JEE-Mains-11-04-2023 [Memory Based] [Evening Shift]

Physics

Question: If $\vec{A} = 2\hat{i} + 36\hat{j} + 2\hat{k}$ is subtracted from $\vec{B}$ then it gives $2j$ then mag of $\left(\vec{B}\right)$?

Options:

(a) $\sqrt{21}$

(b) $\sqrt{33}$

(c) $\sqrt{47}$

(d) $\sqrt{51}$

Answer: (b)

Solution:

$\vec{B} - [2\hat{i} + 3\hat{j} + 2\hat{k}] = 2\hat{j}$

$\vec{B} = 2\hat{j} + 5\hat{j} + 2\hat{k}$

$\sqrt{2^2 + 5^2 + 2^2} = \sqrt{33}$

Question: In projectile motion **θ** = 30° Time of flight 4 sec find velocity at time 2 s

Options:

(a) $20\sqrt{3}$

(b) $2\sqrt{3}$

(c) $30\sqrt{3}$

(d) $20\sqrt{5}$

Answer: (a)

Solution:

$\frac{2u\sin\theta}{g} = 4$

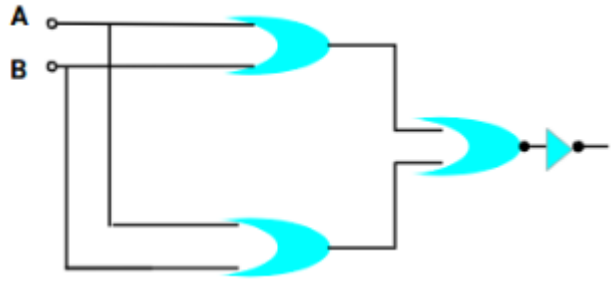
so $\frac{2u}{10} \times \frac{1}{2} = 4$

$u = 40$

Now at t = 2 particle is at p most point so

$V = u\cos\theta = \frac{40\sqrt{3}}{2} = 20\sqrt{3}$

Question: The fact logic gate is:-



Options:
(a) AND
(b) NOR
(c) OR
(d) NAND
Answer: (a)
Solution:
$Y = AB(A + B)$
$Y = A \cdot A \cdot B + AB \cdot B$
$Y = AB + AB = AB$

Question: 8 identical drops are falling in viscous medium with constant velocity of 10 m/s all of them join to form bigger drop, find the velocity of bigger drop
Options:
(a) 10 m/s
(b) 20 m/s
(c) 30 m/s
(d) 40 m/s
Answer: (d)
Solution:
$V \propto r^2$
So
$8\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$
$R = 2r$
$\frac{V_1}{V_2} = \left(\frac{r}{R}\right)^2 = \left(\frac{r}{2r}\right)^2 = \frac{1}{4}$
So $V_2 = 4V_1 = 4 \times 10 = 40$

Question: Mass of 500 gm whose velocity is changing with displacement as v = 10√x. Find force experienced by body
Options:
(a) 15 N
(b) 25 N
(c) 35 N
(d) 45 N
Answer: (b)
Solution:

$$F = ma = mv\frac{dv}{dx}$$
$$= \frac{500}{1000} \times \left(10\sqrt{x}\right) \times \frac{d}{dx}\left(10\sqrt{x}\right)$$
$$= \frac{1}{2} \times 10\sqrt{x} \times 10 \times \frac{1}{2\sqrt{x}}$$
$$= 5 \times 5 = 25\,\text{N}$$

Question: If force, velocity, and time are treated as fundamental quantities then write the dimensional formula of density in terms of F, V, T

Options:

(a) $F^4\ V^4\ T^{-2}$

(b) $F^1\ V^{-4}\ T^{-2}$

(c) $F^{-1}\ V^{-4}\ T^{-2}$

(d) $F^1\ V^4\ T^2$

Answer: (b)

Question: In EM wave that wave moves in +x axis, E = $6.6\,\hat{j}$. Find B

Options:

(a) $-2.2 \times 10^{-8}\ \hat{k}$

(b) $2.2 \times 10^{-8}\ \hat{i}$

(c) $-2.2 \times 10^{-8}\ \hat{i}$

(d) $2.2 \times 10^{-8}\ \hat{k}$

Answer: (d)

Solution:

$$\frac{E}{B} = C$$
$$B = \frac{E}{C}$$

Question: Gravitational potential on the surface of solid sphere V find gravitational potential at the centre of solid sphere

Options:

(a) V/2

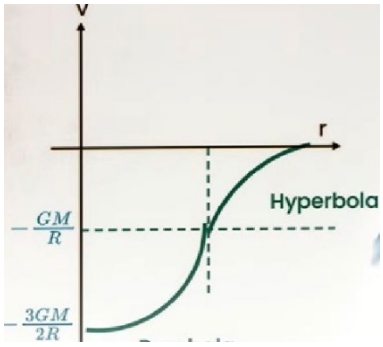
(b) V

(c) 3V/2

(d) 2V

Answer: (c)

Solution:



Question: Ratio of de broglie wavelength of proton and electron if kinetic energy is same ($m_p = 1849m_e$)

Options:

(a) 1/43

(b) 1/107

(c) 1/25

(d) 1/100

Answer: (a)

Solution:

$$\lambda = \frac{h}{\sqrt{2mKE}}$$

so $\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}} = \sqrt{\frac{184gm_e}{m_e}} = \sqrt{1849} = 43$

