44 YEARS OF EXCELLENCE

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Memory Based Question Paper

MATHEMATICS

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. If $x + y + z = 17$ and x, y, z are non-negative integers, then find the number of integral solutions is

(1) 136

(2) 171

(3) 90

(4) 130

Answer (2)

Sol. $x + y + z = 17$

$${}^{17+3-1}C_{3-1} = {}^{19}C_2$$

$$= 171$$

2. Let $M = \left[a_{ij}\right]_{2\times 2}$, $0 \le i, j \le 2$ where $a_{ij} \in \{0, 1, 2\}$ and A be the event such that M is invertible then $P(A)$ is

(1) $\frac{49}{81}$ (2) $\frac{16}{27}$

(3) $\frac{47}{81}$ (4) $\frac{46}{81}$

Answer (2)

Sol. If A is invertible then $|A| \ne 0$

Now, for $|A| = 0$

(1) $\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ or $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ or $\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}$ $\rightarrow$Total matrix = 3

(2) Two 1's and Two 0's $\rightarrow$ Total matrix = 4

(3) Two 2's and Two 0's $\rightarrow$ Total matrix = 4

(4) Two 1's and Two 2's $\rightarrow$ Total matrix = 4

(5) One 1 and Three 0's $\rightarrow$ Total matrix = 4

(6) One 2 and Three 0's $\rightarrow$ Total matrix = 4

$\therefore \quad P(A) = 1 - \frac{23}{81} = \frac{48}{81} = \frac{16}{27}$

3. For a given function $f(x) = \left[x^2 - x\right] + \{x\}$

(1) $f(x)$ is continuous at $x = 0, x = 1$

(2) $f(x)$ is continuous and differentiable at $x = 0$ and $x = 1$

(3) $f(x)$ is continuous but non- differentiable at $x = 0, 1$

(4) $f(x)$ is continuous at $x = 1$ but discontinuous at $x = 0$

Answer (4)

Sol. $f(x) = \begin{cases} x+1 & -0.5 < x < 0\\ 0 & x = 0\\ -1+x & 0 < x < 1\\ 0 & x = 1\\ x-1 & 1 < x < 1.5\end{cases}$

At $x = 0^+$ $\quad f(x) = -1$

$x = 0^-$ $\quad f(x) = 1$

$\therefore$ $f(x)$ is discontinuous at $x = 0$

At $x = 1^+$ $\quad f(x) = 0$

$x = 1^-$ $\quad f(x) = 0$

$f(1) = 0$

$\therefore$ $f(x)$ is continuous at $x = 1$

4. Two complex numbers w_1 and w_2, given by $w_1 = 3 + 5i$ and $w_2 = 5 + 4i$ are both rotated by 90º with respect to origin anti-clockwise and clockwise respectively to get the new complex numbers w_3 and w_4. The principal argument of $w_3 - w_4$ is

(1) $-\pi - \tan^{-1}\frac{8}{9}$ (2) $-\pi - \tan^{-1}\frac{33}{5}$

(3) $\pi - \tan^{-1}\frac{8}{9}$ (4) $\pi - \tan^{-1}\frac{33}{5}$

Answer (3)

Sol : $w_3 = i\,w_1 = i(3 + 5i) = -5 + 3i$

$w_4 = -iw_2 = -i(5 + 4i) = 4 - 5i$

$w_3 - w_4 = -9 + 8i$

$\arg(w_3 - w_4) = \pi - \tan^{-1}\frac{8}{9}$

($\because$ $w_3 - w_4 \in$ II Quadrant)

5. Let a and b are roots of $x^2 - 7x - 1 = 0$. The value of $\frac{a^{21}+b^{21}+a^{17}+b^{17}}{a^{19}+b^{19}}$ is

(1) 29 (2) 49

(3) 53 (4) 51

Answer (4)

