JEE-Mains-11-04-2023 [Memory Based] [Evening Shift]

Mathematics

Question: If the letters of the word MATHS are arranged in all possible orders and these words are written in a dictionary, then rank of the word THAMS is:
Answer: 103.00
Solution:

A H M S T

$A \rightarrow 4!$

$H \rightarrow 4!$

$N \rightarrow 4!$

$S \rightarrow 4!$

$T\ A \rightarrow 3!$

$T\ H\ A\ M\ S \rightarrow 1$

$$\Rightarrow 4! \times 4 + 3! \times 1 = 6(16+1)+1$$
$$= 6 \times 17 + 1$$
$$= 102 + 1$$
$$= 103$$

Question: $\frac{dy}{dx} + \frac{5}{x(1+x^5)} y = \frac{(1+x^5)^2}{x^7}$. If $y(1) = 2,$ then the value of $y(2)$ is:

Answer: $\frac{693}{128}$

Solution:

$$\frac{dy}{dx} + \frac{5}{x(1+x^5)} y = \frac{(1+x^5)^2}{x^9}$$

I.F. $= e^{\int \frac{5}{x(1+x^5)} dx}$

$$\Rightarrow \int \frac{5}{x(1+x^5)} dx = \int \frac{5x^{-6}}{(x^{-5}+1)} dx$$

$$\Rightarrow -\ln\left(x^{-5}+1\right)=\ln\left(\frac{1}{x^5+1}\right)$$

$$\text{I.F.} =\frac{1}{x^{-5}+1}=\frac{x^5}{1+x^5}$$

$$y\left(\frac{x^5}{1+x^5}\right)=\int\frac{x^5}{1+x^5}\cdot\frac{\left(1+x^5\right)^2}{x^7}dx$$

$$=\int\frac{1+x^5}{x^2}dx$$

$$=\int x^{-2}dx+\int x^3dx$$

$$=\frac{-1}{x}+\frac{x^4}{4}+c$$

Put $x=1, y=2$

$$2\left(\frac{1}{2}\right)=-1+\frac{1}{4}+C$$

$$\Rightarrow C=1+1-\frac{1}{4}=\frac{7}{4}$$

$$\Rightarrow y\left(\frac{x^5}{1+x^5}\right)=\frac{-1}{x}+\frac{x^4}{4}+\frac{7}{4}$$

Put $x=2$

$$y\left(\frac{32}{33}\right)=\frac{-1}{2}+\frac{16}{4}+\frac{7}{4}$$

$$y\left(\frac{32}{33}\right)=\frac{-1}{2}+\frac{23}{4}$$

$$y\left(\frac{32}{33}\right)=\frac{21}{4}$$

$$y=\frac{21}{4}\times\frac{33}{32}$$

$$=\frac{693}{128}$$

Question: $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix}=\frac{9}{3}(103x+81)$, then λ and $\frac{\lambda}{3}$ are roots of:

Options:

(a) $4x^2+24x-27=0$

(b) $4x^2-24x+27=0$

(c)

(d)

Answer: (b)

Solution:

Given $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{3}(103x+81)$

Put $x=0$ on both sides

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & \lambda^2 \end{vmatrix} = \frac{9}{8}(81)$$

$$\Rightarrow \lambda^3 = \left(\frac{9}{2}\right)^3$$

$$\Rightarrow \lambda = \frac{9}{2}$$

$$\Rightarrow \text{Roots: } \lambda = \frac{9}{2} \;\; \& \;\; \frac{\lambda}{3} = \frac{3}{2}$$

$$\text{Sum } = \frac{9}{2}+\frac{3}{2} = \frac{12}{2} = 6$$

$$\text{Product } = \frac{9}{2}\times\frac{3}{2} = \frac{27}{4}$$

$$\Rightarrow x^2-6x+\frac{27}{4}=0$$

$$4x^2-24x+27=0$$

Question: $x_1 : 1, 2, 4, 5, x, y$. $\bar{x}=5,\ \sigma^2=10$. Find mean deviation about mean.

