11/04/2023
Evening

Corporate Office : Aakash Tower, 8, Pusa Road, New Delhi-110005 | Ph.: 011-47623456

Memory Based
Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2023 (Online) Phase-2

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Density (ρ) of a body depends on the force applied (F), its speed (v) and time of motion (t) by the relation $\rho = KF^a v^b t^c$, where K is a dimensionless constant. Then
   (1) $a = 1, b = -4, c = -2$
   (2) $a = 2, b = -4, c = -1$
   (3) $a = -1, b = -4, c = 2$
   (4) $a = 1, b = 4, c = -2$

Answer (1)

Sol. $[ML^{-3}] = [MLT^{-2}]^a[LT^{-1}]^b[T]^c$
$= [M^aL^{a+b}T^{-2a-b+c}]$
$a = 1$,
$a + b = -3$,
$\Rightarrow b = -4$,
also $-2a - b + c = 0$
$c = -2$

2. In which of the following process, the internal energy of gas remains constant.
   (1) Isothermal (2) Isochoric
   (3) Isobaric (4) Adiabatic

Answer (1)

Sol. T = constant $\Rightarrow U$ = constant

3. A particle is projected at an angle of 30° with ground with speed 40 m/s. The speed of particle after two seconds is (use $g = 10$ m/s^2)
   (1) $20\sqrt{2}$ m/s (2) $20\sqrt{3}$ m/s
   (3) 20 m/s (4) $10\sqrt{3}$ m/s

Answer (2)

Sol. At $t = 2$ particle is at maximum height moving with 40cos30° m/s.

4. Potential at the surface of a uniformly charged non-conducting sphere is V. Then the potential at its centre is
   (1) 0 (2) $\frac{V}{2}$
   (3) $2V$ (4) $\frac{3V}{2}$

Answer (4)

Sol. $V = \frac{KQ}{2R^3}\left(3R^2 - r^2\right)$ at $r = R \Rightarrow V = \left(\frac{KQ}{R}\right)$

at $r = 0$, $V_0 = \frac{3KQ}{2R} = \left(\frac{3V}{2}\right)$

5. If $\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{A} - \vec{B} = 2\hat{j}$, then find $|\vec{B}|$.
   (1) 3 (2) $3\sqrt{3}$
   (3) 2 (4) $\sqrt{3}$

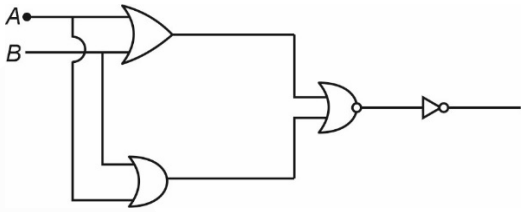
Answer (1)

Sol. $\left(2\hat{i} + 3\hat{j} + 2\hat{k}\right) - \vec{B} = 2\hat{j}$

$\Rightarrow \vec{B} = 2\hat{i} + \hat{j} + 2\hat{k}$

$\Rightarrow |\vec{B}| = 3$

6. The resultant gate is

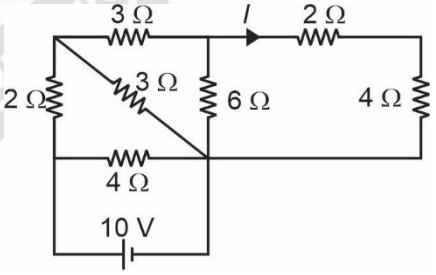


   (1) NAND (2) NOR
   (3) OR (4) AND

Answer (4)

Sol. (A+B) (A·B) = (A·AB) + A·(AB)
= AB + AB
= (AB)

7. For the given circuit diagram, find the current I.



   (1) $\frac{5}{16}$ A (2) $\frac{5}{48}$ A
   (3) $\frac{5}{12}$ A (4) $\frac{1}{16}$ A

Answer (3)

Sol. $i_{battery} = \frac{10}{2} = 5$ A

$I = i_{battery} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{2} = \frac{5}{12}$ A

8. If a nucleus is divided in ratio of $1 : 2^{1/3}$, then find ratio of velocity of the parts is
   (1) 2 (2) $2^{1/3}$
   (3) $2^{2/3}$ (4) $2^{-1/3}$

Answer (2)

Sol. From conservation of momentum,

$$m_0\vec{v}_1 + 2^{1/3}m_0\vec{v}_2 = 0$$

$$\Rightarrow \left|\frac{\vec{v}_1}{\vec{v}_2}\right| = 2^{1/3}$$

9. If electric field $\left(\vec{E}\right)$ at an instant is $6.6\hat{j}$ N/C and the EM wave is propagating along positive *x*-direction then $\vec{B}$ at that instant is given by

(1) $2.2\times10^{-8}\hat{k}$ T (2) $-2.2\times10^{-8}\hat{k}$ T

(3) $-0.5\times10^{-8}\hat{k}$ T (4) $19.8\times10^{8}\hat{k}$ T

Answer (1)

Sol. $\left|\vec{E}\right| = C\left|\vec{B}\right|$

$$\left|\vec{B}\right| = \frac{6.6}{3\times10^8} = 2.2\times10^{-8}\text{ T}$$

Also $\hat{E}\times\hat{B} = \hat{C}$

10. Find average speed of N_2 at 27°C.

(1) 476 m/s (2) 470 m/s

(3) 480 m/s (4) 490 m/s

Answer (1)

Sol. $\bar{v} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8\times8.314\times300}{3.14\times28\times10^{-3}}} = 476$ m/s

11. A charge particle is projected inside along the axis of long solenoid, then

(a) Path will be straight line

(b) There is no effect of magnetic field on charge

(c) Path will be parabolic

(d) Path will be circular

(1) a, d (2) a, b

(3) b, d (4) a, b, d

Answer (2)

