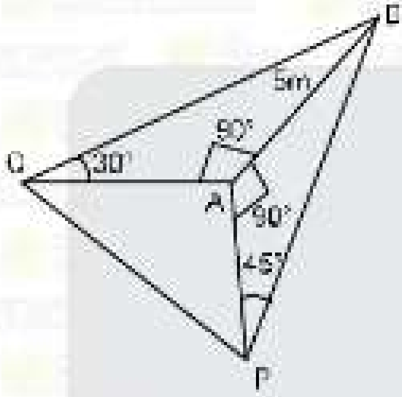
PART : MATHEMATICS

1. A tower of 5m height. From point P, which is south of the tower, angle of elevation of the top of the tower is 45° and from point Q which is west of the tower, angle of elevation of the top of the tower is 30° then distance between point PQ is

(1) 10 m (2) 20 m (3) 30 m (4) 5 m

Ans. (1)

Sol.



Let tower be AB

A is foot of tower

Then AP = 5cot 45°

= 5

& AQ = 5cot 30°

$= 5\sqrt{3}$ m

Now $PQ = \sqrt{AP^2 + AQ^2}$

$= \sqrt{25 + 25 \times 3} = 10$

2. If f(1) + f(2) = f(4) – 1 and a function from A to B is defined where A = {1, 2, 3, 4, 5} B = {1, 2, 3, 4, 5, 6}. Find number of functions.

Ans. (360)

Sol. $f(1) + f(2) + 1 = f(4) \le 6$

$f(1) + f(2) \le 5$

Case I: f(1) = 1 ⇒ f(2) = 1, 2, 3, 4 (4 ways), f(3) = (6 ways), f(5) = (6 ways)

Case II: f(1) = 2 ⇒ f(2) = 1, 2, 3 (3 ways), f(3) = (6 ways), f(5) = (6 ways)

Case III: f(1) = 3 ⇒ f(2) = 1, 2 (2 ways), f(3) = (6 ways), f(5) = (6 ways)

Case IV: f(1) = 4 ⇒ f(2) = 1 (1 ways), f(3) = (6 ways), f(5) = (6 ways)

Total functions = 10 × 6 × 6 = 360

3. Mean & variance of observations 1, 2, 4, 5, x & y are 5 & 10 then the value of mean deviation about mean is

(1) $\frac{8}{3}$ (2) $\frac{10}{3}$ (3) $\frac{5}{3}$ (4) $\frac{7}{6}$

Ans. (1)

Sol. $\frac{1+2+4+5+x+y}{6} = 5 \Rightarrow x + y = 18$ ... (1)

$\frac{1^2+2^2+4^2+5^2+x^2+y^2}{6} - (5)^2 = 10$

$\Rightarrow x^2 + y^2 = 164$ ... (2)

By (1) & (2) $x = 8$ $y = 10$

Now mean deviation about mean $= \frac{|x-\bar{x}|}{6}$

$= \frac{4+3+1+0+3+5}{6} = \frac{8}{3}$

4. If $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x+81)$

Then λ and $\lambda/3$ are roots of equation

(1) $x^2 - 24x - 27 = 0$
(2) $2x^2 - 24x - 27 = 0$
(3) $4x^2 + 24x - 27 = 0$
(4) $4x^2 - 24x + 27 = 0$

Ans. (4)

Sol. Put $x = 0$ $\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^2 \end{vmatrix} = \frac{729}{8} \Rightarrow \lambda = \frac{9}{2}$

The equation whose roots are $\frac{9}{2}$ & $\frac{3}{2}$ is $x^2 - \frac{12x}{2} + \frac{27}{4} = 0 \Rightarrow 4x^2 - 24x + 27 = 0$

5. If angle between plane $x + 2y + 3z = 7$ and line $\frac{x}{1} = \frac{y-1}{-2} = \frac{z-3}{\lambda}$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$ and point of intersection of given line and plane is (α, β, γ) then the value of $\alpha + 2\beta + 6\gamma$ is