So 1 : 43

Question: If energy of Hydrogen atom in ground state is -13.6 eV find energy of He+ in first excited state

Options:

(a) –3.4 eV

(b) –9.6 eV

(c) –13.6 eV

(d) None of these

Answer: (c)

Solution:

$E = \frac{-13.6}{n^2}\left(Z^2\right)$ for H^+

E for (n = 2)

$E = \frac{-13.6}{2^2} \times \left(2^2\right) = -13.6$

Question: Ray of light strikes a plane mirror with angle of incidence 30° find the deviation produced

Options:

(a) 60°

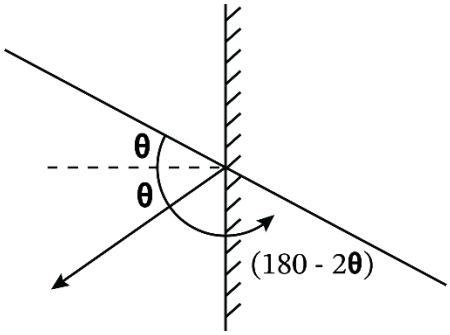
(b) 90°

(c) 120°

(d) 150°

Answer: (c)

Solution:



$=(180-2\times30)=120°$

Question: In which process internal energy is constant
Options:
(a) isothermal
(b) isochoric
(c) isobaric
(d) Adiabatic
Answer: (a)

Question: A nucleus breaks in two nuclei of radius ratio 1: $2^{1/3}$ find the ratio of their velocities
Options:
(a) 2 : 1
(b) 2 : 5
(c) 1 : 2
(d) 3 : 2
Answer: (a)
Solution:

$$V_{ms}=\sqrt{\frac{3kT}{m}}=\sqrt{\frac{3\times1.4\times10^{-23}\times300}{4.6\times10^{-26}}}=523\,\text{m/s}$$

$$m_1v_1=m_2v_2$$

$$\rho\cdot\frac{4}{3}\pi R_1^3v_1=\rho\frac{4}{3}\pi R_2^3\cdot v_2$$

$$\frac{v_1}{v_2}=\left(\frac{R_2}{R_1}\right)^3=\left(\frac{2^{1/3}}{1}\right)^3=\frac{2}{1}$$

Question: RMS velocity of nitrogen molecule at 27°C, k = 1.4×10^{-23} and mass of N_2 = 4.6×10^{-26} Kg (in m/s)
Answer: 523.00
Solution:

$$V_{ms}=\sqrt{\frac{3kT}{m}}=\sqrt{\frac{3\times1.4\times10^{-23}\times300}{4.6\times10^{-26}}}=523\,\text{m/s}$$

Question: S1 : when bar magnet falls in conducting ring is slows down whereas it does not slows down when it falls through a non-conducting ring.
S2 : Eddie currents are induced in conducting ring.

Options:
(a) S1 - True, S2 - False
(b) S1 - False, S2 - False
(c) S1 - True, S2 - True
(d) S1 - False, S2 - True
Answer: (c)

Question: A body is rotating with kinetic energy E. If angular velocity of body is increased to three times of initial angular velocity then kinetic energy become nE. Find n.
Answer: 9.00
Solution:

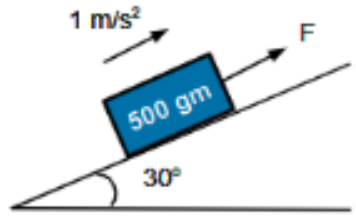
$$KE = \frac{1}{2} I\omega^2$$

$$KE \propto \omega^2$$

$$\text{So } \frac{KE_1}{KE_2} = \left(\frac{\omega_1}{\omega_2}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

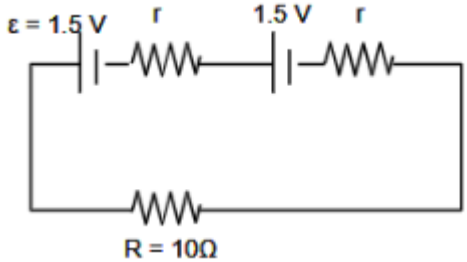
So $KE_2 = 9KE_1$

Question: Find power delivered by F at t = 10 s. If body starts from rest.



Options:
(a) 5 watt
(b) 7.5 watt
(c) 10 watt
(d) 12.5 watt
Answer: (b)
Solution:

Question: If P.D across R is 1.5 volts find internal resistances of cells.



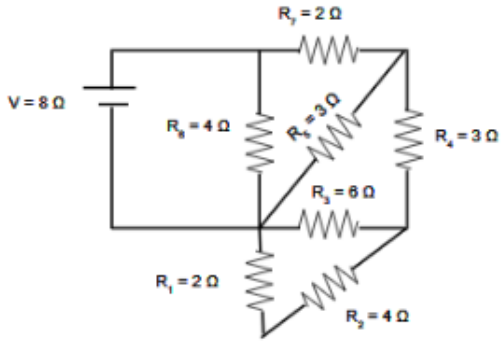
Answer: 5.00
Solution:

Question: A capacitor of capacity C is charged to potential V find the flux through the surface enclosing positive plate of capacitor
Options:
(a) $CV/8\varepsilon_0$