Sol. $\vec{F} = q\vec{v}\times\vec{B} = 0$

12. Six identical small liquid drops are mixed together to form a bigger drop. The terminal velocity of bigger drop if terminal velocity of small drop is 10 m/s, will be

(1) $10\times(6)^{\frac{1}{3}}$ m/s (2) $10\times(6)^{\frac{2}{3}}$ m/s

(3) $5\times(3)^{\frac{2}{3}}$ m/s (4) $10\times(6)^3$ m/s

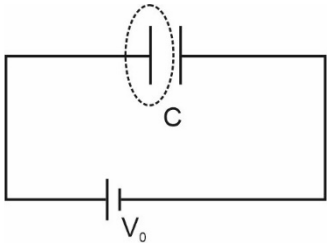
Answer (2)

Sol. $R = 6^{1/3}r$

Also, $\frac{v_b}{v_s} = \frac{R^2}{r^2}$ ($\because v_T \propto$ (Radius)^2)

$V_b = 10\times(6)^{2/3}$

13. A parallel plate capacitor C connected with a battery of voltage V_0. A close gaussian surface is shown by dotted boundary as shown. The electric flux through the surface is



(1) $\frac{2CV}{\epsilon_0}$ (2) $\frac{CV_0}{\epsilon_0}$

(3) $\frac{CV_0}{2\epsilon_0}$ (4) $\frac{3CV}{2\epsilon_0}$

Answer (1)

Sol. $\phi = \frac{Q}{\epsilon_0} = \frac{CV_0}{\epsilon_0}$

14. A satellite is moving around earth surface. How much minimum speed should be increased so that it escapes from earth surface? (g = acceleration due to gravity, R = radius of earth)

(1) $2\sqrt{gR}$ (2) $\left(\sqrt{2}-1\right)\sqrt{gR}$

(3) $\sqrt{\frac{gR}{2}}$ (4) $\left(\sqrt{3}-1\right)\sqrt{gR}$

Answer (2)

Sol. $v_{\text{circular}} = \sqrt{\frac{GM}{R}} = \sqrt{gR}$; $\Delta v = \left(\sqrt{2}-1\right)\sqrt{gR}$

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

15. **A :** Moving magnet in conducting pipe slows down.

R : Because eddy current is formed.

(1) A is correct, R is wrong

(2) A and R both are wrong

(3) A and R both are correct

(4) A is wrong, R is correct

Answer (3)

Sol. Moving magnet in conducting pipe causes change in flux and hence induced emf. This emf causes eddy current in conducting pipe in such a way that it tries to oppose the change in flux, therefore magnet slows down.

16. A source of sound is moving away from a stationary observer with constant velocity 40 m/s. Find frequency heard by observer, if original frequency of source is 400 Hz and speed of sound in air is 360 m/s

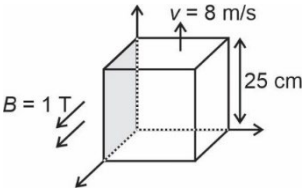
(1) 330 Hz (2) 320 Hz

(3) 360 Hz (4) 280 Hz

Answer (3)

Sol. $f = 400\left(\frac{360}{360+40}\right) = 360$ Hz

17. Find emf induces across the faces of given cube.



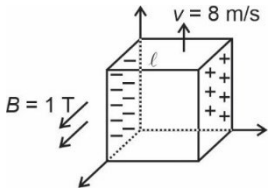
(1) 2*V* (2) 4*V*
(3) 8*V* (4) 6*V*

Answer (1)

Sol. $\varepsilon_{ind} = Bv\ell$

$\varepsilon_{ind} = 1(8)(0.25)$

$\varepsilon_{ind} = 2$ volt



18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. A body is rotating with kinetic energy *E*. If angular velocity of body is increased to three times of initial angular velocity then kinetic energy becomes *nE*. Find *n*.

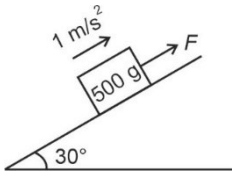
Answer (9)

Sol. K.E. = $\frac{1}{2}I\omega^2 = E$

$E_f = \frac{1}{2}I(3\omega)^2 = 9\times\left(\frac{1}{2}I\omega^2\right)$

$E_f = 9E$

22. Find power delivered by *F* at *t* = 10 s. Body start from rest.



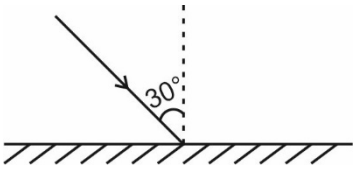
Answer (30)

Sol. $F - 0.5\ g\sin30° = 0.5\ a \Rightarrow F = 0.5 + 2.5 = 3$ N

$v_{10} = u + at \Rightarrow v_{10} = 0 + 1(10) = 10$ m/s

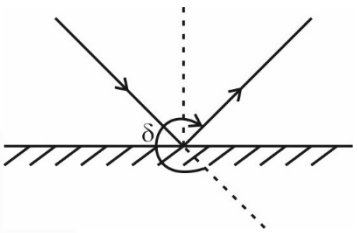
$P_{10} = Fv = 30$ w

23. A ray of light is incident on a plane mirror as shown in figure. Find the deviation of ray (in degree and clockwise direction).



Answer (240)

Sol. $\delta = 180° + 60° = 240°$ (clockwise)



24. Proton and electrons have equal kinetic energy, the ratio of de Broglie wavelength of proton and electron is $\frac{1}{x}$. Find *x*. (Mass of proton = 1849 times mass of electron)

Answer (43)

Sol. $P = \sqrt{2Km}$

$\lambda = \frac{h}{P}$

$\frac{\lambda_p}{\lambda_e} = \frac{P_e}{P_p} = \sqrt{\frac{2Km_e}{2Km_p}} = \sqrt{\frac{m_e}{m_p}} = \sqrt{\frac{1}{1849}} = \frac{1}{43}$

25. Energy of hydrogen in ground state is –13.6 eV. The energy of He^+ in first exited state is –13.6*x*. Find the value of *x*.