Sol. $\frac{a^{17}\left(a^4+1\right)+b^{17}\left(b^4+1\right)}{a^{19}+b^{19}}$

$\alpha^2 - 1 = 7\alpha$

$\Rightarrow \alpha^4 + 1 = 51\alpha^2$ < a, b

$\therefore \frac{51a^{19}+51b^{19}}{a^{19}+b^{19}} = 51$

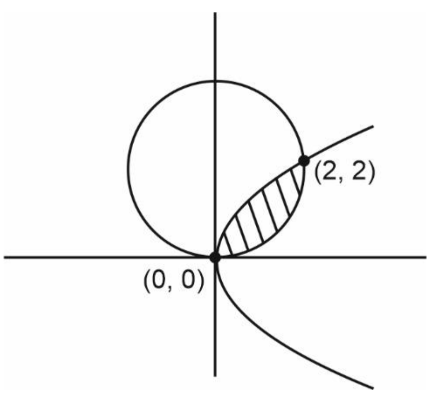
6. Find the area bounded by $\begin{cases} x^2+(y-2)^2 \le 4 \\ y^2 \le 2x \end{cases}$

(1) $\pi + \frac{4}{3}$ (2) $\pi - \frac{4}{3}$

(3) $2\pi + \frac{8}{3}$ (4) $2\pi - \frac{8}{3}$

Answer (2)

Sol.



On solving the curves, (0, 0) and (2, 2) will be the point of intersection

Required area = $\int_0^2 \left(\sqrt{4-(y-2)^2} - \frac{y^2}{2}\right) dy$

$= \left[\frac{1}{2}(y-2)\sqrt{4-(y-2)^2} + \frac{4}{2}\sin^{-1}\left(\frac{y-2}{2}\right) - \frac{y^3}{6}\right]_0^2$

$= -\frac{8}{6} - \left[\frac{1}{2}(-2)\times 0 - \pi\right] = \pi - \frac{4}{3}$

7. The number of solutions of $\cos^4\theta - 2\cos^2\theta + 3\sin^6\theta + 1 = 0$ in $[0, 2\pi]$ is

(1) 1 (2) 2

(3) 3 (4) 4

Answer (3)

Sol. $\left(1-\cos^2\theta\right)^2 + \sin^2\theta = 0$

$\Rightarrow \sin^4\theta + \sin^2\theta = 0$

$\sin^2\theta(\sin^2\theta + 1) = 0$

$\Rightarrow \sin^2\theta = 0$

$\theta = 0, \pi, 2\pi$

$\therefore$ Three solutions in $[0, 2\pi]$

8. A rectangle is drawn by lines $x = 0$, $x = 2$, $y = 0$ and $y = 5$. Points A and B lie on coordinate axes. If line AB divides the area of rectangle in 4 : 1, then the locus of mid-point of AB is

(1) Circle

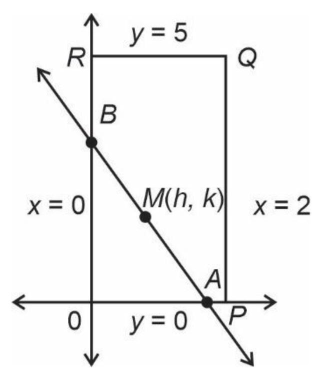
(2) Hyperbola

(3) Ellipse

(4) Straight line

Answer (2)

Sol.



$A(2h, 0)$, $B(0, 2k)$

Area of $\Delta OAB = 8$

$\frac{1}{2}\times 2h \times 2k = 8$

$hk = 4$

Locus is $xy = 4$

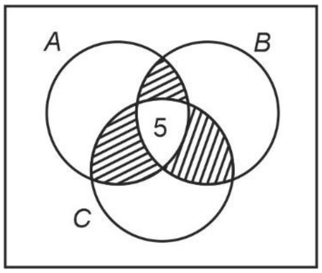
9. Let awards in event A is 48 and awards in event B is 25 and awards in event C is 18 and also $n(A \cup B \cup C) = 60$, $n(A \cap B \cap C) = 5$, then how many got exactly two awards is

(1) 21 (2) 25

(3) 24 (4) 23

Answer (1)

Sol.



