PART : MATHEMATICS

1. There are 6 white and 4 black balls in bag. A dice is thrown and number indicate on dice, number of balls are drawn from the bag, then probability that all drawn balls are white.

(1) $\frac{4}{5}$ (2) $\frac{1}{5}$ (3) $\frac{2}{5}$ (4) $\frac{5}{7}$

Ans. (2)

Sol. Probability of all drawn balls white

$$=\frac{1}{6}\left[\frac{^6C_1}{^{10}C_1}+\frac{^6C_2}{^{10}C_2}+\frac{^6C_3}{^{10}C_3}+\frac{^6C_4}{^{10}C_4}+\frac{^6C_5}{^{10}C_5}+\frac{^6C_6}{^{10}C_6}\right]$$

$$=\frac{1}{6}\left[\frac{6}{10}+\frac{6\times5}{10\times9}+\frac{6\times5\times4}{10\times9\times8}+\frac{6\times5\times4\times3}{10\times9\times8\times7}+\frac{6\times120}{10\times9\times8\times7\times6}+\frac{24}{10\times9\times8\times7}\right]$$

$$=\frac{1}{10}+\frac{1}{18}+\frac{1}{36}+\frac{6}{9\times8\times7}+\frac{1}{36\times7}+\frac{4}{90\times56}$$

$$=\frac{1}{10}+\frac{1}{18}+\frac{1}{36}+\frac{1}{84}+\frac{1}{252}+\frac{1}{90\times14}$$

$$=\frac{1}{10}+\frac{1}{12}+\frac{1}{84}+\frac{1}{252}+\frac{1}{1260}$$

$$=\frac{127}{1260}+\frac{8}{84}+\frac{1}{252}$$

$$=\frac{127}{1260}+\frac{2}{21}+\frac{1}{252}=\frac{254+240+10}{2520}=\frac{504}{2520}=\frac{1}{5}$$

2. If S is the set of values of λ, μ for which $\vec{a}=\lambda\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}+\mu\hat{k}$ and $\vec{c}=3\hat{i}-4\hat{j}+5\hat{k}$ are coplanar and $\lambda-\mu=5$ then find $\sum_{\lambda,\mu\in S}80(\lambda^2+\mu^2)$

Ans. (2290)

Sol.
$$\begin{vmatrix}\lambda & -1 & 1\\ 1 & 2 & \mu\\ 3 & -4 & 5\end{vmatrix}=0$$

$\lambda(10+4\mu)+1(5-3\mu)+1(-4-6)=0$

$10\lambda+4\lambda\mu+5-3\mu-10=0$

$10\lambda+4\lambda\mu-3\mu=5$ ......(i)

And $\lambda=5+\mu$ ....(ii)

$10(5+\mu)+4\mu(5+\mu)-3\mu=5$

$10\mu+20\mu-3\mu+4\mu^2+50=5$

$4\mu^2+27\mu+45=0$

$4\mu^2+12\mu+15\mu+45=0$

$4\mu(\mu+3)+15(\mu+3)=0$

$\mu=-3$ or $-\frac{15}{4}$

$\lambda=5-3=2$ or $\lambda=5-\frac{15}{4}=\frac{5}{4}$

$\sum 80(\lambda^2+\mu^2)=\left(9+4+\frac{225}{16}+\frac{25}{16}\right)80=\left(13+\frac{250}{16}\right)80=1040+1250=2290$

3. Number of 3 digits numbers formed by using digits 1,3, 5, 8 which are divisible by 3, if repetition of digits is allowed in a number, is -

(1) 22 (2) 25 (3) 28 (4) 31

Ans. (1)

Sol. Sum of three digits is 3 then digits are 1, 1, 1
Sum of three digits is 9 then digits are 1, 3, 5 or 3, 3, 3
Sum of three digits is 12 then digits are 1, 3, 8
Sum of three digits is 15 then digits are 5, 5, 5
Sum of three digits is 18 then digits are 5, 5, 8
Sum of three digits is 21 then digits are 5, 8, 8
Sum of three digits is 24 then digits are 8, 8, 8