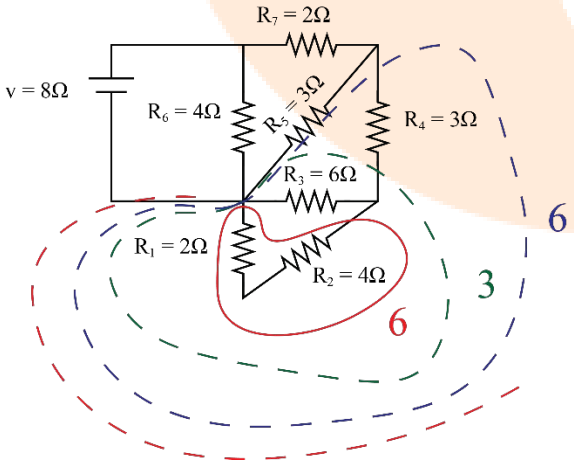
(b) $CV/4\varepsilon_0$
(c) $CV/2\varepsilon_0$
(d) CV/ε_0
Answer: (d)
Solution:

Question: In satellite communication, frequency for uplink is
Answer: 3.7 GHz–4.2 GHz
Solution:
Ground wave propagation - 500 KHz - 1500 KHz
Sky wave propagation - 5 MHz - 100 MHz
Space wave propagation - 100 MHz - 200 MHz
Satellite communication - 3.7 GHz - 4.2 GHz

Question: Current in R_2 resistance is



Options:
(a) ½ A
(b) ¾ A
(c) 3/2 A
(d) ¼ A
Answer: (a)
Solution:



$$\frac{3}{3+6}\times 2=\frac{6}{9}$$

$$=\frac{2}{3}A$$

$$I=\frac{V}{\mathrm{R_{eq}}}=\frac{8}{2}=4\mathrm{A}$$

JEE-Mains-11-04-2023 [Memory Based] [Evening Shift]

Chemistry

Question: Which alkali metal has the lowest melting Point?
Options:
(a) Li
(b) Na
(c) Cs
(d) K
Answer: (c)
Solution:

	Melting point
Li	454
Na	371
K	336
Rb	312
Cs	302 (Lowest melting point)

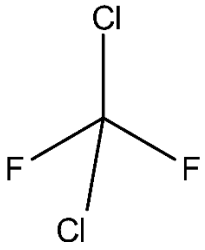
Question: Number of correct statements about modern adsorption theory
Options:
(a) Diffusion of reactants to the surface of the catalyst.
(b) Adsorption of reactant molecules on the surface of the catalyst.
(c) Desorption of reaction products from the catalyst surface, and thereby, making the surface available again for more reaction to occur.
(d) All of these
Answer: (d)
Solution: All options are correct.

Question: 2g of X is dissolved in 1 mol of water. Find mass percentage of X in the solution.
Options:
(a) 10%
(b) 20%
(c) 30%
(d) 40%
Answer: (a)

Solution: The mass % of solute in solution is = $\frac{\text{Mass of solute}}{\text{Mass of soluter}} \times 100$

$\text{mass \%} = \frac{2}{20} \times 100 = 10\%$

Question: Chemical Formula of Freons
Options:
(a) C_2F_4
(b) CCl_2F_2
(c) $C_2H_2F_2$
(d) $C_2H_2Cl_2$
Answer: (b)
Solution:



Question: Statement-1: Low density polymer is formed by polymerisation of ethene in the presence of triethylaluminium and titanium tetrachloride (Ziegler-Natta catalyst) at a temperature of 333 K to 343 K and under a pressure of 6-7 atmospheres.

Statement-2: Nylon 6 is obtained by heating caprolactum with water at 500K.

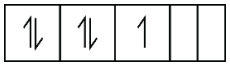
Options:
(a) Both statements I and II are correct
(b) Both statements I and II incorrect
(c) Statement I is correct and II is incorrect
(d) Statement I is incorrect and II is correct
Answer: (d)
Solution: Statement-1 is incorrect, statement-2 is correct.

Question: Magnetic moment $[Fe(CN)_6]^{3-}$ and $[Fe(H_2o)_6]^{3+}$ respectively are :
Options:
(a) 2.92 and 3.73
(b) 1.12 and 4.71
(c) 1.73 and 5.92
(d) 5.92 and 1.73
Answer: (c)
Solution:

$[Fe(CN)_6]^{3-}$

$Fe^{+3} = 3d^5$

S.F.L is present so passing will take place



d^2sp^3

1 unpaired electron

= $\sqrt{1(1+2)}$ B.M

= √3 B.M = 1.73 B.M

$[Fe(H_2o)_6]^{3+}$

$Fe^{+3} = 3d^5$

W.F.L is present so passing will not take place

1	1	1	1	1

sp^3d^2

5 unpaired electrons

= $\sqrt{5(5+2)}$ B.M

= 5.92 B.M

Question: $P_4 + 8SOCl_2 \rightarrow 4A + 2B + x\ SO_2$.

