PART : MATHEMATICS

1. The sum of first fifty coefficients in the expansion of $(1-x)^{100}$ is.

(1) $^{100}c_{50}$ (2) $-^{100}c_{50}$ (3) $\frac{1}{2}{}^{100}c_{50}$ (4) $-\frac{1}{2}{}^{100}c_{50}$

Ans. **(4)**

Sol. $(1-x)^{100} = \sum_{r=0}^{100} {}^{100}c_r(-1)^r x^r$

sum of 1^{st} fifty coefficients

$$= {}^{100}c_0 - {}^{100}c_1 + {}^{100}c_2 - {}^{100}c_3 + \ldots\ldots - {}^{100}c_{49}$$

$$= \frac{1}{2}\left[\left({}^{100}c_0 - {}^{100}c_1 + {}^{100}c_2 - {}^{100}c_3 + \ldots\ldots + {}^{100}c_{100}\right) - {}^{100}C_{50}\right]$$

$$= \frac{1}{2}\left[0 - {}^{100}c_{50}\right] = -\frac{1}{2}{}^{100}c_{50}$$

2. All 5 digits numbers formed using digits 0, 1, 3, 5, 7 and 9, which are greater then 40000 and divisible by 5, then find number of such numbers. (Repetition of digits are not allowed)

(1) 60 (2) 90 (3) 100 (4) 120

Ans. **(4)**

Sol.

$10,000^{th}$	1000^{th}	100^{th}	10^{th}	unit		
3	4	3	2	0	=	72
2	4	3	2	5	=	48

Total = 120

3. If $a\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$, $\hat{i}+\hat{j}+c\hat{k}$ are there coplanar vectors and $a,b,c, \neq 1$ then value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is

(1) 2 (2) 1 (3) 0 (4) –1

Ans. **(2)**

Sol. $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$$\begin{vmatrix} \frac{a}{1-a} & \frac{1}{1-a} & \frac{1}{1-a} \\ \frac{1}{1-b} & \frac{b}{1-b} & \frac{1}{1-b} \\ \frac{1}{1-c} & \frac{1}{1-c} & \frac{c}{1-c} \end{vmatrix} = 0$$

$$\begin{vmatrix} \frac{1}{1-a}-1 & \frac{1}{1-a} & \frac{1}{1-a} \\ \frac{1}{1-b} & \frac{b}{1-b}-1 & \frac{1}{1-b} \\ \frac{1}{1-c} & \frac{1}{1-c} & \frac{1}{1-c}-1 \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} \left(\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}-1\right) & \left(\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}-1\right) & \left(\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}-1\right) \\ \frac{1}{1-b} & \frac{b}{1-b}-1 & \frac{1}{1-b} \\ \frac{1}{1-c} & \frac{1}{1-c} & \frac{1}{1-c}-1 \end{vmatrix}=0$$

$$\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$$

4. Let y = f(x) satisfy differential equation. y(x–y) dx = (1+x²) dy such that y(0) = 1 then y(1) =

(1) $\frac{\sqrt{2}}{1+\ell n\left(1+\sqrt{2}\right)}$ (2) $\frac{2}{1+\ell n\left(1+\sqrt{2}\right)}$ (3) $\frac{\sqrt{2}}{1+\ell n\left(\sqrt{2}-1\right)}$ (4) $\frac{2}{1+\ell n\left(\sqrt{2}-1\right)}$

Ans. (1)

Sol. $y(x-y)\,dx = (1+x^2)\,dy$

$$\frac{y(x-y)}{1+x^2}=\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{x}{1+x^2}y-\frac{y^2}{1+x^2}$$

$$\frac{dy}{dx}-\frac{x}{1+x^2}y=\frac{-y^2}{1+x^2}$$

$$\frac{-1}{y^2}\frac{dy}{dx}+\frac{x}{1+x^2}\frac{1}{y}=\frac{1}{1+x^2}$$

Let $\frac{1}{y}=t$

$$\therefore \frac{-1}{y^2}\frac{dy}{dx}=\frac{dt}{dx}$$

$$\therefore \frac{dt}{dx}+\frac{x}{1+x^2}t=\frac{1}{1+x^2} \text{ linear in t}$$

