PART : MATHEMATICS

1. Statement $(p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$ is equivalent to

(1) $\sim p \vee q$ (2) $p \vee \sim q$ (3) $\sim p \vee \sim q$ (4) $p \vee q$

Ans. (3)

Sol. $(p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

$= (p \wedge \sim q) \vee (\sim p \wedge (q \vee \sim q))$

$= (p \wedge \sim q) \vee (\sim p \wedge t)$

$= (p \wedge \sim q) \vee \sim p$

$= (\sim q \vee p) \wedge (\sim p \vee \sim q)$

$= t \wedge (\sim p \vee \sim q)$

$= \sim p \vee \sim q$

2. Remainder when 7^{103} is divided by 17 is

Ans. (12)

Sol. $7^{103} = 7.(49)^{51} = 7.\ (51-2)^{51}$

$= 7(51\lambda - 2^{51}) = 17k - 7.\ 5^{51}$

$= 17k - 56(17-1)^{12}$

$= 17k - 56(17\lambda + 1)$

$= 17m - 51 - 17 + 12$

So Remainder 12

3. Let $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\hat{a}\,\hat{b} = \frac{\pi}{4}$ then value of $\left|(\vec{a} - 2\vec{b}) \times (\vec{b} - 2\vec{a})\right|^2 =$

And. (162)

Sol. $|\vec{a}| = 2$, $\vec{b} = 3$, $\hat{a}\,\hat{b} = \frac{\pi}{4}$

Now $\left|(\vec{a} - 2\vec{b}) \times (\vec{b} - 2\vec{a})\right|^2 = \left|\vec{a} \times \vec{b} - 2\vec{a} \times \vec{a} - 2\vec{b} \times \vec{b} + 4\vec{b} \times \vec{a}\right|^2$

$|3(\vec{a} \times \vec{b})|^2 = 9|\vec{a}|^2|\vec{b}|^2 \sin^2 \frac{\pi}{4}$

$= 9.4.\ 9.\ \frac{1}{2} = 162$

4. If all words form from letter of word "MONDAY" are arranged in alphabetical order as in dictionary then the rank of word "MONDAY" is

(1) 328 (2) 324 (3) 327 (4) 326

Ans. (3)

Sol. given :- MONDAY

A, D, M, N, O, Y

A – 5! – – – – – – = 120
D 5! = 120
MA 4! = 24
MD 4! = 24
MN 4! = 24
MOA 3! = 6
MOD 3! = 6
MONA 2! = 2
MONDAY = 1

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5. Range of $4\sin^{-1}\left(\frac{x^2}{1+x^2}\right)$ is

(1) $\left[0,\frac{\pi}{2}\right]$ (2) $[0, \pi]$ (3) $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ (4) $[0, 2\pi)$

Ans. (4)

Sol. $\frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}$

$\therefore 0 \le \frac{x^2}{1+x^2} < 1$

$\Rightarrow 0 \le \sin^{-1}\left(\frac{x^2}{1+x^2}\right) < \frac{\pi}{2}$

$\Rightarrow 0 \le 4\sin^{-1}\left(\frac{x^2}{1+x^2}\right) < 2\pi$

6. If α and β are roots of $x^2 - \sqrt{2}x + 2 = 0$ then find the value of $\alpha^{14} + \beta^{14}$

(1) – 128 (2) – 64 (3) – 32 (4) – 16

Ans. (1)

Sol. $\therefore \alpha + \beta = \sqrt{2}$ $\alpha\beta = 2$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = -2$

$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = 4 - 8 = -4$

$\alpha^8 + \beta^8 = (\alpha^4 + \beta^4)^2 - 2(\alpha\beta)^4 = 16 - 2 = -16$

$\alpha^6 + \beta^6 = (\alpha^2 + \beta^2)(\alpha^4 + \beta^4) - (\alpha\beta)^2(\alpha^2 + \beta^2) = -2(-4-4) = 16$