Sum of A, B, x are

Options:
(a) 4
(b) 6
(c) 8
(d) 10
Answer: (d)
Solution: $P_4 + 8SOCl_2 \rightarrow 4PCl_3 + 4SO_2 + 2S_2Cl_2$

4 + 4 + 2 = 10

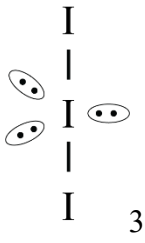
Question: Number of intensive properties are :
E_{cell}, Molarity, Gibbs free energy, Molar mass, Mole, Molar heat capacity?
Options:
(a) 2
(b) 4
(c) 3
(d) 5
Answer: (b)
Solution:

E_{cell}, Molarity, Molar heat capacity

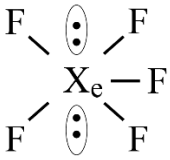
Molar mass, Intensive properties

Question: Which species has maximum number of lone pairs on central atoms?
Options:
(a) I_3^-

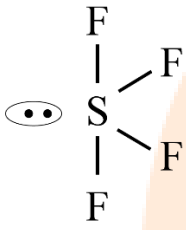
(b) XeF_4
(c) SF_4
(d) PCl_5
Answer: (a)
Solution:



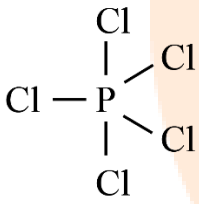
3 lone pair



2 lone pair



1 lone pair



0 lone pair

Question: How many of them will not reacts with benedict's solution?
Sucrose, Glucose, maltose, lactose, amylose, deoxyribose, ribose
Options:
(a) 2
(b) 3
(c) 4
(d) 1
Answer: (a)
Solution: Sucrose, amylose

Not react with Benedict's solution

Question: In $Mg(NO_3)_2.xH_2O$ and $Ba(NO_3)_2$ $.yH_2O$, find x + y.
Options:
(a) 6
(b) 7

(c) 8
(d) 12
Answer: (a)
Solution:

$Mg(NO_3)_2 . 6H_2O$

$Ba(NO_3)_2 . O$

---

6 + 0 = 6

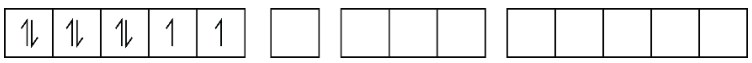
Magnesium nitrate crystallises with six molecules of water, whereas barium nitrate crystallises as the anhydrous salt. This again shows a decreasing tendency to form hydrates.

Question: $H_2 + I_2 \rightleftarrows 2\ HI$
Initial concentration of H_2 and I_2 each 4.5 mole. Find the equilibrium constant when 3 mole of HI is formed at equilibrium.
Options:
(a) 1
(b) 2
(c) 3
(d) 4
Answer: (a)
Solution:

H_2	+	I_2	$\rightleftarrows$	2 HI
1 mol		1 mol		2 mol
(1.5 mol)		(1.5 mol)		(3 mol)

$$k_C = \frac{[HI]^2}{[H_2][I_2]} = \frac{[3]^2}{[3][3]} = 1$$

Question: Which of the following property will change when Ni in $[NiCl_2Br_2]^{2-}$ is changed by Pt.
I. Hybridisation II. Magnetic moment
Options:
(a) Hybridisation,
(b) Magnetic moment
(c) Both
(d) None
Answer: (c)
Solution: $[NiCl_2Br_2]^{2-} = Ni^{+2}$ WFL is attached so no pairing will take place



sp^3 paramagnetic n = 2

$\sqrt{2(4)}$ $\sqrt{8}$ B M

= 2.82 B.M

$[PtCl_2Br2]^{2-}$

In case of pt all ligand work as S.F.L

Then pairing will take place

⇅	⇅	⇅	⇅	

Hybridization = dsp^2

Magnitude moment = 0

Question: 2.4 g of Mg reacts with excess of HCl. Then find the Volume of H_2 formed at STP.

Options:

(a) 1.14 L

(b) 2.24 L

(c) 5.14 L

(d) 6.14 L

Answer: (b)

Solution:

$Mg + 2HCl \rightarrow MgCl_2 + H_2$

1 mol 2 mol 1 mol 1 mol

0.1 mol Excess 0.1 mol 0.1 mol

$$0.1 = \frac{V}{22.4}$$

V = 2.24 Let

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Mathematics

Question: If the letters of the word MATHS are arranged in all possible orders and these words are written in a dictionary, then rank of the word THAMS is:

Answer: 103.00

Solution:

A H M S T

$A \rightarrow 4!$

$H \rightarrow 4!$

$N \rightarrow 4!$

$S \rightarrow 4!$

$T\ A \rightarrow 3!$

$T\ H\ A\ M\ S \rightarrow 1$

$$\Rightarrow 4!\times 4+3!\times 1=6(16+1)+1$$
$$=6\times 17+1$$
$$=102+1$$
$$=103$$

Question: $\frac{dy}{dx}+\frac{5}{x\left(1+x^5\right)}y=\frac{\left(1+x^5\right)^2}{x^7}$. If $y(1)=2,$ then the value of $y(2)$ is:

Answer: $\frac{693}{128}$

Solution:

$$\frac{dy}{dx}+\frac{5}{x\left(1+x^5\right)}y=\frac{\left(1+x^5\right)^2}{x^9}$$

