JEE-Mains-12-04-2023 [Memory Based] [Morning Shift]

Physics

Question: If a planet has mass equal to 16 times the mass of earth, and radius equal to 4 times that of earth. The ratio of escape speed of planet to that of earth is

Options:

(a) 2 : 1

(b) 1 : 2

(c) 2 :1

(d) 4 : 1

Answer: (a)

Solution:

$$V_e = \sqrt{2\frac{GM}{R}}$$

So

$$\frac{U_E}{U_P} = \frac{\sqrt{\frac{2GMe}{R_E}}}{\sqrt{\frac{2G(16M_E)}{4RE}}} = \frac{1}{2}$$

$$\frac{U_P}{U_E} = \frac{2}{1}$$

Question: A particle is thrown vertically upward with initial velocity of 150 m/s. The ratio of speed at t = 3 and t = 5 is $(x + 1)/x$ (Take g = 10 ms^{-2})

Answer: 5.00

Solution:

$$V = u - gt$$

So

$$\frac{V \text{ at } 3}{V \text{ at } 5} = \frac{u - g3}{u - g5} = \frac{150 - 30}{150 - 50} = \frac{120}{100} = \frac{6}{5} = \frac{x+1}{x}$$

So $x = 5$

Question: Find ratio of de-broglie wavelength of a proton and α - particle, when accelerated through a potential difference of 2V and 4V respectively.

Options:

(a) 4 : 1

(b) 2 : 1

(c) 1 : 8

(d) 16 : 1

Answer: (a)

Solution:

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

So

$$\frac{\lambda_P}{\lambda_\alpha} = \sqrt{\frac{4\times 2\times (4V)}{1\times 1\times (2V)}} = \frac{4}{1}$$

Question: If a body of mass 5 kg is in equilibrium due to forces F_1, F_2 and F_3, F_2 and F_3 are perpendicular to each other. If F1 is removed then find the acceleration of body. Given F_2 = 6N and F_3 = 8N

Options:

(a) 2 m/s^2

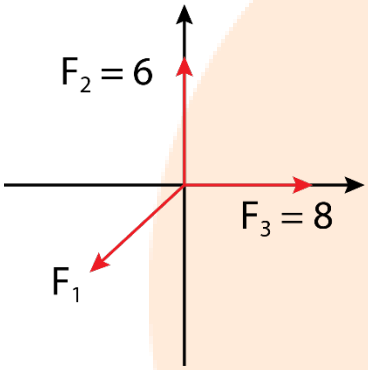
(b) 3 m/s^2

(c) 4 m/s^2

(d) 5 m/s^2

Answer: (a)

Solution:



$F_2 = 6\hat{j} \quad F_3 = 8\hat{i}$

$\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$

$\vec{F_1} = -8\hat{i} - 6\hat{j}$

Now,

$\vec{F_N} = \vec{F_2} + \vec{F_3}$

$\vec{F_N} = 8\hat{i} + 6\hat{j} = m\vec{a}$

$m\vec{a} = 8\hat{i} + 6\hat{j} \Rightarrow 5\,|\,\vec{a}\,| = \sqrt{8^2 + 6^2}\,|$

$|\vec{a}| = 2\,\text{m/s}^2$

Question: 2 planets are revolving around earth in same orbit mass of one satellite is double the mass of other satellite, which of these quantities will be same

Options:

(a) KE

(b) PE

(c) TE

(d) Speed

Answer: (d)

Question: The amplitude of 15 sin(1000 πt) is modulated by 10 sin(4πt) signal. The amplitude modulated signal contains frequency (ies) of

A. 500 Hz
B. 2 Hz
C. 250 Hz
D. 498 Hz
E. 502 Hz
Choose the correct answer from the options given below:
Options:
(a) A only
(b) A, D and E only
(c) B only
(d)A and B only
Answer: (b)
Solution:
$2\pi f_m = 4\pi$
$\Rightarrow f_m = 2$
$2\pi f_c = 1000\pi$
$f_c = 500$
Frequency present are $f_m, f_c - f_m, f_c + f_m$
2, 498, 502
So (b).

Question: A particle is performing SHM with amplitude A find ratio of PE & KE at x = A/2
Options:
(a) 1 : 3
(b) 1 : 2
(c) 2 : 5
(d) 5 : 3
Answer: (a)
Solution:

$$\frac{PE}{KE} = \frac{\frac{1}{2}K\left(x^2\right)}{\frac{1}{2}K\left(A^2 - x^2\right)} = \frac{x^2}{A^2 - x^2}$$

so at $x = \frac{A}{\sqrt{2}}$

$$\frac{PE}{KE} = \frac{(A\ |2)^2}{A^2 - \left(A/2\right)^2} = \frac{\frac{A^2}{4}}{3\frac{A^2}{4}} = \frac{1}{3}$$

Question: Find ratio of rotational kinetic energy to total kinetic energy of hollow sphere
Options:
(a) 3 : 5
(b) 1 : 2
(c) 2 : 5
(d) 3 : 7
Answer: (c)

Solution:

$$\frac{KE_R}{KE_{T+R}} = \frac{\frac{1}{2}I_{cm}\omega^2}{\frac{1}{2}MV_{cm}^2 + \frac{1}{2}I_{cm}\omega^2}$$

Assuming pure rolling for hollow sphere

$I_{cm} = \frac{2}{3}MR^2$ and $V_{cm} = \omega R$

$$\frac{\frac{2}{3}MR^2.\omega^2}{M\omega^2R^2 + \frac{2}{3}M\omega^2R^2} = \frac{\frac{2}{3}}{\frac{5}{3}} = \frac{2}{5}$$