Now possible number are $=1+3!+1+3!+1+\frac{3!}{2!}+\frac{3!}{2!}+1$

$=1+6+1+6+1+3+3+1$

$=22$

4. Let a and b are two positive numbers. Two A.M's b/w them are A_1 and A_2. Three GM's b/w a and b are G_1, G_2 and G_3 then value of $G_1^4+G_2^4+G_3^4+G_1^2G_3^2=$

(1) $G_1G_2(A_1+A_2)^2$ (2) $G_2G_3(A_1+A_2)^2$

(3) $G_1G_3(A_1+A_2)^2$ (4) $G_1G_2G_3(A_1+A_2)$

Ans. (3)

Sol. $A_1=\frac{2a+b}{3}$ and $A_2=\frac{a+2b}{3}\Rightarrow A_1+A_2=(a+b)$

also $G_1=\left(a^3b\right)^{1/4}, G_2=\sqrt{ab}, G_3=\left(ab^3\right)^{1/4}$

$G_1^4=a^3b, G_2^4=a^2b^2, G_3^4=ab^3$

$\Rightarrow G_1^4+G_2^4+G_3^4+G_1^2G_3^2=a^3b+a^2b^2+ab^3+a^2b^2$

$=ab\left(a^2+2ab+b^2\right)$

$=ab(a+b)^2$

$=G_1G_3(A_1+A_2)^2$

5. Number of real solution of the equation $x\,|x| - 5\,|x+2| + 6 = 0$ is/are –

(1) 1 (2) 2 (3) 3 (4) 4

Ans. (3)

Sol. $x\,|x| - 5\,|x+2| + 6 = 0$.

Case-I $x \le -2$, $-x^2 + 5(x+2) + 6 = 0$

$x^2 - 5x - 16 = 0$

$x^2 - 5x + \frac{25}{4} = 16 + \frac{25}{4}$

$\left(x - \frac{5}{2}\right)^2 = \frac{89}{4}$

$x - \frac{5}{2} = \pm\frac{\sqrt{89}}{2}$

$x = \frac{5-\sqrt{89}}{2}$ $\quad x \ne \frac{5+\sqrt{89}}{2}$

Case-II $-2 < x < 0$

$-x^2 - 5x - 10 + 6 = 0$

$x^2 + 5x + 4 = 0 \Rightarrow x = -1, -4$

$x = -1$ $\quad x \ne -4$

Case-III $x \ge 0$

$x^2 - 5x - 10 + 6 = 0$

$x^2 - 5x - 4 = 0$

$x = \frac{5 \pm \sqrt{41}}{2}$

$x = \frac{5+\sqrt{41}}{2}$, $\quad x \ne \frac{5-\sqrt{41}}{2}$

Number of solution = 3.

6. Let ABCD be a quadrilateral E and F are mid points of AC & BD respectively. If $\left(\overrightarrow{AB} - \overrightarrow{BC}\right) + \left(\overrightarrow{AD} - \overrightarrow{DC}\right) = k\,\overrightarrow{ED}$ then value of k is equal to ________

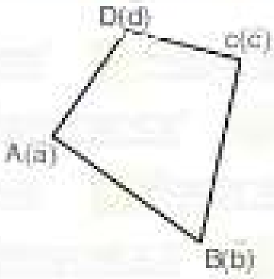
(1) 2 (2) 3 (3) 4 (4) 6

Ans. (3)

Sol. Let P.V. & A, B, C, D are $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ resp. mid-point of AC is E

$\Rightarrow$ P.V. of E are $\left(\frac{\vec{a}+\vec{c}}{2}\right)$

Mid point of BD is F



$\Rightarrow$ P.V of F are $\left(\frac{b+d}{2}\right)$

$\left(\overline{AB}-\overline{BC}\right)+\left(\overline{AD}-\overline{DC}\right)=\left(\vec{b}-\vec{a}\right)-\left(\vec{c}-\vec{b}\right)+\left(\vec{d}-\vec{a}\right)-\left(\vec{c}-\vec{d}\right)$