$\therefore$ Number of persons who get exactly two awards

$= n(A) + n(B) + n(C) - n(A \cup B \cup C) - 2n(A \cap B \cap C)$

$= 48 + 25 + 18 - 60 - 10$

$= 21$

10. Let p : I have fever

q : I do not take medicine

r : I take rest

If I have fever then I take medicine and I take rest is equivalent to

(1) $(\sim p \vee \sim q) \wedge (\sim p \vee r)$

(2) $(\sim p \vee q) \wedge (\sim p \vee \sim r)$

(3) $(\sim p \vee q) \wedge (\sim p \vee r)$

(4) $(\sim p \vee q) \wedge (\sim p \vee \sim r)$

Answer (3)

Sol. Given statement is equivalent to

$p \to (q \wedge r)$

$= \sim p \vee (q \wedge r)$

$= (\sim p \vee q) \wedge (\sim p \vee r)$

11. Let A be a 2 × 2 matrix such that $A^T = \alpha A + 1$ and $|A^2 + 2A| = 4$, then a possible value of α is

(1) 1

(2) –1

(3) 3

(4) 0

Answer (2)

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^T = \alpha A + 1$

$\Rightarrow \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} \alpha a + 1 & \alpha b \\ \alpha c & \alpha d + 1 \end{bmatrix}$

$a = \alpha c + 1 \Rightarrow a = \dfrac{1}{1-\alpha}$ …(i)

$b = \alpha c$ …(ii)

$c = \alpha b$ …(iii)

(ii) and (iii) $c = 0$ or $\alpha = \pm 1 (\alpha \neq 1)$

$\therefore$ $c = 0$ or $\alpha = -1$

Also, $d = \alpha d + 1 \Rightarrow d = \dfrac{1}{1-\alpha}$

Case (I)

$c = 0$

$\Rightarrow b = 0$

$|A^2 + 2A| = 4 \Rightarrow |A| \cdot |A + 2I| = 4$

$$\left(\frac{1}{1-\alpha}\right)^2 \left(\frac{3-2\alpha}{1-\alpha}\right)^2 = 4$$

$$\frac{3-2\alpha}{(1-\alpha)^2} = \pm 2$$

$$\Rightarrow \alpha = \frac{1 \pm \sqrt{3}}{2}$$

Case (II)

$\alpha = -1$

12. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $0 < |a_i| < 1$, $i = \{1, 2, 3\}$

Statement-A : $|a| \geq \max\{|a_1|, |a_2|, |a_3|\}$

Statement-B : $|a| < 3\max\{|a_1|, |a_2|, |a_3|\}$

(1) Both A and B are true

(2) Both A and B are false

(3) A is true, B is false

(4) A is false, B is true

Answer (1)

Sol. $|a|^2 = \sqrt{a_1^2 + a_2^2 + a_3^2} \geq \max\{|a_1|, |a_2|, |a_3|\}$

$|a|^2 = \sqrt{a_1^2 + a_2^2 + a_3^2} < 3 \times \max\{|a_1|, |a_2|, |a_3|\}$

13. Solution of differential equation is $(1 - x^2y^2)dx = xdy + ydx$ if $y(2) = 4$, then $\frac{5y(5)+1}{5y(5)-1} = k$ then k is

(1) $\frac{49}{81}e^2$ (2) $\frac{81}{49}e^2$

(3) e^2 (4) $\frac{9}{7}e^2$

Answer (2)