Question: Statement 1: if a truck and car of same Kinetic energy are stopped by same retarding force then time taken for both of them to stop will be same
Statement 2: if a body going east turns north with same speed then acceleration will be zero
Options:
(a) S1 - Correct, S2 - Correct
(b) S1 - Correct, S2 - False
(c) S1 - Correct, S2 - False
(d) S1 - False, S2 - False
Answer: (a)

Question: A loop is placed perpendicular to magnetic field of 0.4T, if it's radius starts increasing at the rate of 1mm/s then find emf induced when radius is 2 cm
Options:
(a) $16\pi \times 10^{-6}$ V
(b) $8\pi \times 10^{-6}$ V
(c) $4\pi \times 10^{-6}$ V
(d) $32\pi \times 10^{-6}$ V
Answer: (a)
Solution:

$\phi = \beta\pi r^2$

$$|\varepsilon| = \frac{d\phi}{dt} = \beta 2\pi r\frac{dr}{dt}$$

$$= 0.4 \times 2\pi \times \frac{2}{100} \times 1 \times 10^{-3}$$

$$= 16\pi \times 10^{-6}\ \text{V}$$

Question: Potential of one drop = 10 μv. If 64 drops are combined to make bigger drop then find potential of bigger drop.
Options:
(a) 140 μV
(b) 150 μV
(c) 160 μV
(d) 170 μV
Answer: (c)

Solution:

Assuming uniform charge

$$V = \frac{KQ}{R} = \frac{K\rho\frac{4}{3}\pi R^3}{R} = \frac{4}{3}\pi K\rho R^2$$

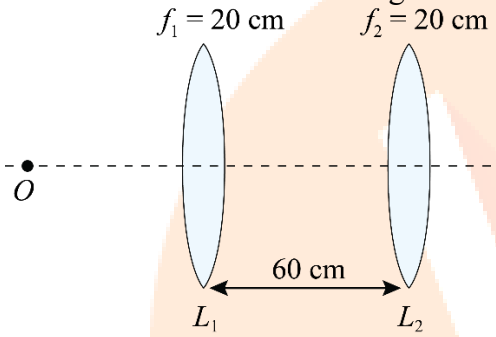
Now, volume remain same

$$64\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow 4r = R$$

$$\frac{V_1}{V_2} = \left(\frac{r}{R}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16} \Rightarrow V_2 = 16V_1$$

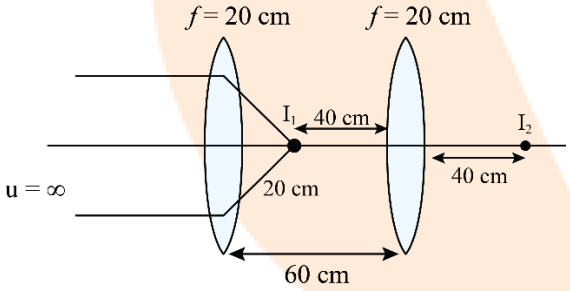
$$V_2 = 160\mu V$$

Question: Two coaxial convex lenses of focal length 20 cm each separated by distance of 60 cm. Then find distance of image from first lens.



Answer: 40.00

Solution:



So u for second lens is (–40)

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{+20} + \frac{1}{-40}$$

$$\frac{1}{v} = \frac{1}{40}$$

Distance between $I_1 - I_2 = 80$

V = +40

Question: The length of a conductor having resistance 160Ω, is compressed by 25% of its initial value. The new resistance will be

Options:

(a) 90 Ω

(b) 20 Ω

(c) 15 Ω

(d) 17 Ω

Answer: (a)

Solution:

Assuming same volume,

$$V = AL \Rightarrow A = \frac{V}{L}$$

So, $R = \rho \frac{L}{A} = \frac{\rho L^2}{V}$

$$R \propto L^2$$

$$\frac{R_1}{R_2} = \left[\frac{(L)}{\left(L - \frac{25}{100}L\right)}\right]^2 = \frac{L^2}{\frac{g}{16}L^2} = \frac{16}{g}$$

So $R_2 = \frac{9}{16}R_1 = \frac{9 \times 160}{16} = 90\Omega$

Question: A photon of energy 12.75 eV falls on a H-atom. Find out no. of spectral lines observed.

Answer: 6.00

Solution:

–13.6, –3.4, –1.5, –0.25, –0.55

13.60 – 0.85 = 12.75

n = 4

$^4C_2 = 6$

Question: If an object cools down from 80°C to 60°C in 5 min is a surrounding of temperature 20°C. The time taken to cool from 60°C to 40°C will be (Assume Newton's law of cooling to be valid)

Options:

(a) $\frac{25}{3}$ min

(b) 5 min

(c) $\frac{25}{4}$ min

(d) 9 min

Answer: (a)

Solution:

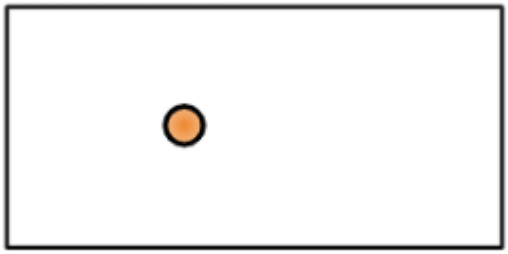
$$\frac{T_i - T_f}{t} = h\left[\frac{T_i + T_f}{2} - T_0\right]$$

$$\frac{80 - 60}{5} = h\left[\frac{80 + 60}{2} - 20\right]$$

$$\frac{60 - 40}{t} = h\left[\frac{60 + 40}{2} - 20\right]$$

$$\frac{t}{5} = \frac{50}{30} \Rightarrow t = \frac{25}{3}$$

Question: In an ice cube of thickness 24 cm, has a bubble trapped in it as shown. If apparent depths are 12 cm & 4 cm from side 1 and side 2 respectively, RI of ice cube is