Answer (1)

Sol. For He^+

$E = \frac{-13.6Z^2}{2^2} = \frac{-13.6\times4}{4} = -13.6$ eV

26.

27.

28.

29.

30.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Which of the following has minimum boiling point?

(1) Na (2) K

(3) Rb (4) Cs

Answer (4)

Sol. Cs has minimum boiling point as boiling point of alkali metals decreases down the group.

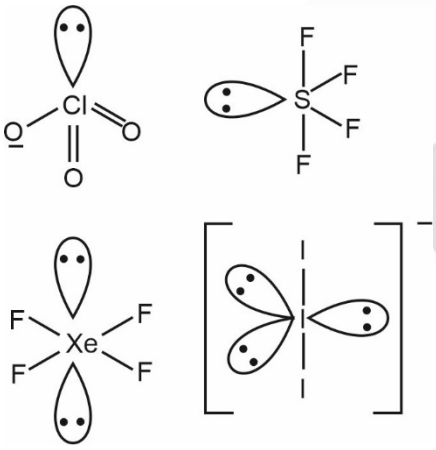
2. Which of the following has maximum number of l.p. at central atom?

(1) ClO_3^- (2) SF_4

(3) XeF_4 (4) I_3^-

Answer (4)

Sol.



From the structures of the given species, it can be clearly seen that I_3^- has maximum number of lone pairs at central atom

3. **Statement-1:** Sulphides are converted into oxide first.

Statement-2: Because oxides can be reduced easily.

(1) Only 1st is correct (2) Only 2nd is correct

(3) Both are correct (4) Both are incorrect

Answer (3)

Sol. Sulphide ores are roasted for conversion to oxides before reduction. Oxides can be easily reduced as compared to sulphides.

4. Red ppt. by Benedict solution is

(1) Glucose

(2) RNA

(3) DNA

(4) Sucrose

Answer (1)

Sol. Benedict solution oxidises aldoses and ketoses to gluconic acid and itself gets reduced to red ppt. of Cu_2O.

Glucose + Benedict solution $\rightarrow$

Gluconic acid + $Cu_2O\downarrow$ (Red)

DNA, RNA and Sucrose do not react with Benedict solution.

5. $\left[Fe(H_2O)_6\right]^{+3}$, $\left[Fe(CN)_6\right]^{-3}$ magnetic spin only magnetic moment is respectively

(1) 8.87 and 6.92 (2) 5.98 and 1.732

(3) 6.92 and 6.92 (4) 3.87 and 1.732

Answer (2)

Sol. Both complexes have d^5 configuration

$\left[Fe(H_2O)_6\right]^{+3} \rightarrow$ 5 unpaired electrons

$\mu = \sqrt{35}$ B.M.

$\left[Fe(CN)_6\right]^{-3} \rightarrow$ 1 unpaired electron

$\mu = \sqrt{3}$ B. M.

6. **Statement 1 :** Nylon-6 is made by Caprolactum

Statement 2 : LDP is made by $TiCl_4$ & $Al(Et)_3$

(1) Only 1st is correct (2) Only 2nd is correct

(3) Both are correct (4) Both are incorrect

Answer (1)

Sol. $TiCl_4 + Al(Et)_3$ is used as a catalyst in preparation HDP

7. Consider the following change :

$[NiBr_2Cl_2]^{2-} \longrightarrow [PtCl_2Br_2]^{2-}$

During the above change, which of the following properties does not change?

(1) Geometrical isomerism

(2) Structure

(3) Optical activity

(4) Splitting energy

Answer (3)

Sol. $[NiBr_2Cl_2]^{2-} \longrightarrow$ This complex species is tetrahedral as $Br^{\ominus}$ & $Cl^{\ominus}$ are weak field ligands.

$[PtBr_2Cl_2]^{2-} \longrightarrow$ As Pt belongs to 5d series, this complex species is square planar.

Splitting energy will be different as central atom is different.

Both the complex species are optically inactive.

$[NiBr_2Cl_2]^{2-}$, being tetrahedral does not show G.I.

$[PtBr_2Cl_2]^{2-}$ shows two G.I.

8. $A \xrightarrow{K} B$

Follows first order kinetics w.r.t. A and B, Both

i.e. $r = K[A]^1[B]^1$

r	[A]	[B]
20	0.1	0.5
(X)	0.4	0.5
40	(0.8)	(Y)

Find out "K" and "Y"

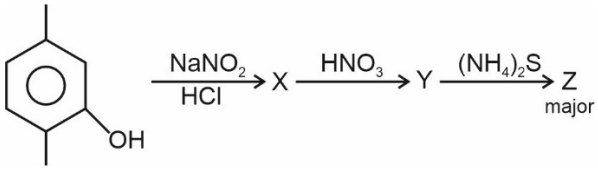
(1) 80, 2 (2) 80, 1

(3) 80, 0.125 (4) 40, 0.125

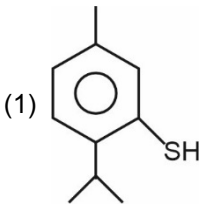
Answer (3)

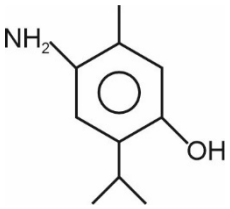
Sol. [A] : 4 times $\Rightarrow$ rate 4 times

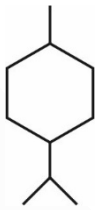
$\Rightarrow$ X = 80

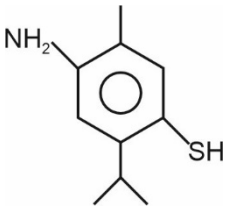
9. 

