JEE-Mains-13-04-2023 [Memory Based] [Evening Shift]

Physics

Question: How a charge 10μC is divided is such that force between those parts is maximum?
Options:
(a) 4μC, 6μC
(b) 5μC, 5μC
(c) 7μC, 3μC
(d) 9μC, 1μC
Answer: (b)
Solution:

$$F' = \frac{k}{r^2}(Q - 2x) = 0$$

$$F = \frac{kx[Q-x]}{r^2} = \frac{k}{r^2}\left[Qz - x^2\right]$$

$$x = \frac{Q}{2}$$

Question: Two identical trains cross each other moving on parallel tracks, opposite in direction. Speed of one of the train is 90 km/hr and second train has a speed of 57 km/hr If it takes 8 s for two trains to cross each other then length of trains is equal to
Options:
(a) 100 m
(b) 200 m
(c) 160 m
(d) 400 m
Answer: (c)
Solution:

$$t = \frac{2l}{40}$$

$$2l = 40 \times 8 = 40 \times 4 = 160 \text{ m}$$

Question: A bi-convex lens of focal length 10 cm is cut perpendicularly to principle axis. Find power (in D) of new lens.
Answer: 5.00
Solution:

$$P = 5D$$

Question: A particle is performing S.H.M whose distance from mean position varies as x = Asin(ωt). Find the position of particle from mean position, where kinetic energy and potential energy is equal.
Options:
(a) A/2

(b) $\frac{A}{\sqrt{2}}$

(c) $\frac{A}{2\sqrt{2}}$

(d) $\frac{A}{4}$

Answer: (b)

Solution:

$\frac{1}{2}m\omega^2\left(A^2-x^2\right)=\frac{1}{2}kx^2$

$x=\frac{A}{\sqrt{2}}$

Question: An electron is moving along positive x direction in xy plane, magnetic field points in negative z direction, then the force due to magnetic field on electron points in the direction

Options:

(a) $\hat{j}$

(b) $-\hat{j}$

(c) $\hat{k}$

(d) $-\hat{k}$

Answer: (b)

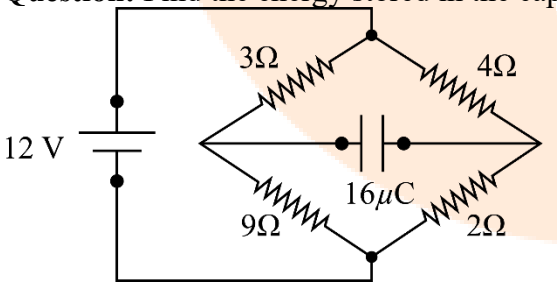
Solution:

$\vec{F}=-e\vec{v}\times\vec{B}$

$=-\hat{i}\times-\hat{k}$

$=-\hat{j}$

Question: Find the energy stored in the capacitor in the given circuit.



Options:

(a) 0.2 mJ

(b) 0.4 mJ

(c) 0.6 mJ

(d) 0.8 mJ

Answer: (a)

Solution:

$$V = \frac{2}{6} \times 12 = 4V$$

$$U = \frac{1}{2} \times 16\mu C \times 5^2$$

$$V = \frac{9}{12} \times 12 = 9V$$

Question: S = (2.5)t² Find instant velocity at t=5 sec
Options:
(a) 10 m/s
(b) 15 m/s
(c) 20 m/s
(d) 25 m/s
Answer: (d)
Solution:

$$x = 2.5t^2$$

$$V = 2.5 \times [2t]$$

$$V = 5t = 5 \times 5$$

$$V = 25 \text{ ms}^{-1}$$

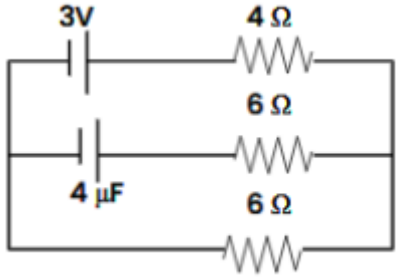
Question: If E is energy to accelerate a car from 0 to u m/s what is the energy required to accelerate car from u to 2u m/s
Options:
(a) E
(b) 2E
(c) 3E
(d) 4E
Answer: (c)
Solution:

$$W = \Delta K = \frac{1}{2}mu^2 - 0$$

$$W' = \frac{1}{2}m(2u)^2 - \frac{1}{2}m(u)^2$$

$$W' = 3W$$

Question: Find charge on 4 μF capacitor



Options:
(a) 7.2 μC

(b) 8.2 μC
(c) 9.2 μC
(d) 10.0 μC
Answer: (a)
Solution:

$$I = \frac{V}{R_{net}} = \frac{3}{10} 0.3A$$

$$V_{cap} = C\Delta V = 4 \times 1.8 = 7.6\mu C$$

Question: A planet has density of half of earth and radius 1.5 times that of earth find the acceleration due to gravity on planet to that on Earth
Options:
(a) 2 : 1
(b) 1 : 2
(c) 4 : 3
(d) 3 : 4
Answer: (d)
Solution:

$$g = \frac{Gm}{R^2}$$

$$g = \frac{4\pi}{3} G\rho R$$

$$g' \propto \frac{1}{2} \times \frac{3}{2} \propto \frac{3}{4}$$

$$g' = \frac{3}{4} g$$

Question: S1: A planet has mass M & radius R, if M/R ratio is increased, Escape velocity also increases.
S2 : escape velocity is independent of R.
Options:
(a) S1: True, S2: False
(b) S1: False, S2: True
(c) S1: True, S2: True
(d) S1: False, S2: False
Answer: (a)
Solution:

$$V_e = \sqrt{\frac{2GM}{R}}$$

Question: What should be the minimum size of antenna required for successful transmission of wave having wavelength λ?
Options:
(a) 2 λ
(b) λ/4
(c) λ/2
(d) λ

Answer: (b)

Solution: $H \propto \frac{\lambda}{y}$

Question: How a charge 10 μC is divided is such that force between those parts is maximum?

Options:

(a) 4μC, 6 μC
(b) 5μC, 5 μC
(c) 7μC, 3 μC
(d) 9μC, 1 μC

Answer: (b)

Solution:

$$F' = \frac{k}{r^2}(Q - 2x)$$

$$F = \frac{kx[Q - x]}{r^2} = \frac{k}{r^2}\left[Qx - x^2\right]$$

$$x = \frac{Q}{2}$$

Question: An electron is moving along positive x direction in xy plane, magnetic field points in negative z direction, then

1. electron experiences force in -y direction
2. Electron moves in a circular path
3. force in experienced in +y direction
4. electron will move in helical path
5. electron moves in straight line

Options:

(a) 1 & 2 are correct
(b) 3 & 4 are correct
(c) None
(d) 5 is correct

Answer: (a)

Solution: $\vec{F} = Q\left[\vec{V} \times \vec{B}\right]$

Question: A particle is performing SHM with amplitude A the location at which the kinetic energy is equal to potential energy

Solution:

$$\frac{1}{2}k\left(A^2 - x^2\right) = \frac{1}{2}kx^2$$

$$x = \frac{A}{\sqrt{2}}$$

Question: Initial pressure P_0, Final pressure if gas is suddenly compressed to $V_0/4$?

Solution:

$$P_0V_0 = P'\left[\frac{V_0}{4}\right]^r$$

$$P' = 4^r P_0$$

Question: $\left(P + \frac{a}{V^2}\right)(V - b) = RT$, $\frac{a}{b}$ Dimension equal to?

Options:

(a) Pressure gradient

(b) Energy

(c) η

(d) $\vec{P}$

Answer: (b)

Solution: $\frac{a}{V^2} = [P]$

$a = PV^2$

$b = V$

Question: In YDSE amplitude ratio is 2:1 find max to min intensity ratio.

Options:

(a) 9:1

(b) 4:1

(c) 16:3

(d) 9:4

Answer: (a)

Solution:

$A_1 = 2$

$A_2 = 1$

$$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{9}{1}$$

Question: A car of mass 200 kg is taking circular turn on circular track of radius 70 m with angular vel 0.2 rad/s find centripetal force

Options:

(a) 250 N

(b) 360 N

(c) 400 N

(d) 560 N

Answer: (d)

Solution:

$F_C = M\omega^2 R$

$F = 200 \times \left(2 \times 10^{-1}\right)^2 \times (70)$

Question: S1 : K_{max} is inversely proportional to frequency.

S2 : If microwave, radio wave & UV waves all incident on metal then K_{max} is maximum from UV rays.

Options:
(a) S1: True, S2: False
(b) S1: False, S2: True
(c) S1: True, S2: True
(d) S1: False, S2: False
Answer: (b)
Solution: $K_{\max} = hf - \phi$

Question: Two rods having same length & cross section are joined find temp at junction.
Answer: 126.00
Solution:

$$I_T = KA\frac{\Delta T}{\Delta X}$$

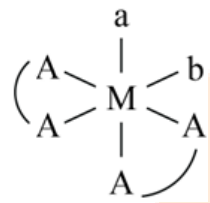
$$\frac{84 \times 4 \times [100 - T]}{L} = \frac{126 \times A \times [T - 0]}{L}$$

$$84(100 - T) = 126T$$

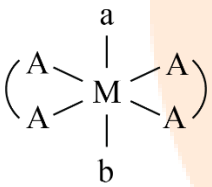
JEE-Mains-13-04-2023 [Memory Based] [Morning Shift]

Chemistry

Question: What is number of stereoisomers of $[Co(ox)_2Cl\ Br]^{3-}$
Options:
(a) 3
(b) 4
(c) 5
(d) 6
Answer: (a)
Solution: $[M(AA)_2\ ab]$