(1) $\frac{115}{9}$
(2) $\frac{119}{3}$
(3) $\frac{40}{3}$
(4) $\frac{41}{3}$

Ans. (3)

Sol. Angle $= \cos^{-1}\left(\sqrt{\frac{5}{14}}\right) = \sin^{-1}\frac{3}{\sqrt{14}}$

Now, $\frac{3}{\sqrt{14}} = \pm\left(\frac{1+(-2)(2)+3(\lambda)}{\sqrt{14}\sqrt{\lambda^2+4+1}}\right)$

$9(\lambda^2+5) = (3\lambda-3)^2$

$9\lambda^2 + 45 = 9\lambda^2 + 9 - 18\lambda$

$\lambda = -2$

Now let point of intersection be $(k, -2k+1, -2k+3)$

$\Rightarrow k - 4k + 2 - 6k + 9 = 7 \Rightarrow k = \frac{4}{9}$

Point of intersection (α, β, γ) is $\left(\frac{4}{9}, \frac{1}{9}, \frac{19}{9}\right)$

$\Rightarrow \alpha + 2\beta + 6\gamma = \frac{40}{3}$

6. Let a, b, c, d are positive numbers and $a + b + c + d = 11$ & maximum value of $a^5b^3c^2d = 3750\,\beta$ then β is

(1) 90 (2) 95 (3) 100 (4) 85

Ans. (1)

Sol. By AM $\geq$ GM

$$\frac{5\left(\frac{a}{5}\right)+3\left(\frac{b}{3}\right)+2\left(\frac{c}{2}\right)+d}{11} \geq \left(\left(\frac{a}{5}\right)^5\left(\frac{b}{3}\right)^3\left(\frac{c}{2}\right)^2 d\right)^{\frac{1}{11}}$$

$$1 \geq \frac{a^5b^3c^2d}{3125 \times 27 \times 4}$$

$3125 \times 108 \geq a^5b^3c^2d$

Now, $\beta(3750) = 3125 \times 108$

$\beta = 90.$

7. $$\int_0^{\pi/2} f(\sin 2x)\sin x\,dx + \alpha\int_0^{\pi/4} f(\cos 2x)\cos x\,dx = 0$$

(1) $\sqrt{3}$ (2) $-\sqrt{3}$ (3) $\sqrt{2}$ (4) $-\sqrt{2}$

Ans. (4)

Sol. $$I = \int_0^{\pi/2} f(\sin 2x)\sin x\,dx$$

$$\int_0^{\pi/4} f(\sin 2x)\sin x\,dx + \int_{\pi/4}^{\pi/2} f(\sin 2x)\sin x\,dx$$

$$\int_0^{\pi/4} f(\cos 2x)\sin\left(\frac{\pi}{4}-x\right)dx + \int_0^{\pi/4} f\left(\sin\left(2\left(\frac{\pi}{4}+x\right)\right)\right)\sin\left(\frac{\pi}{4}+x\right)dx$$

$$I = \int_0^{\pi/4} f(\cos 2x)\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)dx + \int_0^{\pi/4} f(\cos(2x))\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right)dx$$

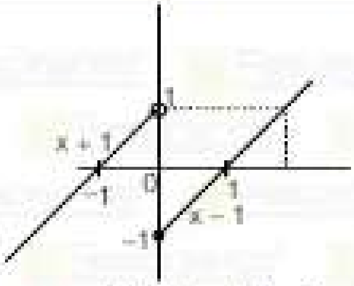
$$I = \int_0^{\pi/4} f(\cos 2x)\left(\sqrt{2}\cos x\,dx\right)$$

$\alpha = -\sqrt{2}$

8. If $f(x) = \begin{cases} x+1 & ; x < 0 \\ x-1 & ; x \geq 0 \end{cases}$ $g(x) = \begin{cases} x+1 & ; x < 0 \\ 1 & ; x \geq 0 \end{cases}$ and the number of discontinuous points of $g(f(x))$ is n and number of non-differentiable points of $g(f(x))$ is m then $m + n$ is equal to