Compound Z is

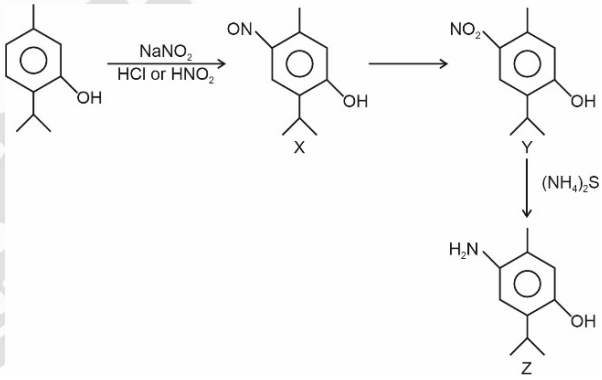
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

10. What is the chemical formula of freon gas?

(1) $C_2Cl_2F_4$ (2) $C_2F_2H_4$

(3) CHF_3 (4) CCl_2F_2

Answer (4)

Sol. The chemical formula of freon gas is CCl_2F_2.

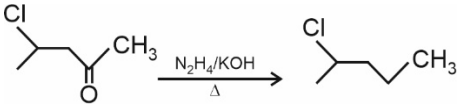
11. 2 gm of x is present in 1 mole of H_2O. Find the mass % of x.

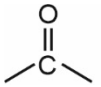
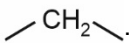
(1) 10% (2) 20%

(3) 5% (4) 7%

Answer (1)

Sol. Mass % of x = $\frac{2}{20} \times 100 = 10$

12. **Assertion:** 

Reason: Wolf Kirshner reduction is used for reduction of  into  .

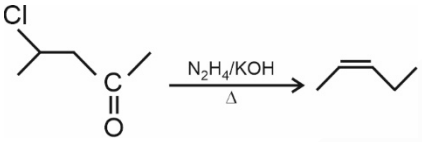
(1) Assertion and Reason both are correct and Reason is correct explanation of Assertion

(2) Assertion and Reason both are correct but the Reason is not correct explanation of Assertion

(3) Assertion and Reason both are incorrect

(4) Assertion is incorrect and reason is correct statement

Answer (4)

Sol. 

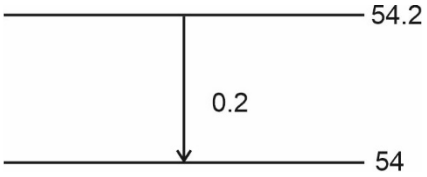
Because heating in the presence of base results in elimination

13. Glucose is added in 100 gm of water. Lowering in vapor pressure is 0.2 mm Hg. Vapour pressure of pure water is 54.2 mm Hg. Then weight of glucose is

(1) 3.70 gm (2) 4.92 gm

(3) 6.73 gm (4) 8.74 gm

Answer (1)

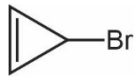
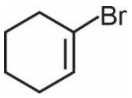
Sol. 

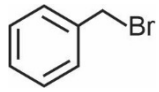
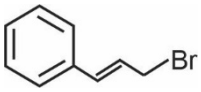
$$\frac{0.2}{54} = \frac{n_{glucose}}{(100/18)}$$

$$n_{glucose}\,\frac{0.2}{54} = \frac{100}{18}$$

Mass of glucose = $\frac{0.2}{54} \times \frac{100}{18} \times 180$ = 3.70 gm

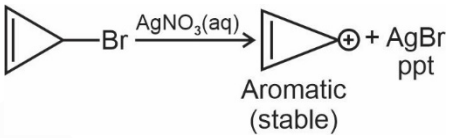
14. Which of the following will not give precipitate with $AgNO_3$(aq.)

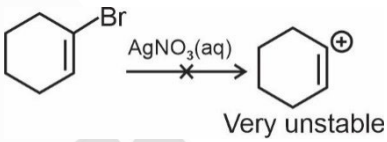
(1)  (2) 

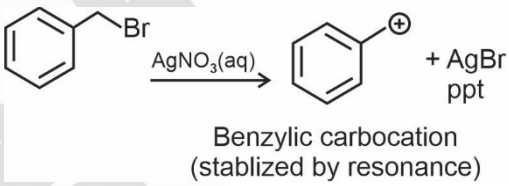
(3)  (4) 

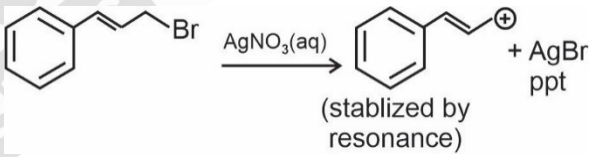
Answer (2)

Sol. Compounds which result in the formation of stable carbocation intermediate will give precipitate with aq. $AgNO_3$









15. Least stable Hydride is

(1) HF (2) LiH

(3) BeH_2 (4) NaH

Answer (3)

Sol. BeH_2 is least stable as it has significant covalent character and is an electron-deficient hydride.

16. Find the root mean square velocity for Nitrogen gas at 27°C (in m/sec)

(1) 426 (2) 517

(3) 327 (4) 646

Answer (2)

Sol. $v = \frac{\sqrt{3RT}}{M} = \sqrt{\frac{3 \times 8.314 \times 300}{28 \times 10^{-3}}}$

= 516.95

$\simeq$ 517 m/sec

17. **Assertion (A) :** Glycine react with Cl_2 in the presence of red P to give optically active compound

Reason (R) : Compound containing two chiral centres is always optically active

(1) Both (A) & (R) are correct & (R) is the correct explanation of (A)

(2) Both (A) & (R) are correct & (R) is not the correct explanation of (A)

(3) (A) is correct, (R) is incorrect statement

(4) (A) & (R), both are incorrect

Answer (3)

Sol. $H_2N-CH_2-COOH \xrightarrow[Cl_2]{RedP} H_2N-\underset{Cl}{\overset{*}{C}H}-COOH$

Contain chiral centre

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. How many of the following are intrinsic properties?