Options:
(a) 4/3
(b) 3/2
(c) 2
(d) 2.4
Answer: (b)
Solution:

$\frac{x}{\mu} = 4$ cm and $\frac{24-x}{\mu} = 12$

Adding up, so

$\frac{24}{\mu} = 16$

$\mu = \frac{3}{2}$

Question: A dipole having dipole moment $\vec{M}$ is placed in two magnetic field of strength B_1 and B_2 respectively. If dipole oscillates 60 times in 20 seconds in B_1 magnetic field and 60 oscillations in 30 seconds in B_2 magnetic field. Then find the $\left(\frac{B_1}{B_2}\right)$

Answer: 9/4
Solution:

$T = 2\pi\sqrt{\frac{L}{m\beta}}$

$T_1 = 20/60$

$T_2 = 30/60$

$\frac{T_1}{T_2} = \sqrt{\frac{\beta_2}{\beta_1}} = \frac{\beta_1}{\beta_2} = \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{\frac{1}{2}}{\frac{1}{3}}\right)^2 = \frac{9}{4}$

Question: Statement (1): I LCR circuit, by increasing frequency, current increases first then decreases.
Statement (2): Power factor of LCR circuit is one at resonance.
Choose the correct option.
Options:
(a) S1 is correct and S2 is incorrect

(b) S1 is incorrect and S2 is correct
(c) Both S1 and S2 are correct
(d) Both S1 and S2 are incorrect
Answer: (a)
Solution: S1 is correct and S2 is incorrect

Question: Which of the following is more energetic between infrared wave and microwave?
Options:
(a) Microwave
(b) IR wave
(c) Both are having same energy
(d) Can't predict
Answer: (a)
Solution: Microwave

Question: Current flowing in a conductor at 0° and 100° is 2A and 1.2A respectively. The current at 80° is
Options:
(a) 1.3 A
(b) 1.5 A
(c) 1.6 A
(d) 1.8 A
Answer: (b)
Solution:

$2 = \frac{V}{R_0}$ and $1.2 = \frac{V}{R_0(1+\alpha 100)}$

So $1.2 = \frac{2}{(1+\alpha 100)}$

$\alpha 100 = \frac{10}{6} - 1 = \frac{4}{6}$

$\alpha = \frac{1}{150}$

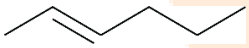
so $i = \frac{V}{R_0(1+50\alpha)} = \frac{2}{1+50\times\frac{1}{150}} = \frac{6}{4} = 1.5$

JEE-Mains-12-04-2023 [Memory Based] [Morning Shift]

Chemistry

Question: The bond order and magnetic property of acetylide ion is same as that of
Options:
(a) NO^+
(b) N_2^+
(c) O_2^-
(d) O_2^+
Answer: (a)
Solution: Both NO^+ and C_2^{2-} has bond order of 3

Question: 2-hexene $\xrightarrow[(ii)H_2O]{(i)O_3}$ products
The two products formed in the above reactive are
Options:
(a) Acetaldehyde and butanal
(b) Acetaldehyde and propanol
(c) Acetic acid and butanoic acid
(d) Acetic acid and propanoic acid
Answer: (c)
Solution:



$\xrightarrow[H_2O]{O_3} CH_3COOH + CH_3CH_2CH_2COOH$

Question: Statement-1: Boron is hard as it has high lattice energy.
Statement-2: Boron has high melting & Boiling point as compared to other group members.
Options:
(a) Both statements are correct.
(b) Both statements are incorrect.
(c) statement 1 is correct and statement 2 is incorrect.
(d) statement 1 is incorrect and statement 2 is correct.
Answer: (a)
Solution: Fact based

Question: Correct order of density of alkali metals?
Li, Na, K, Rb, Cs
Options:
(a) Li < Na < K < Rb < Cs
(b) Li > Na > K > Rb > Cs
(c) Li < Na > K < Rb < Cs

(d) Li < K < Na < Rb < Cs
Answer: (d)
Solution: Potassium is less dense than sodium and down the group density increases.

Question: Statement 1: $SbCl_5$ is more covalent than $SbCl_3$
Statement 2: Higher oxidation states of halides is more stable.
Options:
(a) Both statements are correct.
(b) Both statements are incorrect.
(c) statement 1 is correct and statement 2 is incorrect.
(d) statement 1 is incorrect and statement 2 is correct
Answer: (c)
Solution: More than one oxidation state, the halides in higher oxidation state will be more covalent than the one in lower oxidation state. For example, $SnCl_4$, $PbCl_4$, $SbCl_5$ and UF_6 are more covalent than $SnCl_2$, $PbCl_2$, $SbCl_3$ and UF_4 respectively.

Question: How much volume of water is required to change pH of H_2SO_4 from 1 to 2?
Options:
(a) 10 L
(b) 100 L
(c) 1000 L
(d) 1 L
Answer: (a)
Solution: $M_1V_1 = M_2V_2$
$10^{-1} \times V_1 = 10^{-2} \times V_2$
or $V_2 = 10V_1$

Question: $CaCl_2 + Na_2CO_3 \rightarrow X + Y$
$X + Z \rightarrow CaCl_2 + H_2O + CO_2$
What is X, Y and Z?
Options:
(a) $X = CaCO_3$, Y = NaCl, Z = HCl
(b) X = CaO, $Y = NaCl + CO_2$, Z = KCl
(c) X = CaO, $Y = NaCl + CO_2$, Z = NaCl
(d) $X = CaCO_3$, Y = KCl, Z = NaCl
Answer: (a)
Solution: $CaCl_2 + Na_2CO_3 \rightarrow CaCO_3 + 2NaCl$

$CaCO_3 + 2HCl \rightarrow CaCl_2 + H_2O + CO_2$

Question: Match the following.