$\alpha^{14} + \beta^{14} = (\alpha^8 + \beta^8)(\alpha^6 + \beta^6) - (\alpha\beta)^6(\alpha^2 + \beta^2)$

$= (-16)(16) - 2^6 . (-2)$

$= -256 + 8.16$

$= 8 \times 16 = -128$

7. The coefficient of x^5 in $\left(2x^3 - \frac{1}{3x^2}\right)^5$ is

(1) $\frac{80}{9}$ (2) $\frac{40}{9}$ (3) $\frac{20}{9}$ (4) $\frac{40}{3}$

Ans. (1)

Sol. $T_{r+1} = {}^5C_r (2x^3)^{5-r}\left(\frac{-1}{3x^2}\right)^r$

Power of $x = 15 - 3r - 2r = 5$

$\Rightarrow r = 2$

coefficient $= {}^5C_2(2^3)\left(\frac{1}{9}\right) = \frac{80}{9}$

8. The value of $[\sqrt{1}]+[\sqrt{2}]+[\sqrt{3}]+\text{-----}+[\sqrt{120}]$ is

Ans. (825)

Sol. If $x \in [n^2,(n+1)^2)$ then $[\sqrt{x}] = n$

required value $= \sum_{r=1}^{10}((r+1)^2 - r^2)r = \sum_{r=1}^{10}(2r^2+r)$

$= 2\frac{10.11.21}{6} + \frac{10.11}{2}$

$= 770 + 55 = 825$

9. Let a_1, a_2, a_3 .... is a G.P of positive term such that $a_6 + a_8 = 2$ & $a_3a_5 = \frac{1}{9}$ then value of $6(a_2 + a_4)(a_4 + a_6)$ is

Ans. (3)

Sol. $a_6 + a_8 = 2 \Rightarrow ar^5 + ar^7 = 2$

& $a_3 . a_5 = \frac{1}{9} \Rightarrow a^2 . r^2 r^4 = \frac{1}{9}$

$\Rightarrow ar^3 = \frac{1}{3}$

by (1) $\quad \frac{1}{3}r^2 + \frac{1}{3}r^4 = 2$

$r^2 + r^4 = 6 \Rightarrow r^4 + r^2 - 6 = 0$

$\Rightarrow (r^2 + 3)(r^2 - 2) = 0$

$\Rightarrow r^2 = 2 \therefore ar.2 = \frac{1}{3} \Rightarrow ar = \frac{1}{6}$

Now $6(a_2 + a_4)(a_4 + a_6) = 6(ar + ar^3)(ar^3 + ar^5)$

$= 6\left(\frac{1}{6}+\frac{1}{3}\right)\left(\frac{1}{3}+\frac{1}{3}.2\right)$

$= 6\frac{1+2}{6}.\frac{1+2}{3}$

$= 3$

10. If $x \in (-1, 1]$ then number of solution of equation $\sin^{-1}x = 2\tan^{-1}x$ is

(1) 0 (2) 1 (3) 2 (4) 3

Ans. (3)

Sol. Number of solution = 2

Alternate method

$\sin^{-1}x = \sin^{-1}\frac{2x}{1+x^2} \quad \forall x \in (-1,1]$

$x = \frac{2x}{1+x^2}$

$x = 0, 1$

11. If in a binomial probability distribution if X represent number of successes, if experiment is performed n times and difference of mean and variance is 1 and $2P(X = 2) = 3P(X = 3)$ then find $n^2(P(X > 1)) =$

Ans. (11)

Sol. $np - npq = 1$ and

$2 \times {}^nC_2P^2(1-P)^{n-2} = 3 \times {}^nC_3P^3(1-P)^{n-3}$

$\Rightarrow np^2 = 1$

$\Rightarrow p = \frac{1}{2}, n = 4$

$= n^2(P(X > 1)) = 16\left(1 - {}^4C_0(\frac{1}{2})^4 - {}^4C_1(\frac{1}{2})^4\right)$