Gibbs free energy, E°_{cell}, Volume, Molarity

Answer (02.00)

Sol. E°_{cell} and molarity are intrinsic properties. But Gibb's Free Energy and Volume are extrinsic properties.

22. 2-Chloro-1-butene $\xrightarrow{HCl}$ Number of Isomeric product possible are?

(excluding rearranged products)

Answer (03.00)

Sol. $C=\overset{Cl}{\overset{|}{C}}-C-C \xrightarrow{HCl}$

(i) $C-\overset{Cl}{\overset{|}{C}}-C-C$ (with Cl below C) (2)

(ii) $C-\overset{Cl}{\overset{|}{C}}-C-C$ (with Cl below C) (1)

Total 3 Isomers

23. When 2 gm magnesium reasts with excess of HCl and H_2 gas is produced then the volume of H_2 gas produced is ____ × 10^{-2} liter at STP? (Nearest Integer)

Answer (187)

Sol. $Mg + 2HCl \rightarrow MgCl_2 + H_2(g)$

$\frac{2}{24}$ $\frac{2}{24} \times 22.4$

$= 1.87$ L

$\simeq 187 \times 10^{-2}$ L

24. $P_4 + SOCl_2 \longrightarrow 4PCl_3 + x\ SO_2 + y\ S_2Cl_2$

x + y is _____

Answer (6)

Sol. $P_4 + 8SOCl_2 \longrightarrow 4PCl_3 + 4SO_2 + 2S_2Cl_2$

x = 4

y = 2

x + y = 6

25.

26.

27.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Using all he letters of the word MATHS, then rank of the word THAMS is

(1) 101 (2) 102

(3) 103 (4) 104

Answer (3)

Sol. $\overset{5}{T}\overset{2}{H}\overset{1}{A}\overset{3}{M}\overset{4}{S}$

$\overset{4}{4!}\ \overset{1}{3!}\ \overset{0}{2!}\ \overset{0}{1!}\ \overset{0}{0!}$

$\therefore$ Rank = 4 × 4! + 1 × 3! + 1

= 96 + 6 + 1 = 103

2. $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x+81)$, then λ and $\frac{\lambda}{3}$ are roots of

(1) $4x^2 + 24x - 27 = 0$

(2) $4x^2 - 24x + 27 = 0$

(3) $4x^2 - 24x - 27 = 0$

(4) $4x^2 + 24x + 27 = 0$

Answer (2)

Sol. Put $x = 0$ in the given equation

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^2 \end{vmatrix} = \frac{9}{8} \times 81$$

$$\Rightarrow \lambda^3 = \frac{(3)^6}{2^3}$$

$$\lambda = \frac{9}{2}$$

$$\Rightarrow \frac{\lambda}{3} = \frac{3}{2}$$

$$x^2 - \left(\frac{9}{2} + \frac{3}{2}\right)x + \frac{9}{2} \times \frac{3}{2} = 0$$

$$\boxed{4x^2 - 24x + 27 = 0}$$

3. $\frac{dy}{dx} + \frac{5}{x(1+x^5)}y = \frac{(1+x^5)^2}{x^7}$. If $y(1) = 2$, then the value of $y(2)$ is

(1) $\frac{693}{128}$ (2) $\frac{697}{128}$

(3) $\frac{637}{128}$ (4) $\frac{627}{128}$

Answer (1)

Sol. I.F. $= e^{\int \frac{5}{x(1+x^5)}dx} = e^{\int \frac{5x^{-6}}{(x^{-5}+1)}dx}$

$$= e^{-\ln(x^{-5}+1)} = \frac{1}{x^{-5}+1} = \frac{x^5}{x^5+1}$$

$$y \cdot \frac{x^5}{x^5+1} = \int \frac{(1+x^5)^2}{x^7} \cdot \frac{x^5}{(1+x^5)}dx$$

$$= \int \frac{(1+x^5)}{x^2}dx$$

$$= \frac{-1}{x} + \frac{x^4}{4} + C$$

$$y(1) = 2 \Rightarrow 2\left(\frac{1}{2}\right) = -1 + \frac{1}{4} + C$$

$$\Rightarrow C = \frac{7}{4}$$

Put $x = 2$

$$\Rightarrow y\left(\frac{32}{33}\right) = \frac{-1}{2} + 4 + \frac{7}{4}$$

$$\Rightarrow y = \frac{693}{128}$$

4. The domain of the function $f(x) = \frac{1}{\sqrt{[x]^2 - 3[x] - 10}}$ is

(1) $(-\infty, 3] \cup [6, \infty)$ (2) $(-\infty, -2) \cup (2, \infty)$

(3) $(-\infty, 3] \cup [5, \infty)$ (4) $(-\infty, -2) \cup [6, \infty)$

Answer (4)

Sol. $[x]^2 - 3[x] - 10 > 0$

$([x]+2)([x]-5) > 0$

$[x] < -2$ OR $[x] > 5$

$[x] \le -3$ OR $[x] \ge 6$

$x < -2$ OR $x \ge 6$

$x \in (-\infty, -2) \cup [6, \infty)$

5. Let mean and variance of the data 1, 2, 4, 5, x, y are 5 and 10 respectively. Then mean deviation about the mean of data is