Plane is absent
Optical isomer present



plane present, so no optical isomer.
∴ So total stereoisomer is = 3 for $[Co(ox)_2Cl\ Br]^{3-}$

Question: The orbital angular momentum of an electron in 3s is $xh/2\pi$.
Then value of x is?
Options:
(a) 0
(b) 1
(c) 2
(d) 3
Answer: (a)
Solution:

$$\sqrt{l(l+1)}\frac{h}{2\pi}$$

$$=\sqrt{0\times 1}\frac{h}{2\pi}=0$$

Question: Correct order of enthalpy of formation of NaX, where X = F, Cl, Br, I
Options:
(a) NaI < NaBr < NaCl < NaF
(b) NaI < NaF < NaCl < NaBr
(c) NaBr < NaF < NaCl < NaI
(d) NaI < NaF < NaBr < NaCl
Answer: (a)
Solution: Lattice energy $\propto \frac{\text{change on ion}}{\text{size of ion}}$

NaI < NaBr < NaCl < NaF

Question: Naturally occurring amino acid with one basic functional group.
Options:
(a) Lysine
(b) Arginine
(c) Asparagine
(d) Histidine
Answer: (a)
Solution: Lysine

Question: 1 g of M_2CO_3 is converted to 0.01 mol CO_2 by reacting with excess HCl. Find the molar mass of the carbonate
Options:
(a) 50
(b) 100
(c) 120
(d) 140
Answer: (b)
Solution: $M_2CO_3 + 2HCl \rightarrow CO_2 + 2MCl + H_2O$

Moles of M_2CO_3 $\frac{W}{M.W} = 0.01$

Weight of carbonate = 1 g

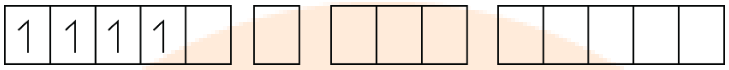
Moles of M_2CO_3 $\frac{1}{M.W} = 0.01$

Molar mass = 100

Question: Number of greenhouse gases are :
Water vapours, ozone, molecular hydrogen, I_2
Options:
(a) 1
(b) 2
(c) 3
(d) 4

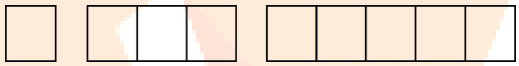
Answer: (b)
Solution: Water vapours, ozone

Question: Which complex shows more attraction towards applied magnetic field**?**
Options:
(a) $[Cr(H_2O)_6]^{+2}$
(b) $[Zn(H_2O)_4]^{+2}$
(c) $[Co(H_2O)_6]^{+3}$
(d) $[Fe(CN)_6]^{-4}$
Answer: (a)
Solution:

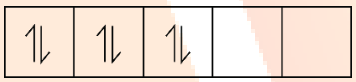
$[Cr(H_2O)_6]^{+2}$ $3d^4$ 

sp^3d^2

4 unpaired electrons

$[Zn(H_2O)_4]^{+2} = 3d^{10}$ 

sp^3 0 unpaired electrons

$[Co(H_2O)_6]^{+3} = 3d^6$ 

d^2sp^3 0 unpaired electrons

$[Fe(CN)_6]^{-4} = 3d^6$ 

d^2sp^3 0 unpaired electrons

Question: Assertion: (1) Para Chlorophenol, (2) Para Fluorophenol (3) Meta Methylphenol Acidity order 1 > 2 > 3
Reason: Fluoride is more electron withdrawing as compared to chlorine
Options:
(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.
(c) Assertion is true, but Reason is false.
(d) Assertion is false, but Reason is true.
Answer: (c)
Solution: Para Chlorophenol > Para Fluorophenol > Meta Methylphenol

Assertion is true but reason is false.

Question: Best method to prepare BeF_2 is
Options:
(a) $Be + F_2$

(b) $BeO + F_2$
(c) $(NH)_3 + F_2$
(d) Thermal decomposition of $(NH_4)_2BeF_4$
Answer: (d)
Solution: Thermal decomposition of $(NH_4)_2BeF_4$ is the best route for the preparation of BeF_2.

Question: Statements I: Ellingham diagram can be made for sulfide, halide and oxides.
Statements II: It shows relation between ΔH vs Temperature.
Options:
(a) Both statements I and II are correct
(b) Both statements I and II incorrect
(c) Statement I is correct and II is incorrect
(d) Statement I is incorrect and II is correct
Answer: (c)
Solution: Statement 1 is correct, and Statement 2 is incorrect.

Question: Borax can be written as $Na_2B_4O_x(OH)_yZ$(water)
The value of x + y + z is
Options:
(a) 12
(b) 13
(c) 15
(d) 17
Answer: (d)
Solution: $Na_2[B_4O_5(OH)_4] \cdot 8H_2O$

x = 5, y = 4, z = 8

x + y + z = 17

Question: Assertion: Isotopes of Hydrogen are chemically similar.
Reason: Isotopes of Hydrogen have different enthalpy of bond dissociation.
Options:
(a) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.
(c) Assertion is true, but Reason is false.
(d) Assertion is false, but Reason is true.
Answer: (b)
Solution: Assertion is correct, and reason is also correct but reason is not correct explanation of Assertion.