Column-I (Polymer)	**Column-II**
(A) Dacron	**(P)** Polyester
(B) Nylon-2-nylon-6	**(Q)** Natural
(C) rubber	**(R)** Synthetic
(D) PAN	**(S)** Biodegradable polymer

Options:

(a) A – R; B – S; C – Q; D – P
(b) A – P; B – S; C – Q; D – R
(c) A – S; B – P; C – Q; D – R
(d) A – Q; B – P; C – S; D – R
Answer: (b)
Solution: Fact based.

Question: Match the following.

Column-I	**Column-II**
(A) MgH_2	**(P)** Saline Hydride
(B) CH_4	**(Q)** Electron rich hydride
(C) B_2H_6	**(R)** Electron precise Hydride
(D) HF	**(S)** Electron deficient Hydride

Options:
(a) A – P; B – R; C – S; D – Q
(b) A – Q; B – P; C – R; D – S
(c) A – S; B – P; C – Q; D – R
(d) A – R; B – P; C – S; D – Q
Answer: (a)
Solution: Fact based

Question: Match the following.

Column-I	**Column-II**
(A) Acid Rain	**(P)** Oxides of nitrogen
(B) Eutrophication	**(Q)** CO_2
(C) Global Warming	**(R)** phosphates

Options:
(a) A – Q; B – P; C – R
(b) A – P; B – Q; C – R
(c) A – P; B – R; C – Q
(d) A – R; B – Q; C – P
Answer: (c)
Solution: Fact based.

Question: Assertion: 5f electrons can participate in bonding to a greater extent as compared to 4f electrons
Reason: Both resemble in their angular part of wave function, but 5f is not as buried as 4f - orbitals
Options:
(a) Both assertion and reason are correct and reason is correct explanation.
(b) Both assertion and reason are correct but it is not correct explanation.
(c) Both assertion and reason are incorrect
(d) Assertion is correct but reason is incorrect.
Answer: (a)
Solution: Fact based

Question: A gas with molecular weight, 42 amu will have same rms velocity at 27°C as that of V_{mps} of which gas at 27°C.

Options:

(a) CO_2

(b) CO

(c) N_2O

(d) NO_2

Answer: (b)

Solution: T = 300 K, $U_{rms} = \sqrt{\frac{3RT}{M}}$

M = 42, $U_{mp} = \sqrt{\frac{2RT}{M}}$

T = 300 K, M = ?

$$\sqrt{\frac{3RT}{42}} = \sqrt{\frac{2RT}{M}}$$

M = 28

Answer = CO

Question: A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. Calculate the total number of spectral lines.

Options:

(a) 3

(b) 2

(c) 4

(d) 1

Answer: (a)

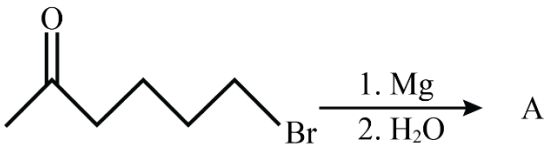
Solution:

$\Delta E = E_3 - E_1 = 12.09$

If 12.5 eV beam is used it will cause transition to 3rd shell

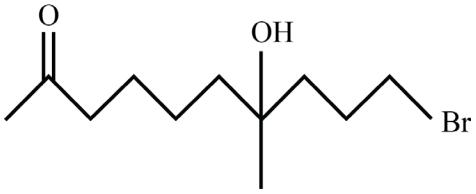
∴ Spectral lines = 3

Question: Find 'A' in the given reaction.

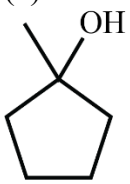


Options:

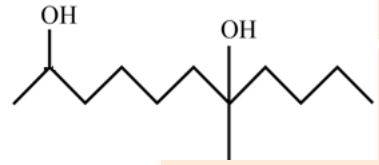
(a)



(b)



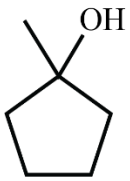
(c)



(d)

Answer: (b)

Solution:



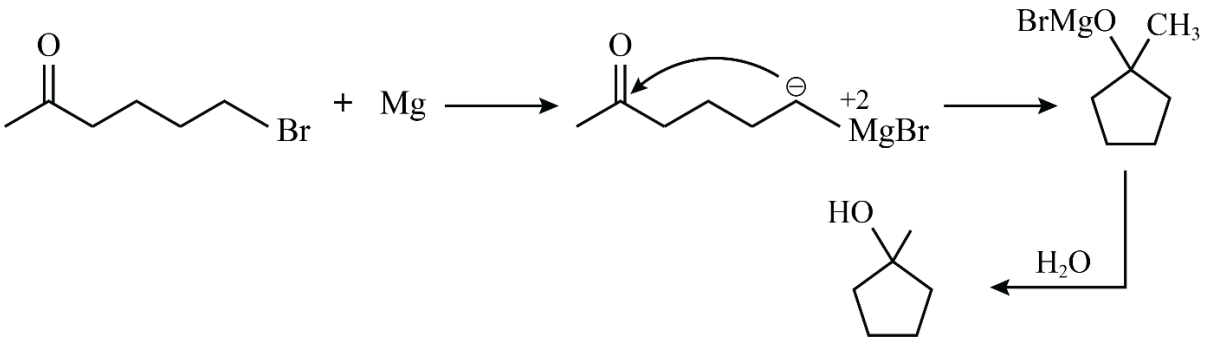
Question: Select correct statement about lead storage battery.