$= 2b - 2a - 2c + 2d$

$= 2(b+d) - 2(a+c)$

$= 4\left[\left(\frac{b+d}{2}\right)-\left(\frac{a+c}{2}\right)\right] = 4\ \overline{EF}$

7. Mean of 50 observation is 10 and standard deviation is 8 but one observation 50 is misread as 40. then correct variance. is

(1) 171.8 (2) 172.8 (3) 192.4 (4) 164.5

Ans. **(1)**

Sol. $\frac{\sum_{i=1}^{50} x_i}{50} = 10 \Rightarrow \sum x_i = 500$

correct $\sum x_i = 500 - 40 + 50 = 510$

$\frac{\sum x_i^2}{50} - (10)^2 = 64$

$\frac{\sum x_i^2}{50} = 164$

$\sum x_i^2 = 8200$

Correct $\sum x_i^2 = 8200 - 40^2 + 50^2$

$= 8200 + 900$

$= 9100$

Correct variance $\sigma^2 = \frac{\sum x_i^2}{50} - (\bar{x})^2$

$= \frac{9100}{50} - \left(\frac{510}{50}\right)^2$

$= 182 - 10.20$

$= 171.8$

8. A person forget PIN of ATM card. PIN has four digits and all digits are different. Greatest digit in PIN is 7. Sum of first two digits of PIN is equal to sum of last two digits of PIN, then maximum number of unsuccessful attempts is

(1) 72 (2) 36 (3) 71 (4) 18

Ans. (3)

Sol. Let the correct ATM PIN is abcd. (a,b,c,d ∈ digits) such that a + b = c + d
Let maximum {a, b} = 7 ⇒ a + b can take 7 ,8, 9, 10, 11
(0, 7) a + b = 7 ⇒ c + d = 7 case (1, 6), (2 ,5), (3, 4), (4, 3), (5, 2), (6, 1)
(1, 7) a + b = 8 ⇒ c + d = 8 case (2, 6), (3 ,5), (5, 3), (6, 2)
(2, 7) a + b = 9 ⇒ c + d = 9 case (3, 6), (4 ,5), (5, 4), (6, 3)
(3, 7) a + b = 10 ⇒ c + d = 10 case (4, 6), (6, 4)
(4, 7) a + b = 12 ⇒ c + d = 11 case (5, 6), (6, 5)
Hence total Number of allow = (18)
Now a, b can be in charged by 2 method and in place of a, b we can be take c, a also.
So total number of attempt = 18 ×2 ×2 = 72
So maximum number of unsuccessful attempt = 72-1= 71

9. f(x) = maximum {1 + x+ [x], x + 2, x +2[x]} 0 ≤ x ≤ 2.
If number of non-differentiability point is 'b' and number of point of discontinuity is 'a' then a + b is equal to (where [.] denotes G.I.F)
(1) 2 (2) 4 (3) 3 (4) 1

Ans. (1)

Sol. f(x) maximum {1+x+[x], x + 2, x+ 2[x]}

$$= \begin{cases} x+2 & 0 \le x < 2 \\ 6 & x = 2 \end{cases}$$

f(x) is discontinuous and non-differentiable
at x = 2 only
⇒ a = 1,b = 1
a + b = 2

10. Let $f(x) = \log(4x^2 + 11x + 9) + \sin^{-1}(4x + 3) + \cos^{-1}\left(\frac{10x+6}{3}\right)$ and domain of f(x) is [α, β] us equal to

(1) 2 (2) 3 (3) 4 (4) 6

Ans. (3)

Sol. $4x^2 + 11x + 9 > 0$ and $-1 \le 4x + 3 \le 1$ and $-1 \le \frac{10x+6}{3} \le 1$

$D < 0$ $\quad -4 \le 4x \le -2$ $\quad -3 \le 10x + 6 \le 3$

So $x \in R$ $\quad -1 \le x \le -\frac{1}{2}$ $\quad -\frac{9}{10} \le x \le -\frac{3}{10}$