Question: NaCl forms BCC structure
If a = 4Å
Find r
Options:
(a) 1.414

(b) 2.324
(c) 3.454
(d) 4.234
Answer: (a)
Solution:

$$r = \frac{a}{2\sqrt{2}}$$

$$r = \frac{4}{2\sqrt{2}}$$

$$r = 1.414$$

Question: Consider the reaction
$Cr_2O_7^{2-} + x\ H^+ + Fe^{2+} \rightarrow yFe^{3+} + 2Cr^{3+} + zH_2O$
Sum of x, y, z?
Options:
(a) 21
(b) 23
(c) 24
(d) 27
Answer: (d)
Solution: The full ionic equation may be obtained by adding the half-reaction for potassium dichromate to the half-reaction for the reducing agent, for e.g.

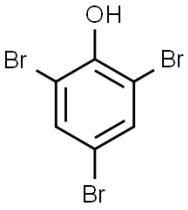
$$Cr_2O_7^{2-} + 14\ H^+ + 6\ Fe^{2+} \rightarrow 2\ Cr^{3+} + 6\ Fe^{3+} + 7\ H_2O$$

Question: The initial pressure & volume of an ideal gas are P_o & V_o. The final pressure of gas when gas is suddenly compressed to volume $v_0/4$.
Options:
(a) $P_0\ (4)^{1/4}$
(b) $4P_0$
(c) P_0
(d) $P_0\ (4)^4$
Answer: (b)
Solution: $P_1V_1 = P_2V_2$

$$P_1V_1 = P_2\frac{V_1}{4}$$

$$P_2 = 4P_1$$

Question: Consider the following reaction and identify the reactant (A).



$$(A) \xrightarrow{Br_2/CS_2} (B) \xrightarrow[(0-5^{\circ}C)]{NaNO_2+HCl} C \xrightarrow{H_3PO_2}$$

Options:

(a) Aniline

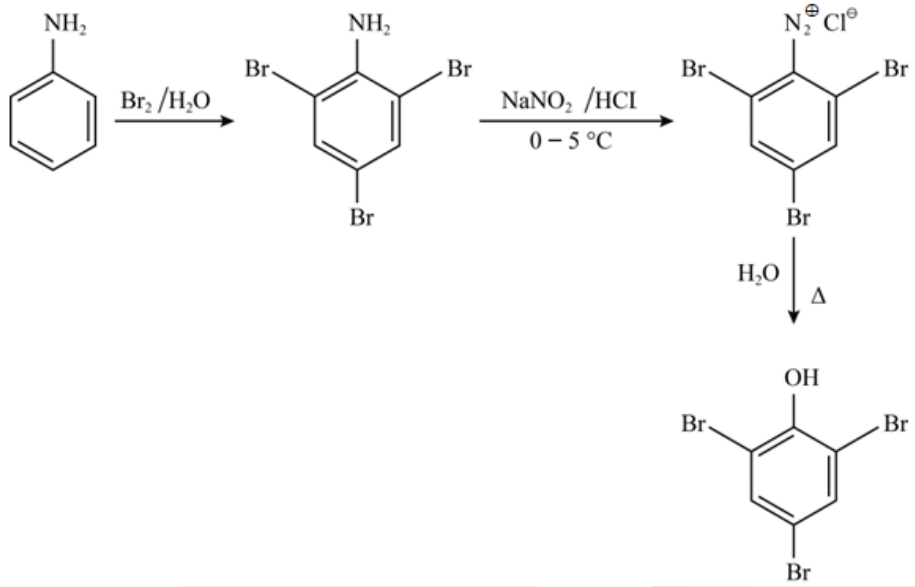
(b) Phenol

(c) Salicylic acid

(d) Acetanilide

Answer: (a)

Solution:



JEE-Mains-13-04-2023 [Memory Based] [Evening Shift]

Mathematics

Question: Number of solutions of $\sin^{-1}x = 2\tan^{-1}x$ in the interval $(-1,1]$ is _____.