Sol. $(1 - x^2y^2)dx = xdy + ydx$

$(1 - x^2y^2)dx = d(xy)$

$$\int dx = \int \frac{d(xy)}{1-x^2y^2}$$

$$x = \frac{1}{2}\log\left|\frac{1+xy}{1-xy}\right| + c$$

$\because \; y(2) = 4$

$$2 = \frac{1}{2}\log\left|\frac{1+4\times 2}{4\times 2-1}\right| + c$$

$$4 = \log\left|\frac{9}{7}\right| + c$$

$$c = 4 - \log\frac{9}{7}$$

$$x = \frac{1}{2}\log\left|\frac{1+xy}{xy-1}\right| + 4 - \log\frac{9}{7}$$

For $y(5)$

$$5 = \frac{1}{2}\log\left|\frac{1+5y}{5y-1}\right| + 4 - \log\frac{9}{7}$$

$$2\left(1+\log\frac{9}{7}\right) = \log\left(\frac{1+5y}{5y-1}\right)$$

$$\frac{5y+1}{5y-1} = e^{2\left(1+\log\frac{9}{7}\right)} \Rightarrow \frac{81}{49}e^2$$

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. 5 boys with allotted roll numbers and seat numbers are seated in such a way that no one sits on the allotted seat. The number of such seating arrangements is

Answer (44)

Sol. Number of seating arrangements

$$= 5!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right)$$

$$= 5!\left(\frac{120 - 120 + 60 - 20 + 5 - 1}{120}\right)$$

$= 44$

22. $(p \vee q) \wedge (p \vee r) \Rightarrow (q \vee r)$. Number of triplets (p, q, r) such that it is true, is ________.

Answer (7)

Sol. $(p \vee q) \wedge (p \vee r) \Rightarrow (q \vee r)$

$\Rightarrow \quad p \vee (q \wedge r) \Rightarrow (q \vee r)$

This is always true if $p \vee (q \wedge r)$ is F.

$\Rightarrow \quad (p,q,r) \equiv$ (F, F, F), (F, F, T), (F, T, F)

This is true if $p \vee (q \wedge r)$ is T and $(q \vee r)$ is T

$\Rightarrow \quad (p,q,r) \equiv$ (T, T, F), (T, F, T), (T, T, T) , (F, T, T)

$\therefore$ 7 triplets are possible

23. The number of rational terms in the expansion of $\left(3^{3/4} + 5^{3/2}\right)^{60}$

Answer (16)

Sol. $T_{r+1} = {}^{60}C_r \left((3)^{3/4}\right)^{60-r} \left(5^{3/2}\right)^r$

$$= {}^{60}C_r (3)^{\frac{180-3r}{4}} 5^{3r/2}$$

Total rational terms = 16

24. Consider the plane $2x + y - 3z = 6$. If (α, β, γ) is the image of point (2, 3, 5) in the given plane, then $\alpha + \beta + \gamma =$ _______

Answer (10)

Sol. $\frac{x-2}{2} = \frac{y-3}{1} = \frac{z-5}{-3} = -2\frac{(-14)}{14}$

$\Rightarrow$ $x = 6, y = 5, z = -1$

$\therefore$ $\alpha = 6, \beta = 5, \gamma = -1$

$\alpha + \beta + \gamma = 6 + 5 - 1 = 10$

25. The mean of coefficients of $x, x^2, \ldots, x^7$ in the binomial expansion of $(2 + x)^9$ is

Answer (2736)

Sol. Sum of coefficients = ${}^9C_1 \cdot 2^8 + {}^9C_2 \cdot 2^7 \ldots + {}^9C_7 \cdot 2^2$

$= (2 + 1)^9 - {}^9C_0 \cdot 2^9 - {}^9C_8 \cdot 2 - {}^9C_9$

$= 3^9 - 2^9 - 19 = 19683 - 572 - 19 = 19152$

$\text{Mean} = \frac{\text{Sum}}{7} = \frac{19152}{7} = 2736$

26. Consider two sets A and B. Set A has 5 elements whose mean & variance are 5 and 8 respectively. Set B has also 5 elements whose mean & variance are 12 & 20 respectively. A new set C is formed by subtracting 3 from each element of set A and by adding 2 to each element of set B. The sum of mean & variance of the set C is _______.