So Domain of f(x) is $x \in \left[-\frac{9}{10}, \frac{1}{2}\right]$

Now $|10(\alpha - \beta)| = \left|10\left(-\frac{9}{10} + \frac{1}{2}\right)\right| = |-9+5| = 4$

11. The negation of the statement
A : p ∧ (q ∧ (~p ∨ q)) is
(1) p ∧ ~ q (2) ~ p ∧ q (3) ~ p ∨ ~ q (4) ~ p ∧ q

Ans. (3)

Sol. $p \wedge (q \wedge (\sim p \vee q))$

$= q \wedge (p \wedge (\sim p \vee q))$

$= q \wedge (p \wedge \sim p) \vee (p \wedge q))$

$= q \wedge (c \vee (p \wedge q))$

$= q \wedge (p \wedge q)$

$= (p \wedge q)$

Hence negation of the above statement is $(\sim p \vee \sim q)$

12. The value of $\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2^2}+\frac{1}{2.3}+\frac{1}{3^2}\right)+\left(\frac{1}{2^3}+\frac{1}{2^2.3}+\frac{1}{2.3^2}+\frac{1}{3^3}\right)+\dots\dots = \frac{\alpha}{\beta}$ (when α, β are coprime) the value of $(\alpha+\beta)$ is equal to

(1) 2 (2) 3 (3) 5 (4) 7

Ans. **(4)**

Sol. **Given** $= \left\{\left(\frac{1}{2}+\frac{1}{3}\right)+\left(\frac{1}{2^2}+\frac{1}{2.3}+\frac{1}{3^2}\right)+\left(\frac{1}{2^3}+\frac{1}{2^2.3}+\frac{1}{2.3^2}+\frac{1}{3^3}\right)+\dots\dots\right\}-\frac{2}{3}$

let $x = \frac{1}{2}$ & $y = \frac{1}{3}$

$= \{(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+\dots\dots\}-\frac{2}{3}$

$= \frac{(x-y)}{(x-y)}\{(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+\dots\dots\}-\frac{2}{3}$

$= \frac{(x^2+x^3+x^4+\dots)-(y^2+y^3+y^4+\dots\dots)}{x-y}-\frac{2}{3}$

$= \frac{\left(\frac{x^2}{1-x}\right)-\left(\frac{y^2}{1-y}\right)}{x-y}-\frac{2}{3}$

put $x = \frac{1}{2}$ and $y = \frac{1}{3}$

we get $\frac{\alpha}{\beta} = \frac{4}{3}$

So $\alpha+\beta = 7$

13. A plane is passes through the points A(1,1,1), B(-2,3,2) and C(0,3,0) and line $\frac{x-1}{-2} = \frac{y+2}{-1} = \frac{z}{4}$ intersect this plane at point P, then distance of P from origin is

(1) $\sqrt{341}$ (2) $\sqrt{241}$ (3) $\sqrt{131}$ (4) $\sqrt{347}$

Ans. **(1)**

Sol. Equation of plane is

$$\begin{vmatrix} x-0 & y-3 & z-0 \\ 1-0 & 1-3 & 1-0 \\ -2-0 & 3-3 & 2-0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y-3 & z \\ 1 & -2 & 1 \\ -2 & 0 & 2 \end{vmatrix} = 0$$

$x(-4-0)-(y-3)(2+2)+z(0-4) = 0$

$-4x - 4y + 12 - 4z = 0$

$x + y + z = 3$ ....(1)

Let point P is $(1-2\lambda, -2-\lambda, 4\lambda)$

$1 - 2\lambda - 2 - \lambda + 4\lambda = 3$

$\lambda = 4$

So $P(-7, -6, 16)$

Distance OP $= \sqrt{49+36+256} = \sqrt{341}$

14. If $\int_0^1 \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})} = \frac{1}{\alpha}\log\left(\frac{\alpha+1}{\beta}\right)$ find $\alpha^4 - \beta^4$