Ans. (4)

Sol.



$$g(f(x)) = \begin{cases} f(x)+1 & f(x) < 0 \\ 1 & f(x) \ge 0 \end{cases}$$

$$g(f(x)) = \begin{cases} x+1+1 & ; x < -1 \\ 1 & ; -1 \le x < 0 \\ x-1+1 & ; 0 \le x < 1 \\ 1 & ; x \ge 1 \end{cases} \Rightarrow h(x) = g(f(x)) = \begin{cases} x+2 & ; x < -1 \\ 1 & ; -1 \le x < 0 \\ x & ; 0 \le x < 1 \\ 1 & ; x \ge 1 \end{cases}$$

$$\left.\begin{matrix} h(-1^-) = -1+2=1 \\ h(-1^+) = 1 \end{matrix}\right\} \text{continuous}$$

$$\left.\begin{matrix} h(0^-) = 1 \\ h(0^+) = 0 \end{matrix}\right\} \text{discontinuous}$$

$$\left.\begin{matrix} h(1^-) = 1 \\ h(1^+) = 1 \end{matrix}\right\} \text{continuous}$$

$h'(-1^-) = 1$
$h'(-1^+) = 0$
$h'(1^-) = 1$
$h'(1^+) = 0$
discontinuous at only $x = 0$
and non differentiable at $x = -1, 0$ and 1

9. The 1011th term from end in the expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$ is equal to 1024 times of 1011th term from beginning. Then the value of x is :

(1) 5/16 (2) 16/5 (3) 8 (4) 5/8

Ans. (1)

Sol. $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$

T_{1011} from end $= 1024\ T_{1011}^{th}$ term from beginning

$${}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010} = 1024 \times {}^{2022}C_{1010}\left(\frac{4x}{5}\right)^{1012}\left(\frac{-5}{2x}\right)^{1010}$$

$$\left(\frac{5}{2x}\right)^2 = \left(\frac{4x}{5}\right)^2 \times 1024$$

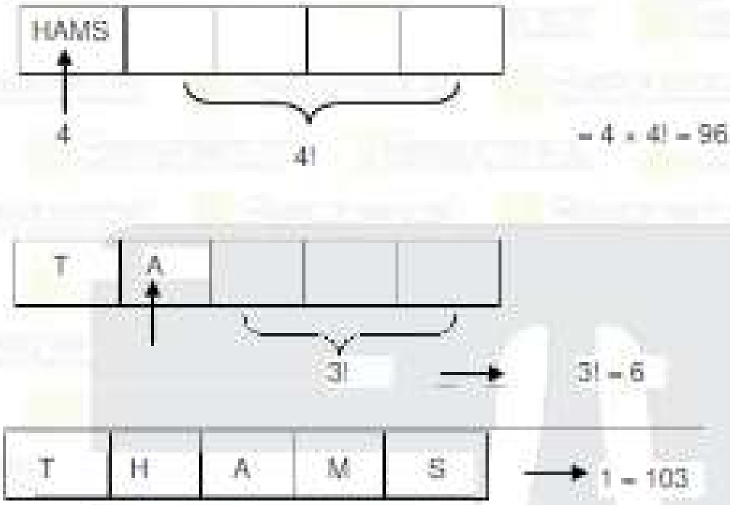
$$x^4 = \frac{25 \times 25}{4 \times 16 \times 1024}$$

$$x = \frac{5}{2^4} = \frac{5}{16}$$

10. The letter of the word MATHS arranged in alphabetic order then the position of 'THAMS' is :

Ans. (103)

Sol.