Answer: $\frac{8}{3}$

Solution:

Mean = 5

Variance = 10

$$\frac{1+2+4+5+x+y}{6}=5$$

$$x+y=30-12$$

$$x+y=18$$

$$\frac{1^2+2^2+4^2+5^2+x^2+y^2}{6}-(5)^2=10$$

$$x^2+y^2=164$$

$$x=10,\ y=8$$

Mean deviation about Mean

$$MD(\bar{X})=\frac{\sum_{i=1}^{6}(x_i-\bar{X})}{6}$$

$$=\frac{4+3+1+0+5+3}{6}$$

$$=\frac{8}{3}$$

Question: If $a+b+c+d=11$ and maximum value of $ab^2c^3d^5=3750\beta$, then $\beta=?$

Answer: 90.00

Solution:

$$a+b+c+d=11$$

$$a+2\left(\frac{b}{2}\right)+3\left(\frac{c}{3}\right)+5\left(\frac{d}{5}\right)=11$$

$$\Rightarrow \frac{a+2\left(\frac{b}{2}\right)+3\left(\frac{c}{3}\right)+5\left(\frac{d}{5}\right)}{1+2+3+5}\geq\left[a\left(\frac{b}{2}\right)^2\left(\frac{c}{3}\right)^3\left(\frac{d}{5}\right)^5\right]^{\frac{1}{11}}$$

$$\frac{11}{11}\geq\left(\frac{ab^2c^3d^5}{4\times27\times3125}\right)$$

$$\Rightarrow ab^2c^3d^5\leq4\times27\times3125$$

$$\Rightarrow 4\times27\times3125=3750\beta$$

$$\beta=\frac{4\times27\times3125}{3750}$$

$$=2\times9\times5$$

$$=90$$

Question: If ratio of 3 consecutive coefficients in expansion of $(1+x)^{n+2}$ is $1:3:5$, then find sum of binomial coefficients.

Answer: 63.00

Solution:

$^{n+1}C_r : {}^{n+1}C_{r+1} : {}^{n+1}C_{r+2}$

$1:3:5$

$$\Rightarrow \frac{^{n+1}C_{r+1}}{^{n+1}C_r} = \frac{3}{1}$$

$$\frac{(n+1)-(r+1)+1}{(r+1)} = \frac{3}{1}$$

$$n-r+1 = 3(r+1)$$

$$n-4r = 2$$

$$\Rightarrow \frac{^{n+1}C_{r+2}}{^{n+1}C_{r+1}} = \frac{5}{3}$$

$$\frac{(n+1)-(r+2)+1}{(r+2)} = \frac{5}{3}$$

$$3(n-r) = 5(r+2)$$

$$3n-8r = 10$$

$$3n-8r = 10$$

$$2n-8r = 4$$

$$n = 6$$

$$\Rightarrow r = 1$$

$$\Rightarrow {}^7C_1 + {}^7C_2 + {}^7C_3 = 7 + \frac{7\times 6}{2} + \frac{7\times 6\times 5}{3\times 2}$$

$$= 7+21+35$$

$$= 63$$

Question: In $\left(\frac{4x}{5}-\frac{5}{2x}\right)^{2022}$, if 1011^{th} term from end is equal to $1024\times$ (1011^{th} term from beginning), then find $|x|$.

Answer: $\frac{5}{16}$

Solution:

$$^{2022}C_{1010}\left(\frac{5}{x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010} = 1011\ {}^{2022}C_{1010}\left(\frac{5}{x}\right)^{1010}$$

$$\left(\frac{5}{2x}\right)^2 = \left(\frac{4x}{5}\right)^2 \times 1024$$

$$1024 \times x^4 = \frac{5^4}{2^2 4^2}$$

$$x^4 = \frac{5^4}{2^{16}}$$

$$x = \pm\frac{5}{16}$$

Question: Find domain of $\frac{1}{\sqrt{[x]^2 - 3[x] - 10}}$.