I. F. = $e^{\int\frac{x}{1+x^2}dx} = e^{\frac{1}{2}\ell n\left(1+x^2\right)} = \sqrt{1+x^2}$

∴ solution is

$$t\sqrt{1+x^2}=\int\frac{\sqrt{1+x^2}}{1+x^2}dx+c$$

$$\frac{1}{y}\sqrt{1+x^2}=\int\frac{1}{\sqrt{1+x^2}}dx+c$$

$$\frac{1}{y}\sqrt{1+x^2}=\ell n\left(x+\sqrt{x^2+1}\right)+c$$

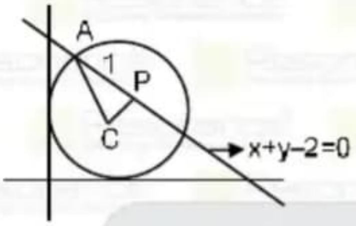
y (0) = 1 ⇒ 1 = 0 + c ⇒ c = 1

So $y(1)=\frac{\sqrt{2}}{1+\ell n\left(1+\sqrt{2}\right)}$

5. Two circles having radius r_1 and r_2 touches both coordinate axis line x + y = 2 makes intercepts. 2 on both the circles, then find the value of $r_1^2 + r_2^2 - r_1r_2$

Ans. (7)

Sol.



Let equation of circle is

$(x - r)^2 + (y - r)^2 = r^2$

CP = $\frac{|r + r - 2|}{\sqrt{2}} = \sqrt{r^2 - 1}$

$\left(\sqrt{2}(r-1)\right)^2 = r^2 - 1$

$r^2 - 4r + 3 = 0$

$r_1 = 1,\ r_2 = 3$

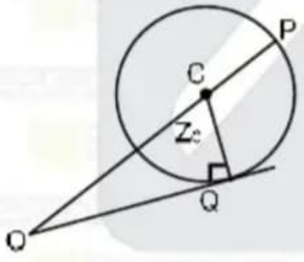
$r_1^2 + r_2^2 - r_1r_2 = 1 + 9 - 3 = 7$

6. Let $Z_0 = \frac{1+3i}{2}$, $Z_1 = 1 + i$ and Z_2 satisfy the relation $|Z_0 - Z_1|.|Z_0 - Z_2| = 1$.If argument of Z_2 is least then value of $4|Z_2|^2$ is equal to.

Ans. (2)

Sol. $|Z_0 - Z_2| = \frac{1}{|Z_0 - Z_1|} = \frac{1}{\left|\frac{i-1}{2}\right|} = \frac{2}{\sqrt{2}}$

$|Z_0 - Z_2| = \sqrt{2}$



⇒ Z_2 lies on circle $|Z - Z_0| = \sqrt{2}$

SO $OQ^2 = OC^2 - CP^2 = \frac{10}{4} - 2 = \frac{5}{2} - 2 = \frac{1}{2}$

$4|Z_2|^2 = 2$

7. Let A = {1, 2, 3} and a relation R on A is such that R = {(1, 2), (2, 3)} then minimum number of elements must be added in R for which R is reflexive and transitive but not symmetric is.

Ans. (04)

Sol. R is reflexive $\Rightarrow$ R have (1, 1), (2, 2), (3, 3)

R is transitive

$\because (1, 2), (2, 3) \in R \qquad \therefore (1, 3) \in R$

$\therefore$ R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}

Clearly R is reflexive & transitive but not symmetric

$\therefore$ minimum number of elements = 04

8. Let $\frac{1}{n+1}\,{}^nC_n + \frac{1}{n}\,{}^nC_{n-1} + ----+\frac{1}{2}\,{}^nC_1 + \frac{1}{1}\,{}^nC_0 = \frac{1023}{10}$ find n.

(where ${}^nC_r = \frac{n!}{r!(n-r)!}$)

Ans. (9)

Sol. since $\frac{1}{1}\,{}^nC_0 + \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 + ----+\frac{1}{n}\,{}^nC_{n-1} + \frac{1}{n+1}\,{}^nC_n = \frac{1023}{10}$

$\Rightarrow \sum_{r=0}^{n} \frac{1}{r+1}\,{}^nC_r = \frac{1023}{10} \qquad (\therefore {}^{n+1}C_{r+1} = \frac{n+1}{r+1}\,{}^nC_r)$

$\Rightarrow \sum_{r=0}^{n} \frac{1}{n+1}\,{}^{n+1}C_{r+1} = \frac{1023}{10}$

$\Rightarrow \frac{1}{n+1}\left[{}^{n+1}C_1 + {}^{n+1}C_2 + ...+ {}^{n+1}C_{n+1}\right] = \frac{1023}{10}$