11. Domain of $f(x) = \dfrac{1}{\sqrt{[x]^2 - 3[x] - 10}}$ Where [.] denotes greatest integer function

(1) $(-\infty, -2) \cup [5, \infty)$ (2) $(-\infty, -3) \cup [5, \infty)$

(3) $(-\infty, -2) \cup [6, \infty)$ (4) $(-\infty, -2)] \cup 6, \infty)$

Ans. (3)

Sol. Let [x] = t

$t^2 - 3t - 10 > 0$

$(t-5)(t+2) > 0$

$t < -2$ or $t > 5$

$[x] < -2$ or $[x] > 5$

$x \in (-\infty, -2) \cup x \in [6, \infty)$

12. Let $S = 1 + \dfrac{4}{k} + \dfrac{13}{k^2} + \dfrac{40}{k^3} + \ldots\ldots = \dfrac{1}{8}$ Then the value of k is where $k > 0$

Ans. (5)

Sol. $S = 1 + \dfrac{4}{k} + \dfrac{13}{k^2} + \dfrac{40}{k^3} + \ldots\ldots$

$\dfrac{S}{k} = \dfrac{1}{k} + \dfrac{4}{k^2} + \dfrac{13}{k^3} + \ldots\ldots$

$\left(\dfrac{k-1}{k}\right) S = 1 + \dfrac{3}{k} + \dfrac{9}{k^2} + \dfrac{27}{k^3} + \ldots\ldots$

$S = \left(\dfrac{1}{1 - \dfrac{3}{k}}\right) \times \dfrac{k}{k-1}$

$S = \dfrac{k^2}{(k-3)(k-1)} = \dfrac{1}{8} \Rightarrow k = \dfrac{3}{7}, -1$

13. If coefficient of any three consecutive terms of expansion $(1+x)^{n+2}$ are in the ratios 1 : 3 : 5 Then the sum of coefficient of these terms is

Ans. (63)

Sol. Coefficients are $^{n+2}C_{r-1}$, $^{n+2}C_r$, $^{n+2}C_{r+1}$

Now $\frac{^{n+2}C_{r-1}}{^{n+2}C_r} = \frac{1}{3} \Rightarrow n+3 = 4r$ ... (1)

$\frac{^{n+2}C_r}{^{n+2}C_{r+1}} = \frac{3}{5} \Rightarrow 3n+1 = 8r$ ... (2)

By (1) & (2) n = 5, r = 2

Now coefficients are $^7C_1$, $^7C_2$, $^7C_3$

sum = 63 Ans.

14. For differential equation $\frac{dy}{dx} + \frac{5}{x(1+x^5)}y = \frac{(1+x^5)^2}{x^7}$. If solution is y(x) and y(1) = 2. Then y(2) is

(1) $\frac{698}{127}$ (2) $\frac{693}{128}$ (3) $\frac{259}{128}$ (4) $\frac{157}{128}$

Ans. (2)

Sol. $I.F = e^{\int \frac{5}{x(1+x^5)}dx}$

$e^{\int \frac{5}{x^6\left(\frac{1}{x^5}+1\right)}dx}$

Put $\frac{1}{x^5} + 1 = t$

$\frac{-5}{x^6}dx = dt$

$e^{\int \frac{-dt}{t}} = e^{-\ln t} = \frac{1}{t}$

$= \frac{x^5}{1+x^5}$

Solution is $y\left(\frac{x^5}{1+n^5}\right) = \int \frac{x^5}{1+x^5} \cdot \frac{(1+x^5)^2}{x^7}dx + c$

$y\left(\frac{x^5}{1+x^5}\right) = \frac{-1}{x} + \frac{x^4}{4} + c$

Now y(1) = 2 then $\frac{2}{2} = \frac{-1}{1} + \frac{1}{4} + c$

$C = 2 - \frac{1}{4} = \frac{7}{4}$

Now $y(2) = \left(\frac{\frac{-1}{2} + 4 + \frac{7}{4}}{1}\right)\frac{33}{32} = \frac{693}{128}$

15. If circle with centre (2,0) and maximum radius r is inscribed in ellipse $x^2 + 4y^2 = 36$ then the value of $12r^2$ is