Answer (58)

Sol. $\overline{x_c} = \text{mean of } c = \frac{(5-3)+(12+2)}{2} = 8$

$\sigma_{12}^2 = \text{variance of } c = \frac{n_1\left(\sigma_1^2 + d_1^2\right) + n_1\left(\sigma_2^2 + d_2^2\right)}{n_1 + n_2}$

$d_1 = \overline{x_{12}} - \overline{x_1}$

$d_2 = \overline{x_{12}} - \overline{x_2}$

$n_1 = 5, \sigma_1^2 = 8, d_1 = 8 - 2 = 6$

$n_2 = 5, \sigma_2^2 = 20, d_2 = 8 - 14 = -6$

$\sigma_{12}^2 = \frac{5(8+36)+5(20+36)}{10} = 50$

$\therefore$ $\sigma_{12}^2 + \overline{x_c} = 50 + 8 = 58$

27. If $S = 109 + \frac{108}{5} + \frac{107}{5^2} + \ldots + \frac{2}{5^{107}} + \frac{1}{5^{108}}$ then the value of $165 - (25)^{-54}$ is

Answer (2175)

Sol. $S = 109 + \frac{108}{5} + \frac{107}{5^2} + \ldots + \frac{2}{5^{107}} + \frac{1}{5^{108}}$

$\frac{S}{5} = \frac{109}{5} + \frac{108}{5^2} + \ldots + \frac{2}{5^{108}} + \frac{1}{5^{109}}$

$\therefore$ $\frac{4}{5}S = 109 - \left[\frac{1}{5} + \frac{1}{5^2} + \ldots \frac{1}{5^{108}} + \frac{1}{5^{109}}\right]$

$= 109 - \left[\frac{1}{5}\frac{\left[1-\left(\frac{1}{5}\right)^{109}\right]}{\left(1-\frac{1}{5}\right)}\right]$

$\therefore$ $S = \frac{5}{4}\left[109 - \frac{1}{4}\left\{1-\left(\frac{1}{5}\right)^{109}\right]\right.$

$\therefore$ $165 = 20 \times 109 - 5 + 5^{-108}$

$\therefore$ $165 - (25)^{-54} = 2180 - 5$

$= 2175$

28. If $\log_{x+\frac{7}{2}}\left(\frac{x+7}{2x+3}\right)^2 \ge 0$, then total number of integral solution(s) is/are____.

Answer (8)

Sol. $x + \frac{7}{2} > 0 \Rightarrow x > -\frac{7}{2}$ …(i)

$x + \frac{7}{2} \ne 1 \Rightarrow x \ne -\frac{5}{2}$ …(ii)

$\frac{x+7}{2x+3} \ne 0$

$\Rightarrow$ $x \ne -7, -\frac{3}{2}$ …(iii)

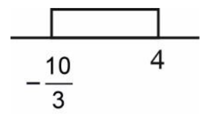
$\log_{x+\frac{7}{2}}\left(\frac{x+7}{2x+3}\right)^2 \ge 0$

$\Rightarrow \left(\frac{x+7}{2x+3}\right)^2 \geq 1$

$\Rightarrow \left(\left(\frac{x+7}{2x+3}\right)-1\right)\left(\frac{x+7}{2x+3}+1\right) \geq 0$

$\Rightarrow \left(\frac{-x+4}{2x+3}\right)\left(\frac{3x+10}{2x+3}\right) \geq 0$

$\Rightarrow \frac{(x-4)(3x+10)}{(2x+3)^2} \leq 0$

$\Rightarrow$ 

$\Rightarrow x \in \left[-\frac{10}{3}, 4\right] \ldots$(iv)

(i) $\cap$ (ii) $\cap$ (iii) $\cap$ (iv)

$x \in \left[-\frac{10}{3}, 4\right] - \left\{-\frac{5}{2}, -\frac{3}{2}\right\}$

$\therefore$ integral values of x = {–3, –2, –1, 0, 1, 2, 3, 4}

= 8 values

29.

30.

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