$\Rightarrow \frac{2^{n+1}-1}{n+1} = \frac{1023}{10} = \frac{2^{10}-1}{10}$

$\Rightarrow n + 1 = 10$

$\Rightarrow n = 9$

9. $\int_{-0.15}^{0.15} |100x^2 - 1|\,dx = \frac{k}{3000}$ then K =

Ans. (575)

Sol. $I = \int_{-0.15}^{0.15} |100x^2 - 1|\,dx = 2\int_{0}^{0.15} |100x^2 - 1|\,dx$

$= 2\left(\int_0^{0.1} (1-100x^2)\,dx + \int_{0.1}^{0.15} (100x^2-1)\,dx\right)$

$= 2\left(\left(x - \frac{100}{3}.x^3\right)_0^{0.1} + \left(\frac{100}{3}.x^3 - x\right)_{0.1}^{0.15}\right)$

$= 4\left(0.1 - \frac{1}{30}\right) + 2\left(\frac{100}{3}(0.15)^3 - 0.15\right)$

$= \frac{8}{30} + 2\,(0.1125 - 0.15)$

$= \frac{8}{30} - 0.075 = \frac{8-2.25}{30} = \frac{5.75}{30} = \frac{575}{3000}$

10. Let $I(x) = \int \sqrt{\frac{x+7}{x}}\, dx$ such that $I(9) = 12 + \ell n7$. If $I(1) = \alpha + 7\ell n\,(1 + 2\sqrt{2}) - 6\ell n7$ then find α^4

Ans. (64)

Sol. $I(x) = \int \sqrt{\frac{x+7}{x}}\, dx$

$= \int \frac{\sqrt{(\sqrt{x})^2 + (\sqrt{7})^2}}{\sqrt{x}}\, dx$

Let $\sqrt{x} = t \therefore \frac{1}{2\sqrt{x}}\, dx = dt$

$= 2\int \sqrt{t^2 + (\sqrt{7})^2}\, dt = 2\left(\frac{t}{2}\sqrt{t^2+7} + \frac{7}{2}\ell n\left(t + \sqrt{t^2+7}\right)\right) + c$

$I(x) = \sqrt{x}\ \sqrt{x+7} + 7\,\ell n\left(\sqrt{x} + \sqrt{x+7}\right) + c$

$I(9) = 12 + \ell n7 = 12 + 7\ell n\,(3 + 4) + c$

$\Rightarrow \ell n7 - 7\ell n7 = c$

$c = -6\,\ell n7$

$\therefore I(x) = \sqrt{x}\ \sqrt{x+7} + 7\ell n\left(\sqrt{x} + \sqrt{x+7}\right) - 6\,\ell n7$

$x = 1$

$I(1) = \sqrt{8} + 7\ell n\,(1 + \sqrt{8}) - 6\ell n7 = 7\ell n\,(1 + 2\sqrt{2}) - 6\ell n7 + \alpha$

$\alpha = \sqrt{8}$

$\alpha^4 = 64$

11. The area bounded by $y = x^3$ and tangent to $y = x^3$ at $(-1, -1)$ is.

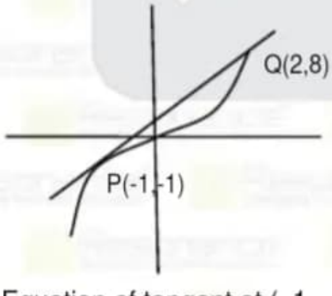
(1) $\frac{51}{2}$ (2) $\frac{51}{4}$ (3) $\frac{49}{4}$ (4) $\frac{27}{4}$

Ans. (4)

Sol. Given $y = x^3$ .... (1)

$\Rightarrow \frac{dy}{dx} = 3x^2$

$\left(\frac{dy}{dx}\right)_{(-1,-1)} = 3$



Equation of tangent at $(-1, -1)$

$(y + 1) = 3\,(x + 1)$

$y = 3x + 2$ .... (2)

Solving (1) & (2)