Ans. **(92)**

Sol. Consider point P $(6\cos\theta, 3\sin\theta)$ on the ellipse $x^2 + 4y^2 = 36$

Now Distance between C(2,0) and P$(6\cos\theta, 3\sin\theta)$ is r

$\Rightarrow r^2 = (6\cos\theta - 2)^2 + (3\sin\theta)^2$

$r^2 = 27\cos^2\theta - 24\cos\theta + 13$

$\because$ Circle touches ellipse so $\cos\theta = \frac{4}{9}$

$r^2 = 27 \times \frac{16}{81} - 24 \times \frac{4}{9} + 13 \Rightarrow 12r^2 = 92$

16. If $e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1 = 0$ then number of solutions

Ans. **(2)**

Sol. $e^{4x} - e^{2x} - 3 - \frac{1}{e^{2x}} + \frac{1}{e^{4x}} = 0$

Let $e^{2x} + \frac{1}{e^{2x}} = t$ then

$t^2 + \frac{1}{t^2} - \left(t + \frac{1}{t}\right) = 3$

$\left(t + \frac{1}{t}\right)^2 - 2 - \left(t + \frac{1}{t}\right) = 3$

Let $u = t + \frac{1}{t}$

Then $u^2 - u - 5 = 0$

Now, $u = \frac{1 + \sqrt{21}}{2}$

$t + \frac{1}{t} = \frac{1 + \sqrt{21}}{2}$

$2t^2 - \left(1 + \sqrt{21}\right)t + 2 = 0$

Here t = 2 values

So, number of solutions = 2

17. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar vectors then the value of $[\vec{a}\ \vec{b}\ \vec{c}]$ is

(1) $[\vec{b}\ \vec{d}\ \vec{c}] + [\vec{a}\ \vec{d}\ \vec{b}] + [\vec{a}\ \vec{d}\ \vec{c}]$

(2) $[\vec{b}\ \vec{c}\ \vec{d}] + [\vec{a}\ \vec{b}\ \vec{d}] + [\vec{a}\ \vec{d}\ \vec{c}]$

(3) $[\vec{b}\ \vec{d}\ \vec{c}] + [\vec{a}\ \vec{b}\ \vec{b}] + [\vec{a}\ \vec{d}\ \vec{c}]$

(4) $[\vec{a}\ \vec{c}\ \vec{d}] + [\vec{a}\ \vec{b}\ \vec{d}] + [\vec{a}\ \vec{d}\ \vec{c}]$

Ans. **(2)**

Sol. $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar then

$\vec{b} - \vec{a},\ \vec{c} - \vec{b},\ \vec{d} - \vec{c}$ are also coplanar

$[\vec{b} - \vec{a}\ \ \vec{c} - \vec{b}\ \ \vec{d} - \vec{c}] = 0$

$[\vec{b}\ \vec{c}\ \vec{d}] - [\vec{a}\ \vec{c}\ \vec{d}] - [\vec{a}\ \vec{b}\ \vec{d}] - [\vec{a}\ \vec{b}\ \vec{c}] = 0$

$[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{d}] + [\vec{a}\ \vec{b}\ \vec{d}] + [\vec{a}\ \vec{d}\ \vec{c}]$

18. $f(x) = \begin{cases} e^{\min(x^2, x)} & x \in (0, 1) \\ e^{[x - \ln x]} & x \in [1, 2) \end{cases}$ then find $\int_0^2 x f(x) dx$ where [·] denote greatest integer function.

(1) $2e - \frac{1}{2}$ (2) $2e + \frac{1}{2}$ (3) $e - \frac{1}{2}$ (4) $3e - \frac{1}{2}$

Ans. **(1)**

Sol. $f(x) = \begin{cases} e^{x^2} & x \in (0, 1) \\ e & x \in [1, 2) \end{cases}$

Now $\int_0^2 x f(x) dx = \int_0^1 x e^{x^2} dx + \int_1^2 x . e \, dx = 2e - \frac{1}{2}$