(1) $\frac{5}{2}$ (2) $\frac{7}{2}$

(3) $\frac{5}{6}$ (4) $\frac{7}{6}$

Answer (1)

Sol. $12 + x + y = 30 \Rightarrow x + y = 18$

and $\frac{x^2+y^2+46}{6} - (5)^2 = 10$

$\therefore \quad \frac{x^2+y^2+46}{6} = 10 + 25$

$x^2 + y^2 = 164$

$\therefore \quad x = 10, y = 8$

Now, mean deviation about mean

$= \frac{4+2+1+0+5+3}{6} = \frac{15}{6} = \frac{5}{2}$

6. If $a + b + c + d = 11$ ($a, b, c, d > 0$) then maximum value of $a^5b^3c^2d = 3750\beta$ the β is

(1) 90 (2) 115

(3) 120 (4) 85

Answer (1)

Sol. Assume numbers to be

$\frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{b}{3}, \frac{b}{3}, \frac{b}{3}, \frac{c}{2}, \frac{c}{2}, d.$

Now apply AM $\geq$ GM

$$\frac{\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{2}+\frac{c}{2}+d}{11} \geq \left(\frac{a^5b^3c^2d}{5^5 3^3 2^2 1}\right)^{\frac{1}{11}}$$

$a^5b^3c^2d \leq 5^5 3^3 2^2$

$\therefore$ Max of $a^5b^3c^2d = 5^5 3^3 2^2 = 3{,}37{,}500$

$= 90 \times 3750$

$\Rightarrow \beta = 90$

7. $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$ then $(1011)^{th}$ term from end is equal to (1024) times $(1011)^{th}$ term from starting then $|x|$ is

(1) $\frac{16}{7}$ (2) $\frac{16}{5}$

(3) $\frac{5}{16}$ (4) $\frac{8}{5}$

Answer (3)

Sol. 1011^{th} term from end = 1011 term from beginning

$\therefore \quad r = 1010 \qquad \left(\frac{5}{2x} - \frac{4x}{5}\right)^{2022}$

$T_{1011} = {}^{2022}C_{1010}\left(\frac{5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$

1011 term from starting $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$

$T_{1011} = {}^{2022}C_{1010}\left(\frac{4x}{5}\right)^{1012}\left(\frac{5}{2x}\right)^{1010}$

Now,

$${}^{2022}C_{1010}\left(\frac{5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010} = 1024$$

$${}^{2022}C_{1010}\left(\frac{4x}{5}\right)^{1012}\left(\frac{5}{2x}\right)^{1010}$$

$$\left(\frac{5\times5}{2x\times4x}\right)^2 = 2^{10}$$

$$\frac{25}{8x^2} = 2^5$$

$$x^2 = \frac{25}{2^8}$$

$$|x| = \frac{5}{2^4}$$

8. A circle with center at (2, 0) and maximum radius "r" is inscribed in the ellipse $\frac{x^2}{36} + \frac{y^2}{9} = 1$. The value of $12r^2$ is

(1) 108 (2) 172

(3) 83 (4) 92

Answer (4)

Sol. Equation of normal at $P(6\cos\theta, 3\sin\theta)$ is

$(6\sec\theta)x - (3\operatorname{cosec}\theta)y = 27$

It passes through (2, 0)

$12\sec\theta = 27$

$\cos\theta = \frac{4}{9}, \quad \sin\theta = \frac{\sqrt{65}}{9}$

$P\left(\frac{8}{3}, \frac{\sqrt{65}}{3}\right)$

$$r = \sqrt{\left(\frac{8}{3} - 2\right)^2 + \left(\frac{\sqrt{65}}{3}\right)^2} = \frac{\sqrt{69}}{3}$$

$$12r^2 = 12 \times \frac{69}{9} = 92$$

9. $f : R \to R$ be a continuous non-constant function and $\int_0^{\pi/2} f(\sin 2x).\sin x\, dx + \alpha \int_0^{\pi/4} f(\cos 2x).\cos x\, dx = 0$ then α is equal to

(1) $\sqrt{2}$ (2) $\sqrt{3}$

(3) $-\sqrt{2}$ (4) $-\sqrt{3}$

Answer (3)

Sol. $\int_0^{\pi/2} f(\sin 2x)\sin x\, dx + \alpha \int_0^{\pi/4} f(\cos 2x).\cos x\, dx = 0$

$$\int_0^{\pi/4} f(\sin 2x)\sin x\, dx + \int_{\pi/4}^{\pi/2} f(\sin 2x)\sin x\, dx + \alpha \int_0^{\pi/4} f(\cos 2x)\cos x\, dx = 0$$

Here $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$

Let $x = t + \frac{\pi}{4}$

$$\Rightarrow \int_0^{\pi/4} f(\cos 2x)\sin\left(\frac{\pi}{4} - x\right) dx + \int_0^{\pi/4} f(\cos 2t)\sin\left(t + \frac{\pi}{4}\right) dx + \alpha \int_0^{\pi/4} f(\cos 2x)\cos x\, dx = 0$$

$$\Rightarrow \int_0^{\pi/4} f(\cos 2x)\left\{\sin\left(\frac{\pi}{4} - x\right) + \sin\left(x + \frac{\pi}{4}\right) + \alpha\cos x\right\} dx = 0$$

$$\Rightarrow \int_0^{\pi/4} f(\cos 2x)\left\{\left(\left(\sqrt{2} + \alpha\right)\cos x\right)\right\} dx = 0$$

$\because \left(\sqrt{2} + \alpha\right)\int_0^{\pi/4} f(\cos 2x).\cos x\, dx = 0$

$\because$ $f(\cos 2x)$ and $\cos x$ is not zero in $\left(0, \frac{\pi}{4}\right)$.