Answer: 2.00

Solution:

$\sin^{-1}x = 2\tan^{-1}x$

$\sin^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$\Rightarrow x = \frac{2x}{1+x^2}$

$x\left(1+x^2\right) = 2x$

$x\left(1+x^2\right) - 2x = 0$

$x\left(1+x^2-2\right) = 0$

$x\left(x^2-1\right) = 0$

$\Rightarrow x = 0 \quad \text{or} \quad x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$

$\Rightarrow x = 0 \quad \text{or} \quad x = 1, -1$

No. of solutions = 2

$\because\ x = -1 \notin (-1,1]$

Question: Coefficient of x^5 in $\left(2x^3 - \frac{1}{3x^2}\right)^5$ is equal to

Answer: $\frac{80}{9}$

Solution:

$\left(2x^3 - \frac{1}{3x^2}\right)^5$

$T_{r+1} = {}^5C_r\left(2r^3\right)^{5-r}\left(\frac{-1}{3x^2}\right)^r$

$= {}^5C_r \cdot 2^{5-r} \cdot \left(\frac{-1}{3}\right)^r \cdot x^{15-3r-2r}$

$\Rightarrow 15-3r-2r=5$

$\Rightarrow 5r=10$

$\Rightarrow r=2$

Coefficient of $x^5={}^5C_2\cdot 2^3\cdot\left(\frac{-1}{3}\right)^2$

$=\frac{5\times 4\times 8}{2\times 3\times 3}$

$=\frac{80}{9}$

Question: If all the letters of the word MONDAY are arranged in any possible orders and written in dictionary then find the rank of the word MONDAY.

Answer: 327.00

Solution:

A, D, M, N, O, Y

A $\rightarrow 5!$

D $\rightarrow 5!$

M A $\rightarrow 4!$

M D $\rightarrow 4!$

M N $\rightarrow 4!$

M O A $\rightarrow 3!$

M O D $\rightarrow 5!$

M O N A $\rightarrow 2!$

M O N D A Y

$\Rightarrow$ Rank $=2(5!)+3(4!)+2(3!)+2!+1$

$=240+72+12+2+1$

$=327$

Question: Find range of $4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$.

Answer: $[0,2\pi)$

Solution:

$4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$

$\frac{x^2}{x^2+1}=\frac{x^2+1-1}{x^2+1}=1-\frac{1}{x^2+1}$

$x^2\geq 0$

$x^2+1\geq 1$

$$0 < \frac{1}{x^2+1} \le 1$$

$$-1 \le \frac{-1}{x^2+1} < 0$$

$$0 \le 1 - \frac{1}{x^2+1} < 1$$

$$\sin^{-1} 0 \le \sin^{-1}\left(1 - \frac{1}{x^2+1}\right) < \sin^{-1} 1$$

$$0 \le \sin^{-1}\left(\frac{x^2}{x^2+1}\right) < \frac{\pi}{2}$$

$$0 \le 4\sin^{-1}\left(\frac{x^2}{x^2+1}\right) < 2\pi$$

Question: $|\bar{a}| = 2$, $|\bar{b}| = 3$, angle between $\bar{a}$ and $\bar{b}$ is $\frac{\pi}{4}$, find $\left|\left(\bar{a} + 2\bar{b}\right) \times \left(2\bar{a} - 3\bar{b}\right)\right|^2$

Answer: 882.00

Solution:

$$\left|\left(\bar{a} + 2\bar{b}\right) \times \left(2\bar{a} - 3\bar{b}\right)\right|^2$$

$$= \left|-3\left(\bar{a} \times \bar{b}\right) + 4\left(\bar{b} \times \bar{a}\right)\right|^2$$

$$= \left|-7\left(\bar{a} \times \bar{b}\right)\right|^2$$

$$= 49|\bar{a}|^2 |\bar{b}|^2 \sin^2\theta$$

$$= 49 \times 4 \times 9 \times \frac{1}{2}$$

$$= 882$$

Question: $A = \begin{bmatrix} 1 & 2 & 1 \\ \alpha & 3 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, $|A| = 2$ and $\left|2\left(adj\left(8\,adj\,2A\right)\right)\right| = 32^n$. Find $3n + \alpha = ?$

Answer: 17.00

Solution:

$$\left|2\left(adj\left(2\,adj\,2A\right)\right)\right|$$

$$= \left|2\left(adj\left(8\,adj\,A\right)\right)\right|$$

$$= \left|2\left(64\,adj\left(adj\,A\right)\right)\right|$$

$$= \left|128\,adj\left(adj\,A\right)\right|$$

$$= (128)^3 \left|adj\left(adj\,A\right)\right|$$

$$= (128)^3 |A|^4$$

$(128)^3(2)^4 = 32^n$

$(2^7)^3(2)^4 = (2^5)^n$

$2^{25} = 2^{5n}$

$25 = 5n$

$\Rightarrow n = 5$

$\Rightarrow |A| = 2$

$1(3-2) - 2(\alpha - 6) + 1(\alpha - 9) = 2$

$1 - 2\alpha + 12 + \alpha - 9 = 2$

$\alpha = 2$

$\Rightarrow 3n + \alpha = 3(5) + 2 = 17$

Question: Find remainder when 7^{103} is divided by 17.