$x^3 = 3x + 2$

$x^3 - 3x - 2 = 0$ → roots -1, -1, α

$-2 + \alpha = 0$

$\alpha = 2$

$Q \equiv (2, 8)$

Required area $= \int_{-1}^{2} \left(3x + 2 - x^3\right) dx$

$= \frac{3}{2}(4-1) + 2(2+1) - \frac{1}{4}(16-1)$

$= \frac{9}{2} + 6 - \frac{15}{4} = \frac{18+24-15}{4}$

$= \frac{27}{4}$

12. If $a_1 + a_2 + a_3 + \ldots\ldots\ldots + a_n = \frac{n^2+3n}{(n+1)(n+2)}$ and $28\sum_{k=1}^{10}\frac{1}{a_k} = P_1 P_2 P_3 \ldots\ldots\ldots P_m$, where P_i, i = 1, 2, 3, ... m are prime number then value of 'm' is–

(1) 2 (2) 4 (3) 6 (4) 8

Ans. **(3)**

Sol. $a_n = \frac{n^2+3n}{(n+1)(n+2)} - \frac{(n-1)^2+3(n-1)}{n(n+1)}$

$a_n = \frac{n^3+3n^2-(n+2)(n^2+n-2)}{n(n+1)(n+2)}$

$a_n = \frac{4}{n(n+1)(n+2)}$

$\sum_{k=1}^{10}\frac{1}{a_k} = \frac{1}{4}\sum_{k=1}^{10}(k(k+1)(k+2))$

$\sum_{k=1}^{10}\frac{1}{a_k} = \frac{1}{16}\sum_{k=1}^{10}(k(k+1)(k+2)(k+3)-(k-1)k(k+1)(k+2))$

$\sum_{k=1}^{10}\frac{1}{a_k} = \frac{1}{16}(10\cdot 11\cdot 12\cdot 13)$

$28\sum_{k=1}^{10}\frac{1}{a_k} = \frac{28}{16}\cdot 2\cdot 5\cdot 11\cdot 3\cdot 4\cdot 13$

$28\sum_{k=1}^{10}\frac{1}{a_k} = 2\cdot 3\cdot 5\cdot 7\cdot 11\cdot 13$

$m = 6$

13. If $A_k = \begin{vmatrix} 1 & 2k-1 & 2k \\ n & n^2+n+1 & n^2 \\ n & n^2+n & n^2+n \end{vmatrix}$ and $\sum_{k=1}^{n} A_k = 64$ find n

Ans. (64)

Sol. $\sum_{k=1}^{n} A_k = \begin{vmatrix} \sum_{k=1}^{n} 1 & \sum_{k=1}^{n} (2k-1) & \sum_{k=1}^{n} 2k \\ n & n^2+n+1 & n^2 \\ n & n^2+n & n^2+n \end{vmatrix}$

$= \begin{vmatrix} n & n^2 & n^2+n \\ n & n^2+n+1 & n^2 \\ n & n^2+n & n^2+n \end{vmatrix}$

$R_1 \to R_1 - R_3$

$R_2 \to R_2 - R_3$

$\begin{vmatrix} 0 & -n & 0 \\ 0 & 1 & -n \\ n & n^2+n & n^2+n \end{vmatrix} = n^3 = 64$

$n = 4$

14. 5 positive number a_1, a_2, a_3, a_4, a_5 are in G.P., their mean & variance are $\frac{31}{10}$ & $\frac{m}{n}$ respectively. The mean of their reciprocals is $\frac{31}{40}$ then m + n =

Ans. (211)

Sol. Let a, ar, ar^2, ar^3, ar^4

gives $\frac{a + ar + ar^2 + ar^3 + ar^4}{5} = \frac{31}{10}$ —— (1)

$\Rightarrow a(1 + r + r^2 + r^3 + r^4) = \frac{31}{2}$ —— (1)

also $\frac{\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} + \frac{1}{ar^4}}{5} = \frac{31}{40}$ —— (2)

$\therefore$ by (1) & (2) $a^2r^4 = 4 \Rightarrow ar^2 = 2$

$ar^2 = 2$ by (1) $r = 2, a = \frac{1}{2}$

$\therefore$ Number are $\frac{1}{2}$, 1, 2, 4, 8

$\sigma^2 = \frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2 = \frac{\frac{1}{4} + 1 + 4 + 16 + 64}{5} - \left(\frac{31}{10}\right)^2$

$= \frac{341}{20} - \frac{961}{100}$

$= \frac{186}{25}$

$\therefore m + n = 211$

15. If a plane $4x - 3y + z = 2$ is rotated by an angle $\frac{\pi}{2}$ at interstation of line of another plane $3x - 4y + 11z = 12$ then distance of P(2, 3, 4) from new plane is.