$\therefore \sqrt{2} + \alpha = 0$

$\Rightarrow \alpha = -\sqrt{2}.$

10. If the ratio of three consecutive terms is 1:3:5 in the expansion of $(1 + x)^{n+2}$. Then sum of consecutive terms is

(1) 41 (2) 64

(3) 63 (4) 43

Answer (3)

Sol. ${}^{n+2}C_{r-1} : {}^{n+2}C_r : {}^{n+2}C_{r+1} :: 1 : 3 : 5$

$$\therefore \frac{(n+2)!}{(r-1)!\,(n-r+3)!} \times \frac{r!(n+2-r)!}{(n+2)!} = \frac{1}{3}$$

$$\Rightarrow \frac{r}{(n-r+3)} = \frac{1}{3} \Rightarrow n - r + 3 = 3r$$

$n = 4r - 3$ ...(i)

and $\frac{(n+1)!}{r!(n+2-r)!} \times \frac{(r+1)!\,(n-r+1)!}{(n+2)!} = \frac{3}{5}$

$$\Rightarrow \frac{(r+1)}{n+2-r} = \frac{3}{5}$$

$\Rightarrow 5r + 5 = 3n + 6 - 3r$

$\Rightarrow 8r - 1 = 3n$ ...(ii)

By (i) and (ii)

$$4r - 3 = \frac{8r-1}{3}$$

$\Rightarrow 4r = 8 \Rightarrow r = 2$

$n = 5$

$\therefore$ Sum $= {}^7C_1 + {}^7C_2 + {}^7C_3 = 7 + 21 + 35 = 63$

11. The converse of the statement $(\sim p \wedge q) \Rightarrow r$ is

(1) $r \Rightarrow (\sim p \wedge q)$ (2) $r \Rightarrow (p \vee \sim q)$

(3) $\sim r \Rightarrow (p \vee \sim q)$ (4) $\sim r \Rightarrow (\sim p \wedge q)$

Answer (1)

Sol. Converse of $(\sim p \wedge q) \Rightarrow r$ is

$r \Rightarrow (\sim p \wedge q)$

12. If $\vec{a}, \vec{b}, \vec{c}, \vec{b}$ are coplanar reactor then value of $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ is

(1) $\left[\vec{b}\ \vec{d}\ \vec{c}\right] + \left[\vec{a}\ \vec{d}\ \vec{b}\right] + \left[\vec{a}\ \vec{d}\ \vec{c}\right]$

(2) $\left[\vec{b}\ \vec{d}\ \vec{c}\right] + \left[\vec{a}\ \vec{b}\ \vec{d}\right] + \left[\vec{a}\ \vec{d}\ \vec{c}\right]$

(3) $\left[\vec{b}\ \vec{c}\ \vec{d}\right] + \left[\vec{a}\ \vec{b}\ \vec{d}\right] + \left[\vec{a}\ \vec{d}\ \vec{c}\right]$

(4) $\left[\vec{b}\ \vec{c}\ \vec{d}\right] + \left[\vec{a}\ \vec{d}\ \vec{b}\right] + \left[\vec{a}\ \vec{d}\ \vec{c}\right]$

Answer (3)

Sol. $\left[\vec{b}-\vec{a}\ \ \vec{c}-\vec{a}\ \ \vec{d}-\vec{a}\right] = 0$

$$\left(\vec{b}-\vec{a}\right)\cdot\left((\vec{c}-\vec{a})\times(\vec{d}-\vec{a})\right) = 0$$

$$\left(\vec{b}-\vec{a}\right)\cdot\left(\vec{c}\times\vec{d} - \vec{c}\times\vec{a} - \vec{a}\times\vec{d}\right) = 0$$

$$\left[\vec{b}\ \vec{c}\ \vec{d}\right] - \left[\vec{b}\ \vec{c}\ \vec{a}\right] - \left[\vec{b}\ \vec{a}\ \vec{d}\right] - \left[\vec{a}\ \vec{c}\ \vec{d}\right] = 0$$

$$\therefore \left[\vec{a}\ \vec{b}\ \vec{c}\right] = \left[\vec{b}\ \vec{c}\ \vec{d}\right] - \left[\vec{b}\ \vec{a}\ \vec{d}\right] - \left[\vec{a}\cdot\vec{c}\ \vec{d}\right]$$

13. $f(x)=\begin{cases} e^{\min(x^2,\alpha x^3),} & x\in(0,1) \\ e^{[x-\ln x],} & x\in[1,2) \end{cases}$ then find $\int_0^2 xf(x)dx$

(1) $2e-\frac{1}{2}$ (2) $2e+\frac{1}{2}$

(3) $4e-\frac{1}{2}$ (4) $4e+\frac{1}{2}$

Answer (1)

Sol. $f(x)=\begin{cases} e^{x^2}, & x\in(0,1) \\ e, & x\in[1,2) \end{cases}$

$$\int_0^2 xf(x)dx=\int_0^1 x\cdot e^{x^2}dx+\int_1^2 x\times e\,dx$$

$x^2 = t$

$2xdx = dt$

$$=\frac{1}{2}\int_0^1 e^t dt+e\int_1^2 xdx$$

$$=\frac{1}{2}\left[e^t\right]_0^1+e\left[\frac{x^2}{2}\right]_1^2$$

$$=\frac{1}{2}\times(e-1)+\frac{3}{2}e$$

$$=2e\frac{-1}{2}$$

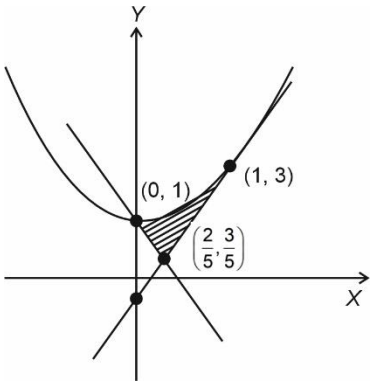
14. The area between the curve $y = 2x^2 + 1$ and tangent to it at (1, 3) and $x + y = 1$ is

(1) $\frac{1}{15}$ (2) $\frac{1}{60}$

(3) $\frac{4}{15}$ (4) $\frac{8}{3}$

Answer (3)

Sol.