Answer: 12.00

Solution:

$7^{103} = 7 \cdot 7^{102}$

$7 \cdot 7^{102} = 7(7^2)^{51} = 7(49)^{51} = 7(51-2)^{51}$

$= 7(-2)^{51} = -7(2)^{51}$

$= -7(2^{48+3}) = -7 \cdot 8(2^{48})$

$= -56(2^4)^{12} = -56(17-1)^{12} = \frac{-56}{17}$

Sum $= -5 + 17 = 12$

Question: $a_1, a_2, a_3 \ldots$ are positive numbers & form increasing G.P. If $a_6 + a_8 = 2$ & $a_3 \times a_5 = \frac{1}{9}$, then find $6(a_2 + a_4)(a_4 + a_6)$.

Answer: 3.00

Solution:

$a_6 + a_8 = 2$

$ar^5 + ar^7 = 2$

$ar^5(1 + r^2) = 2$

$a_3 \times a_5 = \frac{1}{9}$

$ar^2 ar^4 = \frac{1}{9}$

$a^2 r^6 = (ar^3)^2 = \frac{1}{9}$

$ar^3 = \pm\frac{1}{3}$

Now, $ar^5\left(1+r^2\right)=2$

$ar^3r^2\left(1+r^2\right)=2$

$\left(\pm\frac{1}{3}\right)r^2\left(1+r^2\right)=2$

$\frac{r^2\left(1+r^2\right)}{3}=2$

$r^4+r^2-6=0$

$r^2=2$

$r=\sqrt{2}$

$a\left(2\sqrt{2}\right)=\frac{1}{3}$

$a=\frac{1}{6\sqrt{2}}$

$$\begin{aligned}6\left(a_2+a_4\right)\left(a_4+a_6\right)&=6\left(ar+ar^3\right)\left(ar^3+ar^5\right)\\&=6ar\left(1+r^2\right)ar^3\left(1+r^2\right)\\&=6a^2r^4\left(1+r^2\right)^2\\&=6\left(\frac{1}{72}\right)\cdot4\left(1+2\right)^2\\&=3\end{aligned}$$

Question: $\overline{z}=i\left(z^2+\mathrm{Re}\left(\overline{z}\right)\right)$, find $\sum|z|^2$.

Answer: 4.00

Solution:

$\overline{z}=i\left(z^2+\mathrm{Re}\left(\overline{z}\right)\right)$

$x-iy=i\left(x^2-y^2+2ixy+x\right)$

$x-iy=i\left(x^2+y^2+x\right)-2xy$

$\Rightarrow x=-2xy$ & $y=y^2-x^2-x$

$x=0\Rightarrow y=y^2\Rightarrow y=0$ or 1

$y=\frac{-1}{2}\Rightarrow\frac{-1}{2}=\frac{1}{4}-x^2-x\Rightarrow x=\frac{1}{2}$ or $\frac{-3}{2}$

Possible z:

$0+0i$

$0+1i$

$\frac{1}{2}-\frac{1}{2}i$

$\frac{-3}{2}-\frac{1}{2}i$

$$\Rightarrow \sum |z|^2 = 0+1+\left(\frac{1}{4}+\frac{1}{4}\right)+\left(\frac{9}{4}+\frac{1}{4}\right)$$
$$= 1+3$$
$$= 4$$

Question: Solve: $\frac{dy}{dx}+\left(\frac{4x}{1-x^2}\right)y=\frac{x}{\left(1-x^2\right)^{\frac{5}{2}}}$

Answer: ()
Solution:

$$\frac{dy}{dx}+\left(\frac{4x}{1-x^2}\right)y=\frac{x}{\left(1-x^2\right)^{\frac{5}{2}}}$$

$$\text{I.F.} = e^{\int \frac{4x}{1-x^2}dx} = e^{-2\ln\left|1-x^2\right|} = \frac{1}{\left(1-x^2\right)^2}$$

$$\Rightarrow \frac{y}{\left(1-x^2\right)^2} = \int \frac{1}{\left(1-x^2\right)^2}\cdot\frac{x}{\left(1-x^2\right)^{\frac{5}{2}}}dx$$

$$\frac{y}{\left(1-x^2\right)^2} = \int \frac{x}{\left(1-x^2\right)^{\frac{9}{2}}}dx$$

$$\frac{y}{\left(1-x^2\right)^2} = \frac{-1}{2}k.\frac{2}{7}\cdot\frac{1}{\left(1-x^2\right)^{\frac{7}{2}}}+C$$

$$y = \frac{-1}{7}\frac{1}{\left(1-x^2\right)^{\frac{3}{2}}}+C\left(1-x^2\right)^2$$

Question: Foci of hyperbola are $(\pm 2,0), e=\frac{3}{2}$. A tangent is drawn to it, that is perpendicular to the to $2x+3y=36$. If x and y intercepts of tangent are a and b respectively than $|6a|+|5b|=?$

Answer: 12.00
Solution:

$e=\frac{3}{2},\ ae=2$

$\Rightarrow a=\frac{4}{3}$

$\Rightarrow e^2=1+\frac{b^2}{a^2}$

$\frac{a}{4}=1+\frac{9b^2}{16}$

$\frac{5}{4}=\frac{9b^2}{16}$

$b^2=\frac{5}{4}\times\frac{16}{9}=\frac{20}{9}$

Equation of tangent:

$y=mx+\sqrt{a^2m^2-b^2}$

$y=\frac{3}{2}x+\sqrt{\frac{16}{9}\cdot\frac{9}{4}-\frac{20}{9}}$

$y=\frac{3}{2}x+\frac{4}{3}$

y -intercept $=\frac{4}{3}$

x -intercept $=\frac{-8}{9}$

$\Rightarrow\left|6\times\frac{-8}{9}\right|+\left|5\times\frac{4}{3}\right|=\frac{16}{9}+\frac{20}{3}=\frac{36}{3}=12$

Question: Mean and standard deviation of marks of 10 students are 50 and 12 respectively. It was found that two observations 20 and 25 were misread as 45 and 50. Find correct variance.

Answer: 269.00

Solution:

Given, Mean $\bar{x}=50,\ \sigma=12$

$\bar{X}=50-\frac{50}{10}=45$

$\frac{\sum x_i^2}{10}-50^2=144$

$\frac{\sum x_i^2}{10}=2500+144$

$\frac{\sum x_i^2}{10}=2500+144+\frac{400+645-45^2-50^2}{10}$

$=2294$

$\sigma^2=269$

Question: If α, β are the roots of $x^2 - \sqrt{2}x + 2 = 0$, find the value of $\alpha^{14} + \beta^{14}$.

Answer: -128.00

Solution:

$$x^2 - \sqrt{2}x + 2 = 0$$

$$x = \frac{\sqrt{2} \pm \sqrt{-6}}{2}$$

$$x = \frac{1 \pm \sqrt{3}i}{\sqrt{2}}$$

$$\Rightarrow \alpha = -\sqrt{2}\omega,\ \beta = -\sqrt{2}\omega^2$$

$$\Rightarrow \alpha^{14} + \beta^{14} = \left(-\sqrt{2}\omega\right)^{14} + \left(-\sqrt{2}\omega^2\right)^{14}$$

$$= 2^7 \cdot \omega^{14} + 2^7 \omega^{28}$$

$$= 2^7 \left(\omega^2 + \omega\right)$$

$$= -2^7$$

$$= -128$$

Question: Number of 3-digit numbers formed by 1, 2, 3, 4, 5 which are divisible by 6 (repetition is allowed).

Answer: 16.00

Solution:

$\square\ \square\ \boxed{4}$

$$\left.\begin{array}{lll} 1 & 1 \rightarrow & 1 \\ 4 & 4 \rightarrow & 1 \\ 1 & 4 \rightarrow & 2! \\ 2 & 3 \rightarrow & 2! \\ 3 & 5 \rightarrow & 2! \end{array}\right] = 8$$

$\square\ \square\ \boxed{2}$

$$\left.\begin{array}{lll} 2 & 2 \rightarrow & 1 \\ 5 & 5 \rightarrow & 1 \\ 1 & 3 \rightarrow & 2! \\ 2 & 5 \rightarrow & 2! \\ 4 & 3 \rightarrow & 2! \end{array}\right] = 8$$

$\text{Total} = 8 + 8 = 16$

Question: $\sum_{r=1}^{120} \left[\sqrt{r}\right] = ?$, where $[.]$ is GIF.

Answer: 824.00

Solution:

$$\sum_{r=1}^{120}\left[\sqrt{r}\right]=\sum_{r=1}^{3}1+\sum_{r=4}^{8}2+\sum_{r=9}^{15}3+\sum_{r=16}^{25}4+....+\sum_{r=81}^{99}9+\sum_{r=100}^{120}10$$

$$=3(1)+5(2)+7(3)+9(4)+....+21(10)$$

$$=\sum_{r=1}^{10}(2r+1)r=2\times\frac{10\times11\times21}{6}+\frac{10\times11}{2}$$

$$=\frac{10\times11}{2}\left(\frac{2\times21}{3}+1\right)$$

$$=\frac{10\times11}{2}\times15$$

$$=825$$

Question: $\lim\limits_{x\to0}\dfrac{e^{ax}-\cos bx-\dfrac{cx}{2}e^{-cx}}{1-\cos2x}=17$. Find $5a^2+b^2$.