Options:

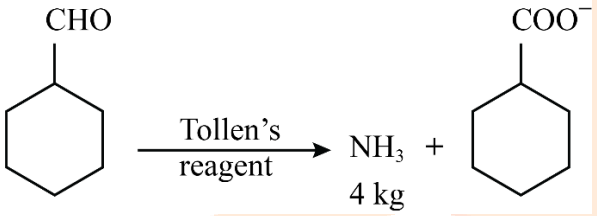
(a) $PbSO_4$ converts into PbO_2 at anode during discharging.
(b) $PbSO_4$ converts into PbO_2 at cathode during discharging.
(c) 38% H_2SO_4 solution is taken as the electrolyte.
(d) H_2SO_4 is produced during discharging.

Answer: (c)

Solution:

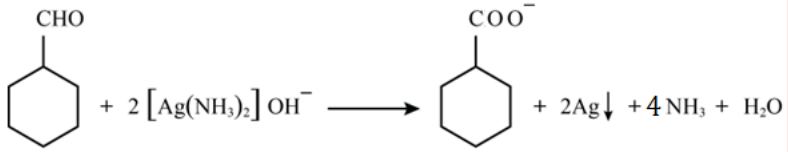


Question:



Calculate mass of Tollen's reagent required

Options:
(a) 18.70 kg
(b) 37.40 kg
(c) 9.35 kg
(d) 55.10 kg
Answer: (a)
Solution:



Moles of NH_3 are double that of T.R

$$\text{Moles} = \frac{4}{17} \times 10^{+3} = 235.29$$

∴ Tollen's reagent is 117.64 moles

or mass = 159 × 117.64 = 18.7 kg

Question: Molality of $MgCl_2$ is 1 m, α = 80%. Calculate vapour pressure of solutions (in torr), if vapour pressure of pure solvent is 100 torr.

Options:
(a) 95.53
(b) 78.23
(c) 68.12
(d) 98.26

Answer: (a)

Solution: m = 1, α = 0.8, P_o = 100 torr

$$\alpha = \frac{i-1}{3-1}$$

$0.8 \times 2 + 1 = i$

$i = 2.6$

$$\frac{P^o - P_s}{P_s} = \frac{i \times n_{solute}}{n_{solvent}}$$

$$\frac{100 - P_s}{P_s} = \frac{1}{55.5} \times 2.6$$

$P_s = 95.524$

Question: How many of the given metals will show photoelectric effect when light of 400 nm falls on below metals?

Metal	**Li**	**Na**	**K**	**Mg**	**Cu**	**Ag**
W (eV)	**2.42**	**2.3**	**2.25**	**3.7**	**4.8**	**4.3**

Answer: 3.00

Solution: Li, Na, K

JEE-Mains-12-04-2023 [Memory Based] [Morning Shift]

Mathematics

Question: $\frac{{}^nC_0}{1}+\frac{{}^nC_1}{2}+\frac{{}^nC_2}{3}+....+\frac{{}^nC_n}{n+1}=\frac{1023}{n+1}$. Find n.

Answer: 9.00

Solution:

$$\frac{{}^nC_0}{1}+\frac{{}^nC_1}{2}+\frac{{}^nC_2}{3}+....+\frac{{}^nC_n}{n+1}=\frac{1023}{n+1}$$

$$\Rightarrow \frac{2^{n+1}-1}{n+1}=\frac{1023}{n+1}$$

$$\Rightarrow 2^{n+1}=1024$$

$$\Rightarrow n+1=10$$

$$\Rightarrow n=9$$

Question: Find sum of first 50 terms in expansion of $(1-x)^{100}$.

Answer: $\frac{-{}^{100}C_{50}}{2}$

Solution:

Sum of first 50 terms in expansion of $(1-x)^{100}$

${}^{100}C_0-{}^{100}C_1+...+{}^{100}C_{49}$

Now, ${}^{100}C_0-{}^{100}C_1+...+{}^{100}C_{100}=0$

$2\left[{}^{100}C_0-{}^{100}C_1+...\right]+{}^{100}C_{50}=0$

Sum of first 50 terms = $\frac{-{}^{100}C_{50}}{2}$

Question: If $\left(x^2+1\right)dy=y\left(x-y\right)dx,\ y\left(0\right)=1,$ then find $y\left(2\sqrt{2}\right)$

Answer:

Solution:

$$\left(x^2+1\right)dy=y\left(x-y\right)dx$$

$$\frac{dy}{dx}+\frac{x}{1+x^2}y=\frac{y^2}{1+x^2}$$

$$\Rightarrow \frac{1}{y^2}\frac{dy}{dx}+\frac{x}{1+x^2}\times\frac{1}{y}=\frac{1}{1=x^2}$$

Let $\frac{1}{y}=t \Rightarrow -\frac{1}{y^2}\frac{dy}{dx}=\frac{dt}{dx}$

$$\Rightarrow -\frac{dt}{dx}+\left(\frac{x}{1+x^2}\right)dt=\frac{1}{1+x^2}$$

$$\Rightarrow \frac{dt}{dx}-\left(\frac{x}{1+x^2}\right)dt=-\frac{1}{1+x^2}$$

I.F. $=e^{-\int\frac{x}{1+x^2}dx}=e^{-\frac{1}{2}\log\left|1+x^2\right|}=\frac{1}{\sqrt{1+x^2}}$

$$\frac{t}{\sqrt{1+x^2}}=-\int\frac{1}{\left(1+x^2\right)\sqrt{1+x^2}}dx$$

Let $x=\tan\theta \Rightarrow dx=\sec^2\theta\, d\theta$

$$I=\int\frac{\sec^2\theta}{\sec^2\theta\cdot\sec\alpha}d\theta=\int\cos\theta=\sin\theta+C$$