$\text{I.F.} = e^{\int \frac{5}{x\left(1+x^5\right)}dx}$

$$\Rightarrow \int \frac{5}{x\left(1+x^5\right)}dx=\int \frac{5x^{-6}}{\left(x^{-5}+1\right)}dx$$

$$\Rightarrow -\ln\left(x^{-5}+1\right)=\ln\left(\frac{1}{x^5+1}\right)$$

$$\text{I.F.}=\frac{1}{x^{-5}+1}=\frac{x^5}{1+x^5}$$

$$y\left(\frac{x^5}{1+x^5}\right)=\int\frac{x^5}{1+x^5}\cdot\frac{\left(1+x^5\right)^2}{x^7}dx$$

$$=\int\frac{1+x^5}{x^2}dx$$

$$=\int x^{-2}dx+\int x^3dx$$

$$=\frac{-1}{x}+\frac{x^4}{4}+c$$

Put $x=1, y=2$

$$2\left(\frac{1}{2}\right)=-1+\frac{1}{4}+C$$

$$\Rightarrow C=1+1-\frac{1}{4}=\frac{7}{4}$$

$$\Rightarrow y\left(\frac{x^5}{1+x^5}\right)=\frac{-1}{x}+\frac{x^4}{4}+\frac{7}{4}$$

Put $x=2$

$$y\left(\frac{32}{33}\right)=\frac{-1}{2}+\frac{16}{4}+\frac{7}{4}$$

$$y\left(\frac{32}{33}\right)=\frac{-1}{2}+\frac{23}{4}$$

$$y\left(\frac{32}{33}\right)=\frac{21}{4}$$

$$y=\frac{21}{4}\times\frac{33}{32}$$

$$=\frac{693}{128}$$

Question: $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix}=\frac{9}{3}(103x+81)$, then λ and $\frac{\lambda}{3}$ are roots of:

Options:

(a) $4x^2+24x-27=0$

(b) $4x^2-24x+27=0$

(c)

(d)

Answer: (b)

Solution:

Given $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{3}(103x+81)$

Put $x=0$ on both sides

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & \lambda^2 \end{vmatrix} = \frac{9}{8}(81)$$

$$\Rightarrow \lambda^3 = \left(\frac{9}{2}\right)^3$$

$$\Rightarrow \lambda = \frac{9}{2}$$

$\Rightarrow$ Roots: $\lambda = \frac{9}{2}$ & $\frac{\lambda}{3} = \frac{3}{2}$

$$\text{Sum} = \frac{9}{2}+\frac{3}{2} = \frac{12}{2} = 6$$

$$\text{Product} = \frac{9}{2}\times\frac{3}{2} = \frac{27}{4}$$

$$\Rightarrow x^2-6x+\frac{27}{4}=0$$

$$4x^2-24x+27=0$$

Question: $x_1: 1, 2, 4, 5, x, y$. $\bar{x}=5$, $\sigma^2=10$. Find mean deviation about mean.

Answer: $\frac{8}{3}$

Solution:

Mean = 5

Variance = 10

$$\frac{1+2+4+5+x+y}{6}=5$$

$$x+y=30-12$$

$$x+y=18$$

$$\frac{1^2+2^2+4^2+5^2+x^2+y^2}{6}-(5)^2=10$$

$$x^2+y^2=164$$

$$x=10,\ y=8$$

Mean deviation about Mean

$$MD(\bar{X})=\frac{\sum_{i=1}^{6}(x_i-\bar{X})}{6}$$

$$=\frac{4+3+1+0+5+3}{6}$$

$$=\frac{8}{3}$$

Question: If $a+b+c+d=11$ and maximum value of $ab^2c^3d^5=3750\beta$, then $\beta=?$

Answer: 90.00

Solution:

$$a+b+c+d=11$$

$$a+2\left(\frac{b}{2}\right)+3\left(\frac{c}{3}\right)+5\left(\frac{d}{5}\right)=11$$

$$\Rightarrow \frac{a+2\left(\frac{b}{2}\right)+3\left(\frac{c}{3}\right)+5\left(\frac{d}{5}\right)}{1+2+3+5}\geq\left[a\left(\frac{b}{2}\right)^2\left(\frac{c}{3}\right)^3\left(\frac{d}{5}\right)^5\right]^{\frac{1}{11}}$$

$$\frac{11}{11}\geq\left(\frac{ab^2c^3d^5}{4\times27\times3125}\right)$$

$$\Rightarrow ab^2c^3d^5\leq4\times27\times3125$$

$$\Rightarrow 4\times27\times3125=3750\beta$$

$$\beta=\frac{4\times27\times3125}{3750}$$

$$=2\times9\times5$$

$$=90$$

Question: If ratio of 3 consecutive coefficients in expansion of $(1+x)^{n+2}$ is $1:3:5$, then find sum of binomial coefficients.

Answer: 63.00

Solution:

$${}^{n+1}C_r : {}^{n+1}C_{r+1} : {}^{n+1}C_{r+2}$$

$1:3:5$

$$\Rightarrow \frac{{}^{n+1}C_{r+1}}{{}^{n+1}C_r} = \frac{3}{1} \qquad \Rightarrow \frac{{}^{n+1}C_{r+2}}{{}^{n+1}C_{r+1}} = \frac{5}{3}$$

$$\frac{(n+1)-(r+1)+1}{(r+1)} = \frac{3}{1} \qquad \frac{(n+1)-(r+2)+1}{(r+2)} = \frac{5}{3}$$

$$n-r+1 = 3(r+1) \qquad 3(n-r) = 5(r+2)$$

$$n-4r = 2 \qquad 3n-8r = 10$$

$$3n-8r = 10$$

$$\underline{2n-8r = 4}$$

$$n = 6$$

$$\Rightarrow r = 1$$

$$\Rightarrow {}^7C_1 + {}^7C_2 + {}^7C_3 = 7 + \frac{7\times 6}{2} + \frac{7\times 6\times 5}{3\times 2}$$

$$= 7 + 21 + 35$$

$$= 63$$

Question: In $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$, if 1011^{th} term from end is equal to $1024\times$ (1011^{th} term from beginning), then find $|x|$.