Tangent at (1, 3) $\frac{y+3}{2}=2x+1$

$y = 4x - 1$

$\therefore$ Area

$$\int_0^{2/5}\left(2x^2+1-(1-x)\right)dx+\int_{2/5}^1\left(2x^2+1\right)-(4x-1)dx$$

$$=\int_0^{2/5}\left(2x^2+x\right)dx+\int_{2/5}^1\left(2x^2-4x+2\right)dx$$

$$=\left(\frac{2x^3}{3}+\frac{x^2}{2}\right)_0^{2/5}+\left[\frac{2x^3}{3}-\frac{4x^2}{2}+2x\right]_{2/5}^1$$

$$=\frac{92}{750}+\frac{144}{1000}=\frac{368+432}{3000}=\frac{800}{3000}=\frac{4}{15}$$

15. Angle between line $x=\frac{y-1}{2}=\frac{z-3}{r}$ and plane $x + 2y + 3z + 4 = 0$ is $\cos^{-1}\sqrt{\frac{5}{14}}$ then point of intersection of line and plane is

(1) (–15, –23, –11) (2) $\left(\frac{15}{7},\frac{-23}{7},\frac{11}{7}\right)$

(3) (15, 23, 11) (4) $\left(\frac{-15}{7},\frac{-23}{7},\frac{11}{7}\right)$

Answer (4)

Sol. $\sin\theta=\frac{1+4+3r}{\sqrt{14}\sqrt{5+r^2}}$

$$\cos^{-1}\frac{\sqrt{5}}{\sqrt{14}}=\sin^{-1}\frac{3}{\sqrt{14}}=\sin^{-1}\left(\frac{5+3r}{\sqrt{14}\sqrt{5+r^2}}\right)$$

$$\frac{3}{\sqrt{14}}=\frac{5+3r}{\left(\sqrt{5+r^2}\right)\sqrt{14}}$$

$3\sqrt{5+r^2}=5+3r$

$9(5 + r^2) = 25 + 9r^2 + 30r$

$\Rightarrow$ $45 = 25 + 30r$

$\Rightarrow$ $30r = 30$

$r=\frac{2}{3}$

Let the point on line is $P(3k, 6k + 1, 2k + 3)$

$3k + 12k + 2 + 6k + 9 + 4 = 0$

$\Rightarrow$ $21k = -15$

$\Rightarrow$ $k=-\frac{5}{7}$

$\therefore$ $P\left(\frac{-15}{7},\frac{-23}{7},\frac{11}{7}\right)$

16.
17.
18.
19.
20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. If $e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1 = 0$, then number of solutions of above equation is

Answer (2)

Sol. $e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1 = 0$,

$$\Rightarrow \left(e^{4x} + \frac{1}{e^{4x}}\right) - \left(e^{2x} + \frac{1}{e^{2x}}\right) = 0$$

$$\Rightarrow \left(e^{2x} + \frac{1}{e^{2x}}\right)^2 - \left(e^{2x} + \frac{1}{e^{2x}}\right) = 5$$

$\Rightarrow t^2 - t - 5 = 0$

$$t = \frac{1 \pm \sqrt{1+20}}{2}$$

$$= \frac{1 \pm \sqrt{21}}{2}$$

$\frac{1-\sqrt{21}}{2}$ is rejected

$$\therefore \quad t = \frac{1+\sqrt{21}}{2}$$

$$\Rightarrow \quad e^{2x} + \frac{1}{e^{2x}} = \frac{1+\sqrt{21}}{2} \Rightarrow 2 \text{ values of } e^{2x} \text{ possible}$$

$\therefore$ 2 real solution

22. If $f(1) + f(2) = f(4) - 1$ and a function from A to B is defined where $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 2, 3, 4, 5, 6\}$. Find the numbers of function with such relation.

Answer (360)

Sol. $f(4) = f(1) + f(2) + 1$

$\Rightarrow f(1) + f(2) + 1 \le 6$

$f(1) + f(2) \le 5$

Possible cases

1	{1,2,3,4}	$\rightarrow$	4
2	{1,2,3}	$\rightarrow$	3
3	{1,2}	$\rightarrow$	2
4	{1}	$\rightarrow$	$\frac{1}{10}$

$f(5)$, $f(3)$ can be filled in 6 ways

Total functions = $10 \times 6 \times 6 = 360$

23. For a biased coin, the probability of getting head is $\frac{1}{4}$. It is tossed n times till we get head. Given a quadratic equation $64x^2 + 2nx + 1 = 0$. If the probability that the quadratic equation has no real roots is $\frac{P}{Q}$ (where P and Q are coprime), then the value of $Q - P$ is

Answer (2187)

Sol. $(2n)^2 - 4 \times 64 < 0 \Rightarrow n < 8 \Rightarrow n \le 7$

Required probability

$$= \frac{1}{4} + \frac{3}{4} \cdot \frac{1}{4} + \left(\frac{3}{4}\right)^2 \cdot \frac{1}{4} + \ldots + \left(\frac{3}{4}\right)^6 \cdot \frac{1}{4}$$

$$= \frac{1}{4} \frac{\left(1 - \left(\frac{3}{4}\right)^7\right)}{1 - \frac{3}{4}} = \frac{4^7 - 3^7}{4^7} = \frac{P}{Q}$$

$Q - P = 3^7 = 2187$

24.
25.
26.
27.
28.
29.
30.

❑ ❑ ❑