-54}$ is

Answer (2175)

Sol. $S = 109 + \frac{108}{5} + \frac{107}{5^2} + \dots + \frac{2}{5^{107}} + \frac{1}{5^{108}}$

$\frac{S}{5} = \frac{109}{5} + \frac{108}{5^2} + \dots + \frac{2}{5^{108}} + \frac{1}{5^{109}}$

$\therefore \frac{4}{5}S = 109 - \left[\frac{1}{5} + \frac{1}{5^2} + \dots + \frac{1}{5^{108}} + \frac{1}{5^{109}} \right]$

$= 109 - \left[\frac{1}{5} \left[\frac{1 - \left(\frac{1}{5}\right)^{109}}{1 - \frac{1}{5}} \right] \right]$

$\therefore S = \frac{5}{4} \left[109 - \frac{1}{4} \left[1 - \left(\frac{1}{5}\right)^{109} \right] \right]$

$\therefore 165 = 20 \times 109 - 5 + 5^{-108}$

$\therefore 165 - (25)^{-54} = 2180 - 5 = 2175$

28. If $\log_{x+\frac{7}{2}} \left(\frac{x+7}{2x+3} \right)^2 \geq 0$, then total number of integral solution(s) is/are _____.

Answer (8)

Sol. $x + \frac{7}{2} > 0 \Rightarrow x > -\frac{7}{2}$... (i)

$x + \frac{7}{2} \neq 1 \Rightarrow x \neq -\frac{5}{2}$... (ii)

$\frac{x+7}{2x+3} \neq 0$

$\Rightarrow x \neq -7, -\frac{3}{2}$... (iii)

$\log_{x+\frac{7}{2}} \left(\frac{x+7}{2x+3} \right)^2 \geq 0$

$$\Rightarrow \left(\frac{x+7}{2x+3}\right)^2 \geq 1$$

$$\Rightarrow \left(\left(\frac{x+7}{2x+3}\right) - 1\right)\left(\frac{x+7}{2x+3} + 1\right) \geq 0$$

$$\Rightarrow \left(\frac{-x+4}{2x+3}\right)\left(\frac{3x+10}{2x+3}\right) \geq 0$$

$$\Rightarrow \frac{(x-4)(3x+10)}{(2x+3)^2} \leq 0$$

$$\Rightarrow \frac{\boxed{}}{-\frac{10}{3} \quad 4} \Rightarrow x \in \left[-\frac{10}{3}, 4\right] \dots \text{(iv)}$$

$$(i) \cap (ii) \cap (iii) \cap (iv)$$

$$x \in \left[-\frac{10}{3}, 4\right] - \left\{-\frac{5}{2}, -\frac{3}{2}\right\}$$

\therefore integral values of $x = \{-3, -2, -1, 0, 1, 2, 3, 4\}$
 = 8 values

29.

30.