Answer: $\frac{5}{16}$

Solution:

$${}^{2022}C_{1010}\left(\frac{5}{x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010} = 1011\,{}^{2022}C_{1010}\left(\frac{5}{x}\right)^{1010}$$

$$\left(\frac{5}{2x}\right)^2 = \left(\frac{4x}{5}\right)^2 \times 1024$$

$$1024\times x^4=\frac{5^4}{2^2 4^2}$$

$$x^4=\frac{5^4}{2^{16}}$$

$$x=\pm\frac{5}{16}$$

Question: Find domain of $\frac{1}{\sqrt{[x]^2-3[x]-10}}$.

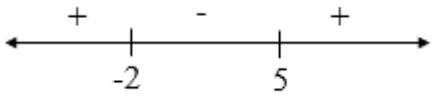
Answer: $(x<-2)\cup(x\geq 6)$

Solution:

$$[x]^2-3[x]-10>0$$

$$[x]^2-5[x]+2[x]-10>0$$

$$([x]-5)([x]+2)>0$$



$[x]<-2$ or $[x]>5$

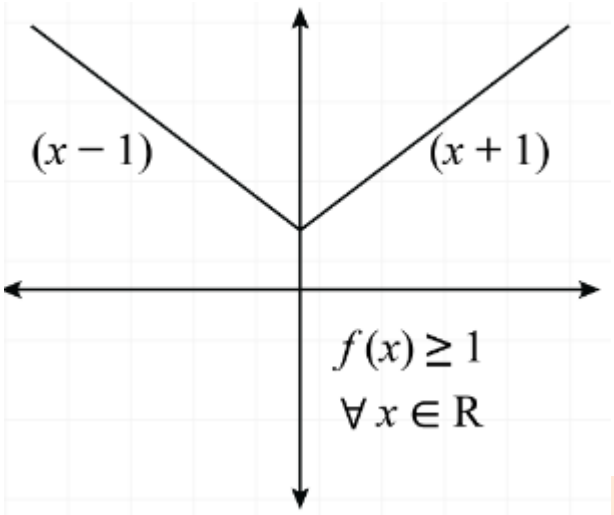
$x<-2$ or $x\geq 6$

$\Rightarrow x\in(-\infty,-2)\cup[6,\infty)$

Question: $f(x)=\begin{cases}x+1; & x>0\\ |x-1|; & x\leq 0\end{cases}$; $g(x)=\begin{cases}x+1; & x>0\\ 1; & x<0\end{cases}$. Find points of discontinuity of $g(f(x))$.

Answer: 0.00

Solution:



$g(f(x)) = f(x)+1;\ \ f(x) > 0$

$$= \begin{cases} x+1+1; & x > 0 \\ |x-1|+1; & x \le 0 \end{cases}$$

$$= \begin{cases} x+2; & x > 0 \\ |x-1|+1; & x \le 0 \end{cases}$$

$$= \begin{cases} x+2; & x > 0 \\ -x+2; & x \le 0 \end{cases}$$

LHL = RHL = $f(0)$

Question: R is maximum possible radius of a circle centered at $(2,0)$ which is enclosed in $x^2+4y^2=36$. Find $12R^2$.

Answer: 92.00

Solution:

$$\frac{x^2}{36}+\frac{y^2}{9}=1$$

$$(x-2)^2+y^2=r^2$$

$$x^2+4y^2=36$$

$$x^2+4\left(r^2-(x-2)^2\right)=36$$

$$x^2+4r^2-4\left(x^2-4x+4\right)=36$$

$$-3x^2+16x+4r^2-52=0$$

$$\Delta=0$$

$$16^2+12\left(4r^2-52\right)=0$$

$$16+3\left(r^2-13\right)=0$$

$$3r^2-23=0$$

$$3r^2=23$$

$$r^2=\frac{23}{3}$$

$$\therefore 12R^2=12\times\frac{23}{3}=92$$

Question: $I=\int_0^{\frac{\pi}{2}} f\left(\sin 2x\right)\sin x\,dx+\alpha\int_0^{\frac{\pi}{4}} f\left(\cos 2x\right)\cos x\,dx$. If $I=0$, then α

Answer: $-\sqrt{2}$

Solution:

$$I_1=\int_0^{\frac{\pi}{2}} f\left(\sin 2x\right)\cdot\sin x\cdot dx$$

Apply king's rule:

$$I_1=\int_0^{\frac{\pi}{2}} f\left(\sin 2x\right)\cdot\cos x\cdot dx$$

$$2I_1=\int_0^{\frac{\pi}{2}} f\left(\sin 2x\right)\cdot\left[\sin x+\cos x\right]dx$$

$$2I_1=\sqrt{2}\int_0^{\frac{\pi}{2}} f\left(\sin 2x\right)\cdot\cos\left(x-\frac{\pi}{4}\right)dx$$

$$\sqrt{2}I_1=2\int_0^{\frac{\pi}{4}} f\left(\sin 2x\right)\cdot\cos\left(x-\frac{\pi}{4}\right)dx$$

$$I_1=\sqrt{2}\int_0^{\frac{\pi}{4}} f\left(\sin 2x\right)\cdot\cos\left(x-\frac{\pi}{4}\right)dx$$

Apply king's rule again:

$$I_1=\sqrt{2}\int_0^{\frac{\pi}{4}} f\left(\cos 2x\right)\cdot\cos x\,dx$$

$$\Rightarrow\alpha=-\sqrt{2}$$

Question: Consider $y = e^{8x} - e^{6x} + 3e^{4x} - e^{2x} + 1$. At how many points it cuts x-axis.