$$\therefore \frac{1}{y\sqrt{1+x^2}}=-\frac{x}{\sqrt{1+x^2}}+C$$

$$\because y(0)=1\Rightarrow C=\sqrt{2}$$

$$\frac{1}{y\sqrt{1+x^2}}+\frac{x}{\sqrt{1+x^2}}=\sqrt{2}$$

$$1+xy=\sqrt{2}y\sqrt{1+x^2}$$

Now $y\left(2\sqrt{2}\right)$

$$1+2\sqrt{2}y=3\sqrt{2}y$$

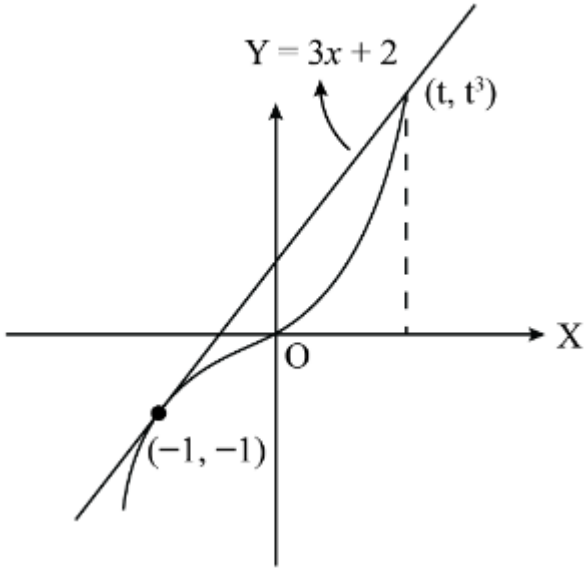
$$\sqrt{2}y=1$$

$$\Rightarrow y=\frac{1}{\sqrt{2}}$$

Question: Find area between $y=x^3$ and its tangent at $(-1,-1)$.

Answer: $\frac{27}{4}$

Solution:



$$\frac{t^3+1}{t+1}=3(-1)^2$$

$$t^2-t+1=3$$

$$t=2$$

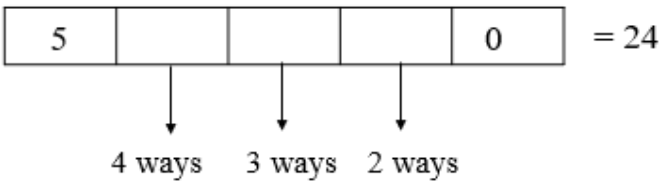
$$\text{Area}=\int_{-1}^{2}(3x+2)-x^3\,dx$$

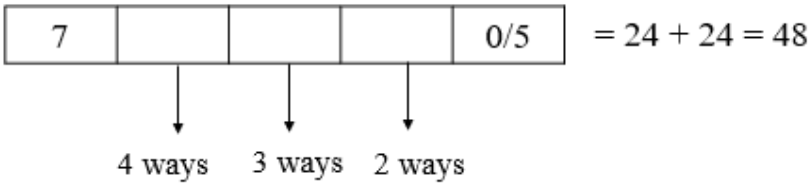
$$=\frac{3}{2}(3)+2(3)-\frac{1}{4}\times 15$$

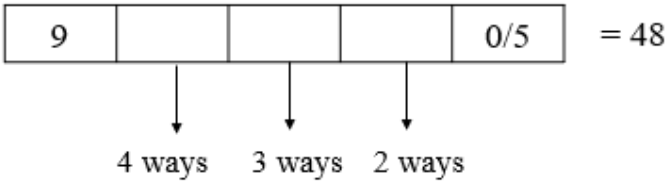
$$=\frac{27}{4}$$

Question: Find number of 5 digit numbers greater than 40000 which are divisible by 5 that can be made using $\{1,0,3,5,7,9\}$. Repetition is not allowed.

Answer: 120.00
Solution:







Total = 24 + 48 + 48 = 120

Question: In ΔABC, $\cos A+2\cos B+\cos C=2$ and $a=3, c=7$. $\cos A-\cos C=?$

Answer: $\frac{10}{7}$

Solution:

$\cos A+\cos C=2(1-\cos B)$

$\Rightarrow 2\cos\frac{A+C}{2}\cos\frac{A-C}{2}=4\sin^2\frac{B}{2}$

$\Rightarrow \cos\frac{A-C}{2}=2\sin\frac{B}{2}$

$\Rightarrow 2\cos\frac{B}{2}\cos\frac{A-C}{2}=4\sin\frac{B}{2}\cos\frac{B}{2}$

$\Rightarrow 2\sin\frac{A+C}{2}\cos\frac{A-C}{2}=2\sin B$

$\Rightarrow \sin A+\sin C=2\sin B$

$\Rightarrow a, b, c$ are in AP

$\Rightarrow b=5$

$\cos A-\cos C=\frac{b^2+c^2-a^2}{2bc}-\frac{a^2+b^2-c^2}{2ab}$

$=\frac{25+49-9}{70}-\frac{9+25-49}{30}$

$=\frac{65}{70}+\frac{1}{2}$

$$= \frac{13}{14} + \frac{1}{2}$$
$$= \frac{20}{14}$$
$$= \frac{10}{7}$$

Question: $P_1 : 4x - y + z = 4$ and $P_2 : x - y + z = 13$. P_1 is rotated by $90°$ about line of intersection to get P_1'. Find distance of $(-2, 3, 4)$ from P_1'.