Answer: 68.00

Solution:

$$\lim_{x\to0}\frac{ae^{ax}+b\sin bx-\frac{c}{2}e^{-cx}+\frac{c^2x}{2}e^{-cx}}{2\sin2x}=17$$

$$\Rightarrow a-\frac{c}{2}=0$$

$$\lim_{x\to0}\frac{a^2e^{ax}+b^2\cos bx+\frac{c^2}{2}e^{-cx}-\frac{c^3x}{2}e^{-cx}+\frac{c^2}{2}e^{-cx}}{2\cos2x}=17$$

$$\Rightarrow\frac{a^2+b^2+\frac{c^2}{2}+\frac{c^2}{2}}{2}=17$$

$$a^2+b^2+c^2=34$$

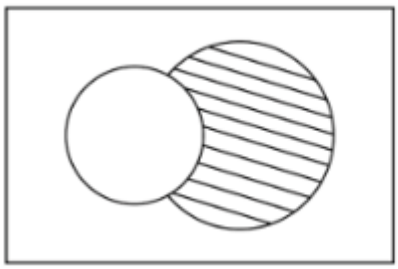
$$a^2+b^2+4a^2=34$$

$$5a^2+b^2=34$$

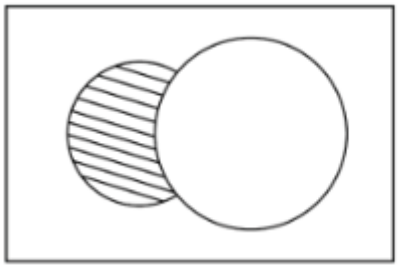
Question: $(\sim p\wedge q)\vee(p\wedge\sim q)\vee(\sim p\wedge(\sim q))$ is equivalent to

Answer: $\sim(p\wedge q)$

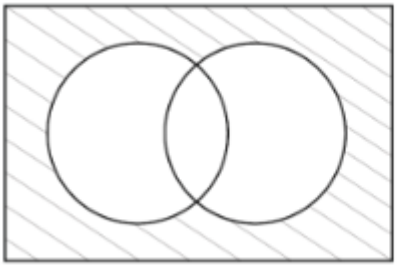
Solution:



$\sim p\wedge q$



$p \wedge \sim q$



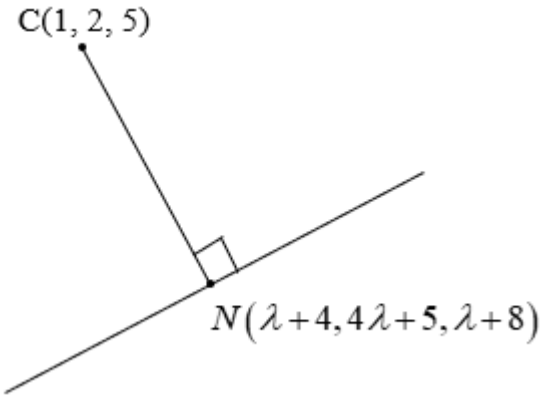
$\sim p \wedge \sim q$

$\equiv \sim (p \wedge q)$

Question: A perpendicular is drawn from C(1, 2, 5) on the line joining A(4, 5, 8) and B(1, -7, 5). Find distance of foot of perpendicular from $2x-2y+2z-3=0$.

Answer: $\frac{15}{2\sqrt{3}}$

Solution:



$L: \frac{x-4}{3}=\frac{y-5}{12}=\frac{z-8}{3}$, $L; \frac{x-4}{1}=\frac{y-5}{4}=\frac{z-8}{1}$

Now $NC \perp AB$

$\langle \lambda+3, 4\lambda+3, \lambda+3 \rangle \cdot \langle 1,4,1 \rangle = 0$

$\lambda+3+16\lambda+12+\lambda+3=0$

$\Rightarrow \lambda=-1$

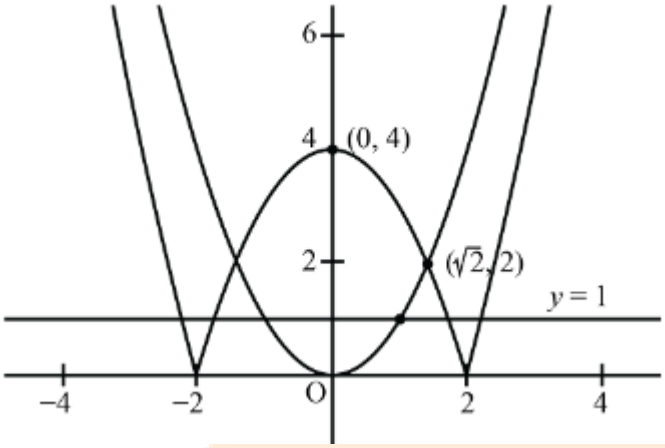
$\therefore N(3,1,7)$

Distance $= \left| \frac{6-1+14-3}{\sqrt{2^2+2^2+2^2}} \right| = \frac{15}{2\sqrt{3}}$ units.

Question: Find area between $y = x^2, y = |x^2 - 4|$ and $y = 1$.

Answer: ()

Solution:



Required area $= 2\left(\int_1^2 \sqrt{y}dy + \int_2^4 \sqrt{4-y}dy\right)$

$$= 2\left(\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^2 - \left[\frac{(4-y)^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4\right)$$

$$= \frac{4}{3}\left[\left(2\sqrt{2}-1\right)-\left(0-2\sqrt{2}\right)\right]$$

$$= \frac{4}{3}\left(4\sqrt{2}-1\right)$$