$= 16\left(1 - \frac{5}{16}\right) = 11$

12. How many 3 digit number can be formed by using the digit 1,2,3,4, 5 which are divisible by 6, repetition of digit are allowed

Ans. (16)

Sol. For number to be divisible by '6' unit digit should be even and sum of digit is divisible by 3.
(2,1,3) , (2,3,4) , (2,5,5) , (2,2,5) , (2,2,2) (4,1,1), (4,4,1), (4,4,4), (4,3,5)
number of such number by using (2,1,3) , (2,5,5) , (4,1,1), (4,3,5)
= 2 + 1 + 1 + 2 = 6
by using (2,3,4) , (2,2,5) (4,4,1) = 2 + 2 + 2 + 2 = 8
by using (2,2,5), (4,4,4) = 1 + 1 = 2

13. $\dfrac{e^{-\pi/4} + \int_0^{\pi/4} e^{-x}(\tan^{50} x)dx}{\int_0^{\pi/4} e^{-x}\left(\tan^{49} x + \tan^{51} x\right)dx}$ is equal to

Ans. (50)

Sol. $\dfrac{e^{-\pi/4} + \int_0^{\pi/4} e^{-x}(\tan^{50} x)dx}{\int_0^{\pi/4} e^{-x}\left(\tan^{49} x \sec^2 x\right)dx} = \dfrac{e^{-\pi/4} + \int_0^{\pi/4} e^{-x}\left(\tan^{50} x\right)dx}{\left(\left(\frac{e^{-x}\tan^{50} x}{50}\right)_0^{\pi/4} + \int_0^{\pi/4} \frac{e^{-x}\tan^{50} x}{50}dx\right)} = 50$

14 The mean & standard deviation of 50 number is 50 & 12 respectively. If 20 and 25 are wrongly read as 45 & 50 find correct variance

Ans. (173)

Sol. $\bar{x} = \frac{\sum_{i=1}^{50} x_i}{50} = 50$

$\therefore \sum x_i = 2500$

correct sum $= \sum x_i - (45 + 50) + (20 + 25)$

$= 2450$

$\sigma^2 = \frac{\sum x^2}{50} - \bar{x}^2$

$\Rightarrow 144 = \frac{\sum x^2}{50} - 2500$

$\Rightarrow \sum x^2 = 50 \times 2644 = 132200$

$\therefore$ correct $\sum x^2 = \sum x^2 - (45^2 + 50^2) + (20^2 + 25^2)$
$= 132200 - (2025 + 2500) + (400 + 625)$
$= 128700$

correct Variance $= \dfrac{\sum x^2}{n} - \left(\dfrac{\sum x}{n}\right)^2$

$= \dfrac{128700}{50} - \left(\dfrac{2450}{50}\right)^2$

$= 2574 - 2401$
$= 173$

15. If 'Z' is a complex number satisfying $\bar{z} = i(z^2 + \text{Re}(z))$ then the sum of all values of $|z|^2$ is

Ans. (4)

Sol. Let $Z = x + iy$

$x - iy = i((x^2 - y^2 + (2xy)i + x)$
$= (-2xy) + i(x^2 + x - y^2)$

$x = -2xy$ ---------(1)
$x(1 + 2y) = 0$
$x = 0,\ y = -\dfrac{1}{2}$
$-y = x^2 + x - y^2$ ---------(2)

If $x = 0$, $\quad y^2 - y = x^2 + x \Rightarrow y = 0, 1$
If $y = -\dfrac{1}{2}$, $\quad \dfrac{1}{4} + \dfrac{1}{2} = x^2 + x \Rightarrow x^2 + x - \dfrac{3}{4} = 0$

$x = \dfrac{1}{2}, -\dfrac{3}{2}$

Possible $Z = 0 + i0,\ 0 + i,\ \dfrac{1}{2} - \dfrac{i}{2}, -\dfrac{3}{2} - \dfrac{i}{2}$