Answer: $(x < -2) \cup (x \geq 6)$

Solution:

$$[x]^2 - 3[x] - 10 > 0$$

$$[x]^2 - 5[x] + 2[x] - 10 > 0$$

$$([x] - 5)([x] + 2) > 0$$

$[x] < -2$ or $[x] > 5$

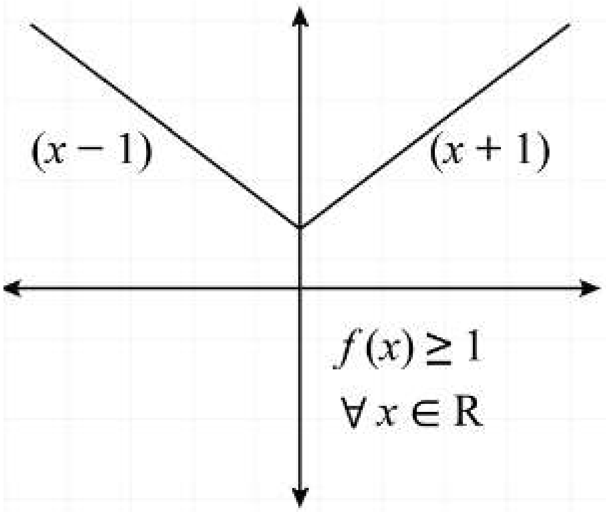
$x < -2$ or $x \geq 6$

$$\Rightarrow x \in (-\infty, -2) \cup [6, \infty)$$

Question: $f(x) = \begin{cases} x+1; & x > 0 \\ |x-1|; & x \leq 0 \end{cases}$; $g(x) = \begin{cases} x+1; & x > 0 \\ 1; & x < 0 \end{cases}$. Find points of discontinuity of $g(f(x))$.

Answer: 0.00

Solution:



$g\left(f\left(x\right)\right)=f\left(x\right)+1\ ;\ f\left(x\right)>0$

$=\begin{cases} x+1+1; & x>0 \\ \left|x-1\right|+1; & x\le 0 \end{cases}$

$=\begin{cases} x+2; & x>0 \\ \left|x-1\right|+1; & x\le 0 \end{cases}$

$=\begin{cases} x+2; & x>0 \\ -x+2; & x\le 0 \end{cases}$

LHL = RHL = $f\left(0\right)$

Question: R is maximum possible radius of a circle centered at $\left(2,0\right)$ which is enclosed in $x^2+4y^2=36$. Find $12R^2$.

Answer: 92.00

Solution:

$\frac{x^2}{36}+\frac{y^2}{9}=1$

$\left(x-2\right)^2+y^2=r^2$

$x^2+4y^2=36$

$x^2+4\left(r^2-\left(x-2\right)^2\right)=36$

$x^2+4r^2-4\left(x^2-4x+4\right)=36$

$-3x^2+16x+4r^2-52=0$

$\Delta=0$

$16^2+12\left(4r^2-52\right)=0$

$16+3\left(r^2-13\right)=0$

$3r^2-23=0$

$3r^2=23$

$r^2=\frac{23}{3}$

$\therefore 12R^2=12\times\frac{23}{3}=92$

Question: $I=\int_0^{\frac{\pi}{2}} f\left(\sin 2x\right)\sin x\,dx+\alpha\int_0^{\frac{\pi}{4}} f\left(\cos 2x\right)\cos x\,dx$. If $I=0$, then α

Answer: $-\sqrt{2}$

Solution:

$I_1=\int_0^{\frac{\pi}{2}} f\left(\sin 2x\right)\cdot\sin x\cdot dx$

Apply king's rule:

$I_1=\int_0^{\frac{\pi}{2}} f\left(\sin 2x\right)\cdot\cos x\cdot dx$

$2I_1=\int_0^{\frac{\pi}{2}} f\left(\sin 2x\right)\cdot\left[\sin x+\cos x\right]dx$

$2I_1=\sqrt{2}\int_0^{\frac{\pi}{2}} f\left(\sin 2x\right)\cdot\cos\left(x-\frac{\pi}{4}\right)dx$

$\sqrt{2}I_1=2\int_0^{\frac{\pi}{4}} f\left(\sin 2x\right)\cdot\cos\left(x-\frac{\pi}{4}\right)dx$

$I_1=\sqrt{2}\int_0^{\frac{\pi}{4}} f\left(\sin 2x\right)\cdot\cos\left(x-\frac{\pi}{4}\right)dx$

Apply king's rule again:

$I_1=\sqrt{2}\int_0^{\frac{\pi}{4}} f\left(\cos 2x\right)\cdot\cos x\,dx$

$\Rightarrow \alpha=-\sqrt{2}$

Question: Consider $y=e^{8x}-e^{6x}+3e^{4x}-e^{2x}+1$. At how many points it cuts x-axis.
Answer: 0.00
Solution:

$y=e^{8x}-e^{6x}+3e^{4x}-e^{2x}+1$

$e^{x}=t$

$y=t^{8}-t^{6}+3t^{4}-t^{2}+1;\ t>0$

$y>0$ for $t>0$

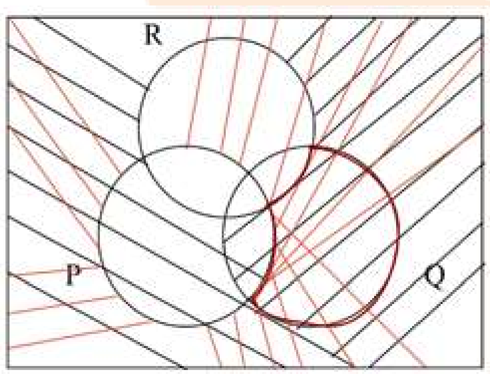
No real root

Question: Converse of: $(p\vee\sim q)\rightarrow r$ is ____.
Answer: ()
Solution:

$(p\vee\sim q)\rightarrow r$

$r\rightarrow(\sim p\wedge q)$

i.e., $\sim r\vee(\sim p\wedge q)$



Question: $\vec{a}=\hat{i}+2\hat{j}+3\hat{k},\ \vec{b}=\hat{i}+\hat{j}-\hat{k},\ \vec{a}\cdot\vec{c}=11,\ \vec{b}\cdot(\vec{a}\times\vec{c})=27$, $\vec{b}\cdot\vec{c}=-\sqrt{3}|\vec{b}|$. Find $|\vec{a}\times\vec{c}|^{2}$.
Answer: 285.00
Solution:

$\vec{b}\times(\vec{a}\times\vec{c})=(b\cdot c)\vec{a}-(b\cdot a)\vec{c}$

$|\vec{b}\cdot(\vec{a}\times\vec{c})|^{2}+|\vec{b}\times(\vec{a}\times\vec{c})|^{2}=|b|^{2}|\vec{a}\times\vec{c}|^{2}$

$$27^2+\left|\left(\vec{b}\cdot\vec{c}\right)^2\vec{a}^2+\left(\vec{b}\cdot\vec{a}\right)^2\vec{c}^2-2\left(\vec{a}\cdot\vec{b}\right)\left(\vec{b}\cdot\vec{c}\right)\left(\vec{a}\cdot\vec{c}\right)\right|=3\left(\vec{a}\times\vec{c}\right)^2$$

$$27^2+\left|14\times3\times3-0\right|=3\left|\vec{a}\times\vec{c}\right|^2$$

$$729+126=3\left|\vec{a}\times\vec{c}\right|^2$$

$$\left|\vec{a}\times\vec{c}\right|^2=285$$

Question: If $10=1+\frac{4}{k}+\frac{8}{k^2}+\frac{13}{k^3}+\frac{19}{k^4}+\ldots\infty$, then find k.