Answer: 0.00

Solution:

$y = e^{8x} - e^{6x} + 3e^{4x} - e^{2x} + 1$

$e^x = t$

$y = t^8 - t^6 + 3t^4 - t^2 + 1;\ t > 0$

$y > 0$ for $t > 0$

No real root

Question: Converse of: $(p \vee \sim q) \to r$ is ____.

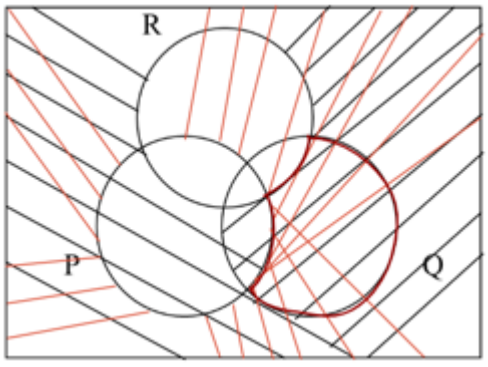
Answer: ()

Solution:

$(p \vee \sim q) \to r$

$r \to (\sim p \wedge q)$

i.e., $\sim r \vee (\sim p \wedge q)$



Question: $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k},\ \vec{b} = \hat{i} + \hat{j} - \hat{k},\ \vec{a} \cdot \vec{c} = 11,\ \vec{b} \cdot (\vec{a} \times \vec{c}) = 27,\ \vec{b} \cdot \vec{c} = -\sqrt{3}|\vec{b}|$. Find $|\vec{a} \times \vec{c}|^2$

.

Answer: 285.00

Solution:

$\vec{b} \times (\vec{a} \times \vec{c}) = (b \cdot c)\vec{a} - (b \cdot a)\vec{c}$

$|\vec{b} \cdot (\vec{a} \times \vec{c})|^2 + |\vec{b} \times (\vec{a} \times \vec{c})|^2 = |b|^2 |\vec{a} \times \vec{c}|^2$

$$27^2+\left|\left(\vec{b}\cdot\vec{c}\right)^2\vec{a}^2+\left(\vec{b}\cdot\vec{a}\right)^2\vec{c}^2-2\left(\vec{a}\cdot\vec{b}\right)\left(\vec{b}\cdot\vec{c}\right)\left(\vec{a}\cdot\vec{c}\right)\right|=3\left(\vec{a}\times\vec{c}\right)^2$$

$$27^2+\left|14\times3\times3-0\right|=3\left|\vec{a}\times\vec{c}\right|^2$$

$$729+126=3\left|\vec{a}\times\vec{c}\right|^2$$

$$\left|\vec{a}\times\vec{c}\right|^2=285$$

Question: If $10=1+\frac{4}{k}+\frac{8}{k^2}+\frac{13}{k^3}+\frac{19}{k^4}+...\infty$, then find k.

Answer: 2.00

Solution:

$$10=1+\frac{4}{k}+\frac{8}{k^2}+\frac{13}{k^3}+\frac{19}{k^4}+...\infty$$

$$\frac{10}{k}=\quad\frac{1}{k}+\frac{4}{k^2}+\frac{8}{k^3}+\frac{13}{k^4}+....\infty$$

$$10-\frac{10}{k}=1+\frac{3}{k}+\frac{4}{k^2}+\frac{5}{k^3}+\frac{6}{k^4}+....\infty$$

$$\frac{1}{k}\left(10-\frac{10}{k}\right)=\quad\frac{1}{k}+\frac{3}{k^2}+\frac{4}{k^2}+\frac{5}{k^4}+....\infty$$

$$10\left(1-\frac{2}{k}+\frac{1}{k^2}\right)=1+\frac{2}{k}+\frac{1}{k^2}+\frac{1}{k^3}+\frac{1}{k^4}+....\infty$$

$$10\left(1-\frac{1}{k}\right)^2=1+\frac{1}{k}+\frac{1}{k^2}+\frac{1}{k^3}+\frac{1}{k^4}+....\infty+\frac{1}{k}$$

$$10\left(1-\frac{1}{k}\right)^2=\frac{1}{1-\frac{1}{k}}+\frac{1}{k}$$

$$10\left(\frac{k-1}{k}\right)^2=\frac{k}{k-1}+\frac{1}{k}$$

$$10\left(\frac{(k-1)^2}{k^2}\right)=\frac{k^2+k-1}{k(k-1)}$$

$$10(k-1)^3=k^3+k^2-k$$

$$\Rightarrow 9k^3-31k^2+31k-10=0$$

$$\Rightarrow k=2$$

Question: If $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ are coplanar vectors then the value of $\left[\vec{a}\quad\vec{b}\quad\vec{c}\right]$ is:

Answer: ()
Solution:

$$\left[\vec{a}-\vec{d}\quad \vec{b}-\vec{d}\quad \vec{c}-\vec{d}\right]=0$$

$$\left[\vec{a}\quad \vec{b}-\vec{d}\quad \vec{c}-\vec{d}\right]-\left[\vec{d}\quad \vec{b}-\vec{d}\quad \vec{c}-\vec{d}\right]=0$$

$$\left[\vec{a}\quad \vec{b}\quad \vec{c}-\vec{d}\right]-\left[\vec{a}\quad \vec{d}\quad \vec{c}-\vec{d}\right]-\left[\vec{d}\quad \vec{b}\quad \vec{c}-\vec{d}\right]=0$$

$$\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]-\left[\vec{a}\quad \vec{b}\quad \vec{d}\right]-\left[\vec{a}\quad \vec{d}\quad \vec{c}\right]-\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]=0$$

$$\Rightarrow\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]=\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]+\left[\vec{a}\quad \vec{d}\quad \vec{c}\right]+\left[\vec{d}\quad \vec{b}\quad \vec{c}\right]$$

Question: f is defined from $A=\{1,2,3,4,5\}$ to $B=\{1,2,3,4,5,6\}$, such that $f(1)+f(2)=f(4)-1$. Find number of such functions.
Answer: 360.00
Solution:

$$f(1)+f(2)+1=f(4)\le 6$$

$$f(1)+f(2)\le 5$$

Case-1:

$$f(1)=1$$
$$f(2)=1,2,3,4 \rightarrow 4 \text{ mappings}$$

Case-2:

$$f(1)=2$$
$$f(2)=1,2,3 \rightarrow 3 \text{ mappings}$$

Case-3:

$$f(1)=3$$
$$f(2)=1,2 \rightarrow 2 \text{ mappings}$$

Case-4:

$$f(1)=4$$
$$f(2)=1 \rightarrow 1 \text{ mapping}$$

No. of functions $=(4+3+2+1)\times 6\times 6$

$$=10\times 6\times 6$$
$$=360$$

Question: For a biased coin, $P(H)=\frac{1}{4}$. Its tossed n times, till we get H. If probability that $64x^2+5nx+1=0$ has no real roots is $\frac{p}{q}$ (p,q co-primes) then $q-p=?$

Answer: 27.00
Solution:

$64x^2+5nx+1=0$

$D<0$

$(5n)^2-4(64)<0$

$(5n)^2<(2\times 8)^2$

$5n<2\times 8$

$n<\frac{16}{5}$

$\Rightarrow n=1,2 \text{ or } 3$

Probability $=\frac{1}{4}+\frac{3}{4}\cdot\frac{1}{4}+\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{4}$

$=\frac{1}{4}+\frac{3}{4}\cdot\frac{1}{4}\left(1+\frac{3}{4}\right)$

$=\frac{1}{4}\left(1+\frac{3}{4}\cdot\frac{7}{4}\right)$

$=\frac{1}{4}\left(\frac{16+21}{16}\right)$

$=\frac{37}{64}$

$\Rightarrow q-p=64-37=27$

Question: The area between the curves $y=2x^2+1$ and tangent to it at $(1,3)$ and $x+y=1$ is

Answer: ()
Solution:

$y=2x^2+1$

$y'=4x$

Point is (1, 3)

$m = 4$

$\Rightarrow y - 3 = 4(x - 1)$

$y - 3 = 4x - 4$

$y = 4x - 1$

$\Rightarrow x + y = 1$

$y = x - 1$

$\Rightarrow 4x - 1 = 1 - x$

$5x = 2$

$x = \frac{2}{5}$

$$\text{A} = \int_0^{\frac{2}{5}} (2x^2 + 1) - (1 - x)\,dx + \int_{\frac{2}{5}}^{1} (2x^2 + 1) - (4x - 1)\,dx$$

$$= \int_0^{\frac{2}{5}} 2x^2 + x\,dx + \int_{\frac{2}{5}}^{1} 2x^2 - 4x + 2\,dx$$

$$= \frac{2x^3}{3} + \frac{x^2}{2}\Big]_0^{\frac{2}{5}} + \frac{2x^3}{3}\Big]_{\frac{2}{5}}^{1} - 2x^2\Big]_{\frac{2}{5}}^{1} + 2x\Big]_{\frac{2}{5}}^{1}$$

$$= \frac{2}{25} + \frac{2}{5} - 2 + \frac{8}{25} + 2 - \frac{4}{5}$$

$$= -\frac{2}{5} + \frac{2}{3} = \frac{4}{15}$$

Question: $f(x) = \begin{cases} e^{\min(x^2 \cdot \{x\})} & ; \quad x \in (0,1) \\ e^{[x - \ln x]} & ; \quad x \in (1,2) \end{cases}$, $\int_0^2 x\, f(x)\,dx = ?$

Answer: $2e - \frac{1}{2}$

Solution:

$$f(x) = \begin{cases} e^{x^2} & ; \quad x \in (0,1) \\ e & ; \quad x \in [1,2) \end{cases}$$

$$\int_0^2 x f(x)\,dx = \int_0^1 x \cdot e^{x^2}\,dx + \int_1^2 x \cdot e\,dx$$

Substitute $x^2 = t \Rightarrow 2x\,dx = dt$

$$\int_0^2 x\, f(x)\, dx = \frac{1}{2}\int_0^1 e^t dt + e\int_1^2 x\, dx$$

$$= \frac{1}{2}\left[e^t\right]_0^1 + e\left[\frac{x^2}{2}\right]_1^2$$

$$= \frac{1}{2}(e-1) + \frac{3}{2}e = 2e - \frac{1}{2}$$