Answer: $\frac{31}{3}$

Solution:

$P_1' : (4x - y + z - 4) + \lambda(x - y + z - 13) = 0$

$\Rightarrow (4 + \lambda)x - (1 + \lambda)y + (1 + \lambda)z - 4 - 13\lambda = 0$

$P_1 \perp P_1'$

$4(4 + \lambda) + 1 + \lambda + 1 + \lambda = 0$

$\Rightarrow 6\lambda + 18 = 0$

$\rightarrow \lambda = -3$

$P_1' : x + 2y - 2z + 35 = 0$

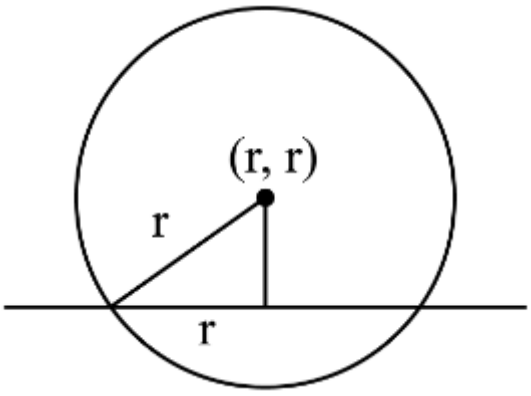
Distance $= \left| \frac{-2 + 6 - 8 + 35}{\sqrt{1 + 4 + 4}} \right| = \frac{31}{3}$

Question: Two circles with radius r_1 and r_2 touches both axes in first quadrant and make intercept of length 2 on $x + y = 2$. Find the value of $r_1^2 + r_2^2 - r_1 r_2$.

Answer: 7.00

Solution:

Circle: $(x - r)^2 + (y - r)^2 = r^2$



$$\left|\frac{r+r-2}{\sqrt{2}}\right| = \sqrt{r^2-1}$$

$$\Rightarrow \left(\sqrt{2}(r-1)\right)^2 = r^2-1$$

$$\Rightarrow 2\left(r^2-2r+1\right) = r^2-1$$

$$\Rightarrow r^2-4r+3=0$$

$$\Rightarrow r = 1,3$$

$$r_1^2 + r_2^2 - r_1 r_2 = 1+9-3 = 7$$

Question: $f(x) = [\![x]\!] - \sqrt{x-[x]}$. Find number of points of discontinuity in $(-2,1)$.

Answer: ()

Solution:

$$f(x) = [\![x]\!] - \sqrt{x-[x]}$$

Let $n \in z$, $f(n) = |n| - \sqrt{n-n} = |n|$

RHL $= |n| - 0 = |n|$

LHL $= |n-1| - \sqrt{n-(n-1)} = |n-1|-1$

$= -(n-1)-1$ as $n<1$

$= -n$

RHL = LHL if $n = 0$ or -1

So, no. of points of discontinuity = 0

Question: $f(x) = \int \sqrt{\frac{x+7}{x}}dx,\ f(9) = 12+7\ln(7)$

Answer:

Solution:

$$f(x) = \int \sqrt{\frac{x+7}{x}}\,dx$$

$$I = \int \frac{x+7}{\sqrt{x^2+7x}}dx$$

$$= \int \frac{x\,dx}{\sqrt{x^2-7x}} + 7\int \frac{dx}{\sqrt{x^2+7x}}$$

$$= \frac{1}{2}\int \frac{2x+7-7\,dx}{\sqrt{x^2+7x}} + 7\int \frac{dx}{\sqrt{x^2+7x}}$$

Let $x^2+7x = t^2$

$(2x+7)dx = 2t\,dt$

$$= \frac{1}{2}\int \frac{2x+7dx}{\sqrt{x^2+7x}} - \frac{7}{2}\int \frac{dx}{\sqrt{x^2+7x}} + 7\int \frac{dx}{\sqrt{x^2+7x}}$$

$$= \frac{1}{2}\int \frac{(2x+7)\,dx}{\sqrt{x^2+7x}} + \frac{7}{2}\int \frac{dx}{\sqrt{x^2+7x}}$$

$$= \frac{1}{2}\int \frac{2tdt}{t} + \frac{7}{2}\int \frac{dx}{\sqrt{\left(x+\frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2}}$$

$$= \sqrt{x^2+7x} + \frac{7}{2}\int \frac{dx}{\sqrt{\left(x+\frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2}}$$

$$= \sqrt{x^2+7x} + \frac{7}{2}\ln\left|\left(x+\frac{7}{2}\right) + \sqrt{\left(x+\frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2}\right| + C$$

Question: $\int_{-0.15}^{0.15} \left|100x^2 - 1\right| dx$

Answer: $\frac{575}{3000}$

Solution:

$$\int_{-0.15}^{0.15} \left|100x^2 - 1\right| dx$$

$$= 2\int_{0}^{0.15} \left|100x^2 - 1\right| dx$$

$$= 2\int_{0}^{0.1} 1 - 100x^2 + 2\int_{0.1}^{0.15} 100x^2 - 1\,dx$$

$$= 2\left[0.1 - \frac{100}{3}(0.001)\right] + 2\left[\frac{100}{3}\left((0.15)^3 - (0.1)^3\right) - 0.05\right]$$

$$= 0.2 - \frac{0.2}{3} + \frac{0.475}{3} - 0.1$$

$$= 0.192$$

Question: $S_1 : (p \to q) \wedge (p \wedge \sim q)$ is contradiction

$S_2 : (p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$ is tautology.