Answer: 2.00

Solution:

$$10=1+\frac{4}{k}+\frac{8}{k^2}+\frac{13}{k^3}+\frac{19}{k^4}+\ldots\infty$$

$$\frac{10}{k}=\quad\frac{1}{k}+\frac{4}{k^2}+\frac{8}{k^3}+\frac{13}{k^4}+\ldots.\infty$$

$$10-\frac{10}{k}=1+\frac{3}{k}+\frac{4}{k^2}+\frac{5}{k^3}+\frac{6}{k^4}+\ldots.\infty$$

$$\frac{1}{k}\left(10-\frac{10}{k}\right)=\quad\frac{1}{k}+\frac{3}{k^2}+\frac{4}{k^2}+\frac{5}{k^4}+\ldots.\infty$$

$$10\left(1-\frac{2}{k}+\frac{1}{k^2}\right)=1+\frac{2}{k}+\frac{1}{k^2}+\frac{1}{k^3}+\frac{1}{k^4}+\ldots.\infty$$

$$10\left(1-\frac{1}{k}\right)^2=1+\frac{1}{k}+\frac{1}{k^2}+\frac{1}{k^3}+\frac{1}{k^4}+\ldots.\infty+\frac{1}{k}$$

$$10\left(1-\frac{1}{k}\right)^2=\frac{1}{1-\frac{1}{k}}+\frac{1}{k}$$

$$10\left(\frac{k-1}{k}\right)^2=\frac{k}{k-1}+\frac{1}{k}$$

$$10\left(\frac{\left(k-1\right)^2}{k^2}\right)=\frac{k^2+k-1}{k\left(k-1\right)}$$

$$10\left(k-1\right)^3=k^3+k^2-k$$

$$\Rightarrow 9k^3-31k^2+31k-10=0$$

$$\Rightarrow k=2$$

Question: If $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ are coplanar vectors then the value of $\left[\vec{a}\quad\vec{b}\quad\vec{c}\right]$ is:

Answer: ()
Solution:

$$\left[\vec{a}-\vec{d}\quad \vec{b}-\vec{d}\quad \vec{c}-\vec{d}\right]=0$$

$$\left[\vec{a}\quad \vec{b}-\vec{d}\quad \vec{c}-\vec{d}\right]-\left[\vec{d}\quad \vec{b}-\vec{d}\quad \vec{c}-\vec{d}\right]=0$$

$$\left[\vec{a}\quad \vec{b}\quad \vec{c}-\vec{d}\right]-\left[\vec{a}\quad \vec{d}\quad \vec{c}-\vec{d}\right]-\left[\vec{d}\quad \vec{b}\quad \vec{c}-\vec{d}\right]=0$$

$$\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]-\left[\vec{a}\quad \vec{b}\quad \vec{d}\right]-\left[\vec{a}\quad \vec{d}\quad \vec{c}\right]-\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]=0$$

$$\Rightarrow\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]=\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]+\left[\vec{a}\quad \vec{d}\quad \vec{c}\right]+\left[\vec{d}\quad \vec{b}\quad \vec{c}\right]$$

Question: f is defined from $A=\{1,2,3,4,5\}$ to $B=\{1,2,3,4,5,6\}$, such that $f(1)+f(2)=f(4)-1$. Find number of such functions.