$|z|^2 = 0, 1, \dfrac{1}{2}, \dfrac{10}{4}$

Sum of values of $|z|^2 = 1 + \dfrac{1}{2} + \dfrac{10}{4} = 1 + 3 = 4$

16. If 'N' is foot of perpendicular drawn form A(1,2,5) to line joining B(4,5,8) and C(1, –7, 5) then distance of N from the plane $2x - 2y + 2z - 3 = 0$ is

(1) $\dfrac{5\sqrt{2}}{2}$ (2) $\dfrac{5\sqrt{3}}{2}$ (3) $\dfrac{5\sqrt{5}}{2}$ (4) $5\sqrt{3}$

Ans. (2)

Sol. $N = \left(\dfrac{\lambda + 4}{\lambda + 1}, \dfrac{-7\lambda + 5}{\lambda + 1}, \dfrac{5\lambda + 8}{\lambda + 1}\right)$

distance of A N $= < \dfrac{3}{\lambda + 1}, \dfrac{-9\lambda + 3}{\lambda + 1}, \dfrac{3}{\lambda + 1} >$

distance of BC $= < 3, 12, 3 >$

$\because AN \perp BC$

$\frac{-9}{\lambda+1}+\frac{12(3-9\lambda)}{\lambda+1}+\frac{9}{\lambda+1}=0$

$9+36-108\lambda+9=0$

$\lambda=\frac{1}{2}$

$N=(3,1,7)$

Perpendicular distance of N form $2n-2y+2z-3=0$

$=\left|\frac{6-2+14-3}{\sqrt{4+4+4}}\right|=\frac{15}{2\sqrt{3}}=\frac{5\sqrt{3}}{2}$

17. Given $\frac{x+3}{-3}=\frac{y-2}{2}=\frac{z-5}{5}$ then which of the following lines in option is coplanar with the given line

(1) $\frac{x+1}{-1}=\frac{y-1}{1}=\frac{z-5}{5}$ (2) $\frac{x+1}{1}=\frac{y+1}{-1}=\frac{z-5}{5}$

(3) $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-5}{5}$ (4) $\frac{x-1}{-1}=\frac{y+3}{-2}=\frac{z-5}{4}$

Ans. (1)

Sol. Let general Pt. be $(-3\lambda-3, 2\lambda+2, 5\lambda+5)$

clearly which lies on (1)

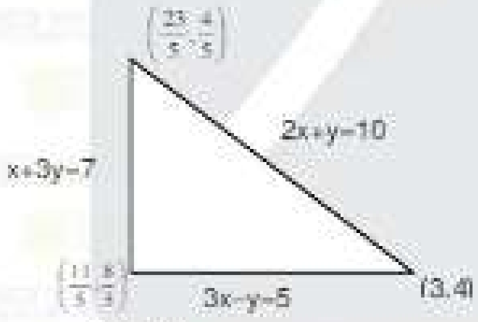
18. If centroid of triangle formed by the lines $2x+y=10$, $x+3y$ and $3x-y=5$ is (α,β) the quadratic equation whose roots are $\alpha+2\beta$ and $2\alpha+\beta$ is

(1) $45x^2-729x+2938=0$ (2) $45x^2-729x+2838=0$

(3) $45x^2-729x+2738=0$ (4) $45x^2-119x+2638=0$

Ans. (1)

Sol.



$(\alpha,\beta)=\left(\frac{49}{15},\frac{32}{15}\right)$

$\alpha+2\gamma=\frac{113}{15}, 2\alpha+\beta=\frac{130}{15}=\frac{26}{3}$

Sum $=\frac{243}{15}=\frac{81}{5}$

eq^n. $x^2-\frac{81}{5}x+\frac{2938}{45}=0$

$45x^2-729x+2938=0$