(1) $\frac{643}{\sqrt{66846}}$ (2) $\frac{641}{\sqrt{66848}}$ (3) $\frac{641}{\sqrt{66846}}$ (4) $\frac{643}{\sqrt{66848}}$

Ans. (3)

Sol. Family of plane is $4x - 3y + z - 2 + \lambda(3x - 4y + 11z - 12) = 0$

$\Rightarrow (4 + 3\lambda)x - (3 + 4\lambda)y + (1 + 11\lambda)z - 2 - 12\lambda = 0$

Plane is $\perp$ to $4x - 3y + z = 2$

$\therefore 4(4 + 3\lambda) + 3(3 + 4\lambda) + 1(1 + 11\lambda) = 0$

$\Rightarrow 16 + 12\lambda + 9 + 12\lambda + 1 + 11\lambda = 0$

$\Rightarrow 35\lambda + 26 = 0 \qquad \Rightarrow \lambda = \frac{-26}{35}$

$\therefore$ Plane is $62x - y - 251z + 242 = 0$

$\therefore$ Distance of P (2, 3, 4) $= \frac{|62 \times 2 - 3 - 251 \times 4 + 242|}{\sqrt{62^2 + 251^2 + 1^2}} = \frac{641}{\sqrt{66846}}$

16. Given A, B, C represents angles of ΔABC. If cosA + 2cosB + cosC = 2 & AB = 3, BC = 7, then cosA – cosC =

(1) $\frac{10}{7}$ (2) $-\frac{10}{7}$ (3) $\frac{7}{10}$ (4) $-\frac{7}{10}$

Ans. (2)

Sol. cosA + 2cosB + cosC = 2, c = 3, a = 7

$\frac{b^2 + c^2 - a^2}{2bc} + \frac{2(c^2 + a^2 - b^2)}{2ac} + \frac{a^2 + b^2 - c^2}{2ab} = 2$

$\frac{b^2 + 9 - 49}{6b} + \frac{2(49 + 9 - b^2)}{2.3.7} + \frac{49 + b^2 - 9}{14b} = 2$

$\frac{b^2 - 40}{6b} + \frac{58 - b^2}{21} + \frac{40 + b^2}{14b} = 2$

$\Rightarrow b = 5$

$\cos A - \cos C = \frac{b^2 + c^2 - a^2}{2bc} - \frac{a^2 + b^2 - c^2}{2ab}$

$= \frac{ab^3 + ac^2 - a^3 - ca^2 - cb^2 + c^3}{2abc}$

$= -\frac{10}{7}$

17. Statement $((p \wedge q) \vee (p \wedge (\sim q)) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$ is equivalent to

(1) Tautology (2) Fallacy (3) p (4)q

Ans. (1)

Sol. Given $((p \wedge q) \vee (p \wedge \sim q)) \vee ((\sim p \wedge q) \vee (\sim p \wedge \sim q))$

$\equiv (p \wedge (q \vee \sim q)) \vee (\sim p \wedge (q \vee \sim q))$

$\equiv (p \wedge t)) \vee (\sim p \wedge t)$

$\equiv p \vee \sim p$

$= t$

18. Let $\vec{a} = \lambda i + j - k$, $\vec{b} = 3i - j + 2k$ and $\vec{c}$ is a vector such that $(\vec{a}+\vec{b}+\vec{c})\times\vec{c} = \vec{o}$ and $\vec{a}\cdot\vec{c} = -17$, $\vec{b}\cdot\vec{c} = -20$. Then $|\vec{c}\times(\lambda i + j + k)|^2 =$ $(\lambda > 0)$

Ans. (46)

Sol. $(\vec{a}+\vec{b}+\vec{c})\times\vec{c} = 0$

$\Rightarrow (\vec{a}+\vec{b})\times\vec{c} = \vec{0}$

$\vec{c} = \alpha(\vec{a}+\vec{b})$, α is scalar

Now $\vec{a}\cdot\vec{c} = -17$

$\therefore (\lambda i + j - k) . \alpha ((\lambda + 3) i + k) = -17$

$(\lambda(\lambda + 3) - 1) \alpha = -17$-----------(1)

also $\vec{b}\cdot\vec{c} = -20 \Rightarrow \alpha ((\lambda + 3) i + k).(3i - j + 2k) = -20$