Answer: 360.00
Solution:

$$f(1)+f(2)+1=f(4)\le 6$$

$$f(1)+f(2)\le 5$$

Case-1:

$f(1)=1$
$f(2)=1,2,3,4$ $\rightarrow$ 4 mappings

Case-2:

$f(1)=2$
$f(2)=1,2,3$ $\rightarrow$ 3 mappings

Case-3:

$f(1)=3$
$f(2)=1,2$ $\rightarrow$ 2 mappings

Case-4:

$f(1)=4$
$f(2)=1$ $\rightarrow$ 1 mapping

No. of functions $=(4+3+2+1)\times 6\times 6$

$=10\times 6\times 6$

$=360$

Question: For a biased coin, $P(H)=\frac{1}{4}$. Its tossed n times, till we get H. If probability that $64x^2+5nx+1=0$ has no real roots is $\frac{p}{q}$ (p,q co-primes) then $q-p=?$

Answer: 27.00
Solution:

$64x^2+5nx+1=0$

$D<0$

$(5n)^2-4(64)<0$

$(5n)^2<(2\times 8)^2$

$5n<2\times 8$

$n<\frac{16}{5}$

$\Rightarrow n=1,2$ or 3

Probability $=\frac{1}{4}+\frac{3}{4}\cdot\frac{1}{4}+\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{4}$

$=\frac{1}{4}+\frac{3}{4}\cdot\frac{1}{4}\left(1+\frac{3}{4}\right)$

$=\frac{1}{4}\left(1+\frac{3}{4}\cdot\frac{7}{4}\right)$

$=\frac{1}{4}\left(\frac{16+21}{16}\right)$

$=\frac{37}{64}$

$\Rightarrow q-p=64-37=27$

Question: The area between the curves $y=2x^2+1$ and tangent to it at $(1,3)$ and $x+y=1$ is

Answer: ()
Solution:

$y=2x^2+1$

$y'=4x$

Point is (1, 3)

$m = 4$

$\Rightarrow y - 3 = 4(x - 1)$

$y - 3 = 4x - 4$

$y = 4x - 1$

$\Rightarrow x + y = 1$

$y = x - 1$

$\Rightarrow 4x - 1 = 1 - x$

$5x = 2$

$x = \frac{2}{5}$

$$\text{A} = \int_0^{\frac{2}{5}} (2x^2 + 1) - (1 - x)\, dx + \int_{\frac{2}{5}}^{1} (2x^2 + 1) - (4x - 1)\, dx$$

$$= \int_0^{\frac{2}{5}} 2x^2 + x\, dx + \int_{\frac{2}{5}}^{1} 2x^2 - 4x + 2\, dx$$

$$= \left.\frac{2x^3}{3} + \frac{x^2}{2}\right]_0^{\frac{2}{5}} + \left.\frac{2x^3}{3}\right]_{\frac{2}{5}}^{1} - \left.2x^2\right]_{\frac{2}{5}}^{1} + \left.2x\right]_{\frac{2}{5}}^{1}$$

$$= \frac{2}{25} + \frac{2}{5} - 2 + \frac{8}{25} + 2 - \frac{4}{5}$$

$$= -\frac{2}{5} + \frac{2}{3} = \frac{4}{15}$$

Question: $f(x) = \begin{cases} e^{\min(x^2, \{x\})} & ; \quad x \in (0,1) \\ e^{[x - \ln x]} & ; \quad x \in (1,2) \end{cases}, \int_0^2 x f(x)\, dx = ?$

Answer: $2e - \frac{1}{2}$

Solution:

$$f(x) = \begin{cases} e^{x^2} & ; \quad x \in (0,1) \\ e & ; \quad x \in [1,2) \end{cases}$$

$$\int_0^2 xf(x)\, dx = \int_0^1 x \cdot e^{x^2}\, dx + \int_1^2 x \cdot e\, dx$$

Substitute $x^2 = t \Rightarrow 2xdx = dt$

$$\int_0^2 x\, f(x)\, dx = \frac{1}{2}\int_0^1 e^t\, dt + e\int_1^2 x\, dx$$

$$= \frac{1}{2}\left[e^t\right]_0^1 + e\left[\frac{x^2}{2}\right]_1^2$$

$$= \frac{1}{2}(e-1) + \frac{3}{2}e = 2e - \frac{1}{2}$$