Options:

(a) Both true

(b) Both false

(c) Only S_1 is true

(d) Only S_2 is true

Answer: (a)

Solution:

$S_1 : (p \to q) \wedge (p \wedge \sim q)$

$\equiv(\sim p\vee q)\wedge(p\wedge\sim q)$

S_1 is contradiction (True)

$S_2:(p\wedge q)\vee(\sim p\wedge q)\vee(p\wedge\sim q)\vee(\sim p\wedge\sim q)$

S_2 is Tautology (True)

Both are true

Question: $a\hat{i}+\hat{j}+\hat{k},\hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$, are coplanar where $a,b,c\neq 1$, then $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=?$

Answer: 1.00

Solution:

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}=0$$

$\Rightarrow a\big(b(-1)\big)-(c-1)+(1-b)=0$

$\Rightarrow abc-a-c+1+1-b=0$

$\Rightarrow a+b+c=abc+2$

$1-a=x,\ 1-b=y,\ 1-c=z$

$\Rightarrow a=1-x,\ b=1-y,\ c=1-z$

$3-(x+y+z)=(1-x)(1-y)(1-z)+2$

$\Rightarrow 1-x-y-z=(1-x-y+xy)(1-z)$

$\Rightarrow 1-x-y-z=1-x-y+xy-z+xz+yz-xyz$

$\Rightarrow xy+yz+zx=xyz$

$\Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$

Question: If α,β are the roots of $x^2+\sqrt{6}x+3=0$, then find $\frac{\alpha^{23}+\beta^{23}+\alpha^{14}+\beta^{14}}{\left(\alpha^{15}+\beta^{15}\right)+\left(\alpha^{10}+\beta^{10}\right)}$

Answer: 81.00

Solution:

$\alpha^2+3=-\sqrt{6}\alpha$

$\alpha^4+9+6\alpha^2=6\alpha^2$

$\Rightarrow \alpha^4=-9$

$$=\frac{(-9)^5\alpha^3+(-9)^5\beta^3+(-9)^3\alpha^2+(-9)^3\beta^2}{(-9)^3\alpha^3+(-9)^3\beta^3+(-9)^2\left(\alpha^2+\beta^2\right)}$$

$$=\frac{(-9)^3\left[81\left(\alpha^3+\beta^3\right)+\left(\alpha^2+\beta^2\right)\right]}{(-9)^2\left[-9\left(\alpha^3+\beta^3\right)+\left(\alpha^2+\beta^2\right)\right]}$$

$$\alpha^3+\beta^3=\left(-\sqrt{6}\right)^3-3(3)\left(-\sqrt{6}\right)$$
$$=-6\sqrt{6}+9\sqrt{6}=3\sqrt{6}$$
$$\alpha^2+\beta^2=6-2(3)=0$$
$$=\frac{-9\times\left(81\times3\sqrt{6}\right)}{-9\times3\sqrt{6}}$$
$$=81$$

Question: $A=\begin{bmatrix}1 & \frac{1}{51}\\ 0 & 1\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}\times A\times\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$. Find $\sum_{n=1}^{50}B^n$.

Answer: $25\begin{bmatrix}3 & 1\\ -1 & 1\end{bmatrix}$

Solution:

$$B^2=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}A\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}A\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$$
$$B^2=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}A^2\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$$
$$\sum B^n=P\sum A^nP^{-1}$$
$$A=\begin{bmatrix}0 & \frac{1}{51}\\ 0 & 0\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$A^n=(C+I)^n=n\begin{bmatrix}0 & \frac{1}{51}\\ 0 & 0\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$=P\begin{bmatrix}\sum n & \sum\frac{n}{51}\\ 0 & \sum1\end{bmatrix}P^{-1}=P\begin{bmatrix}50 & 25\\ 0 & 50\end{bmatrix}P^{-1}$$
$$=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}\begin{bmatrix}50 & 25\\ 0 & 50\end{bmatrix}\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$$
$$=25\begin{bmatrix}2 & 5\\ -2 & -3\end{bmatrix}\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$$
$$=25\begin{bmatrix}3 & 1\\ -1 & 1\end{bmatrix}$$

Question: $\sum a_n=\frac{n^2+3n}{(n+1)(n+2)}$

$28\sum_{n=1}^{10}\frac{1}{a_n}=p_1p_2p_3....p_n$, where p_i are first m prime numbers. Find m .

Answer: ()

Solution:

$$\sum a_n=\frac{n^2+3n}{(n+1)(n+2)}=2-\frac{2}{n+1}+\frac{2}{n+2}$$

$$a_n=2-\frac{2}{n+1}+\frac{2}{n+2}-\left(2-\frac{2}{n}+\frac{2}{n+1}\right)$$

$$a_n=\frac{-4}{n+1}+\frac{2}{n}+\frac{2}{n+2}$$

$$a_n=\frac{2}{n(n+1)}-\frac{2}{(n+1)(n+2)}$$

$$a_n=\frac{4}{n(n+1)(n+2)}$$

$$\sum\frac{1}{a_n}=\frac{1}{8\times2}[10\times11\times12\times13]$$

$$28\sum\frac{1}{a_n}=7\times15\times11\times6\times13$$

$$=2\times3\times5\times7\times11\times13$$

$$m=6$$

Question: $\Delta_k=\begin{vmatrix}1 & 2k & 2k-1\\ n & n^2+n+1 & n^2\\ n & n^2+n & n^2+n\end{vmatrix}$, $\sum\Delta_k=96$. Find n .

Answer: $\sqrt{96}$

Solution:

Given, $\Delta_k=\begin{vmatrix}1 & 2k & 2k-1\\ n & n^2+n+1 & n^2\\ n & n^2+n & n^2+n\end{vmatrix}$

$$\sum\Delta_k=\begin{vmatrix}\sum1 & \sum2k & \sum2k-1\\ n & n^2+n+1 & n^2\\ n & n^2+n & n^2+n\end{vmatrix}$$

$$96=\begin{vmatrix}n & n^2+n & n^2\\ n & n^2+n+1 & n^2\\ n & n^2+n & n^2+n\end{vmatrix}$$

$R_3\to R_3-R_1$

$$96 = \begin{vmatrix} n & n^2+n & n^2 \\ n & n^2+n+1 & n^2 \\ 0 & 0 & n \end{vmatrix}$$

$$= n\left[n\left(n^2+n+1\right) - n\left(n^2+n\right) \right]$$

$$= n^2$$

$$n^2 = 96$$

$$n = \sqrt{96}$$