$\Rightarrow \quad \alpha(3 (\lambda + 3) + 2) = -20$ __________(2)

by (1) and (2) $\dfrac{\lambda^2 + 3\lambda - 1}{3\lambda + 11} = \dfrac{17}{20}$

$\Rightarrow \quad 20\lambda^2 + 60 \lambda - 20 = 51 \lambda + 187$

$\Rightarrow \quad 20\lambda^2 + 9\lambda - 207 = 0$

$\Rightarrow \quad \lambda = 3$

$\therefore \quad$ by (2) $\alpha (20) = -20 \Rightarrow \alpha = -1$

$\vec{c} = -(6i + k)$

Now $\vec{c} \times (\lambda i + j + k) = \begin{vmatrix} i & j & k \\ -6 & 0 & -1 \\ 3 & 1 & 1 \end{vmatrix}$

$= i (0+1) - j (-6 + 3) + k (-6 - 0) = i + 3j - 6k$

$\therefore \quad |\vec{c}\times(\lambda i + j + k)|^2 = 1 + 9 + 36 = 46$

19. Let f(x) = |[x]| + $\sqrt{x-[x]}$, then number of points of discontinuity of f(x) in [-2, 1], is ([$\cdot$] is denoted greatest integer function)

(1) 2 (2) 3 (3) 4 (4) 0

Ans. (1)

Sol. f(x) = |[x]| + $\sqrt{x-[x]}$, $x \in [-2, 1]$

$$f(x) = \begin{cases} 2+\sqrt{x+2} & -2 \le x < -1 \\ 1+\sqrt{x+1} & -1 \le x < 0 \\ \sqrt{x} & 0 \le x < 1 \\ 1 & x = 1 \end{cases}$$

f(x) is dc at x = –1, 0,

Clearly number of discontinuity points = 2

20. Let $A = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}$ is a matrix and $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} A \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$ then $\sum_{n=1}^{50} B^n =$

(1) $\begin{bmatrix} 75 & -25 \\ -25 & 25 \end{bmatrix}$ (2) $\begin{bmatrix} 75 & 25 \\ -25 & 25 \end{bmatrix}$ (3) $\begin{bmatrix} -75 & -25 \\ -25 & -25 \end{bmatrix}$ (4) $\begin{bmatrix} 75 & -25 \\ -25 & -25 \end{bmatrix}$

Ans. (2)

Sol. Let $C = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$ and $D = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

$DC = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} -1+2 & -2+2 \\ 1-1 & 2-1 \end{bmatrix}$

$DC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$\because B = (CAD)$

$B^n = (CAD)^n = CA\,(DC)\cdot A(DC)\cdot A(DC)\cdot \ldots\ldots\ldots\ldots A\cdot D$

$B^n = C\cdot A^n\cdot D$

$\because A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{51} \\ 0 & 0 \end{bmatrix} = I + E$ $\left[E = \begin{bmatrix} 0 & \frac{1}{51} \\ 0 & 0 \end{bmatrix}\right]$

$E^2 = \begin{bmatrix} 0 & \frac{1}{51} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & \frac{1}{51} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

$I\cdot E = E$

$A^n = (I+E)^n = I^n + {}^nC_1\, E\cdot I + \ldots\ldots\ldots\ldots$

$A^n = (I+E)^n = I + nE = \begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}$

$B^n = C\cdot A^n\cdot D = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

$B^n = C\cdot A^n\cdot D = \begin{bmatrix} 1+\frac{n}{51} & \frac{n}{51} \\ \frac{-n}{51} & 1-\frac{n}{51} \end{bmatrix}$

$\sum_{n=1}^{50} B^n = \begin{bmatrix} 50+\frac{50\cdot 51}{2\cdot 51} & \frac{50\cdot 51}{2\cdot 51} \\ \frac{-50\cdot 51}{2\cdot 51} & 50-\frac{50\cdot 51}{2\cdot 51} \end{bmatrix}$

$\sum_{n=1}^{50} B^n = \begin{bmatrix} 50+25 & 25 \\ -25 & 50-25 \end{bmatrix}$

$\sum_{n=1}^{50} B^n = \begin{bmatrix} 75 & 25 \\ -25 & 25 \end{bmatrix}$