Ans. (21)

Sol. $I = \int_0^1 \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})}$ ....(1)

$$I = \int_0^1 \frac{dx}{\left[5+2(1-x)-2(1-x)^2\right]\left[1+e^{2-4(1-x)}\right]}$$

$$I = \int_0^1 \frac{dx}{(5-2x+4x-2x^2)(1+e^{-2+4x})}$$

$$I = \int_0^1 \frac{dx}{(5+2x-2x^2)\left(1+\frac{1}{e^{2-4x}}\right)}$$

$$I = \int_0^1 \frac{e^{2-4x}}{(5+2x-2x^2)(1+e^{2-4x})}dx \text{ ......(2)}$$

From (1) and (2)

$$2I = \int_0^1 \frac{dx}{(5+2x-2x^2)}$$

$$2I = \frac{1}{2}\int_0^1 \frac{dx}{\left(\frac{5}{2}+x-x^2\right)} = \frac{1}{2}\int_0^1 \frac{dx}{\frac{11}{4}-\left(x-\frac{1}{2}\right)^2}$$

$$2I = \frac{1}{2}\cdot\frac{1}{2\sqrt{\frac{11}{4}}}\left[\ln\left|\frac{\frac{\sqrt{11}}{2}+x-\frac{1}{2}}{\frac{\sqrt{11}}{2}-x+\frac{1}{2}}\right|\right]_0^1$$

So $I = \frac{1}{\sqrt{11}}\ln\left(\frac{\sqrt{11}+1}{\sqrt{10}}\right)$

Hence $\alpha = \sqrt{11}, \beta = \sqrt{10}$

$\Rightarrow \alpha^4 - \beta^4 = 121 - 100 = 21$

15. Let $A = \{x : x = 3^n - 3$ is a multiple of 7, $10 \le n \le 100\}$ then number of elements in the set A is equal to

Ans. 15

Sol. $3^n - 3$ div by 7. $\Rightarrow 3^n = 7\lambda + 3$

$\Rightarrow 3^n \equiv 3 \pmod 7$ form

Now $\Rightarrow 3^1 \equiv 3 \pmod 7$

$\Rightarrow 3^2 \equiv 2 \pmod 7$

$\Rightarrow 3^3 \equiv 6 \pmod 7$

$\Rightarrow 3^4 \equiv 4 \pmod 7$
$\Rightarrow 3^5 \equiv 5 \pmod 7$
$\Rightarrow 3^6 \equiv 1 \pmod 7$
$\Rightarrow 3^7 \equiv 3 \pmod 7$

So we can say that $3^n - 3$ is divisible by 7 iff $n = 1, 7, 13, 19, 25, \ldots\ldots\ldots$

as $10 \le n \le 100$

$\Rightarrow n = 13, 19, 25, 31, \ldots\ldots, 97$

Hence number of element in set A is 15

16. Matrix A having order m such that $|A| = (m)^{-n}$ then value of $|n \text{ adj } (\text{adj} mA)| =$________.

(1) $n^m (n)^{(m-n)(m-1)^2}$ (2) $n^m (m)^{(m-n)(m-1)^2}$

(3) $m^n (n)^{(m-n)(m-1)^2}$ (4) $n^m (m)^{(m-n)(m-1)^2}$

Sol. $|A| = m^{-n}$

$|n \text{ adj}(\text{adj } mA)| = n^m |\text{adj}(\text{adj } mA)|$

$= n^m |mA|^{(m-1)^2}$

$= n^m m^{m(m-1)^2} |A|^{(m-1)^2}$

$= n^m m^{m(m-1)^2} m^{-n(m-1)^2}$

$= n^m m^{(m-n)(m-1)^2}$

17. Number of common tangents of circles $x^2 + y^2 - 13x - 15y + 13 = 0$ and $x^2 + y^2 - 6x - 6y - 7 = 0$ is

(1) 3 (2) 4 (3) 2 (4) 1

Ans. (3)

Sol. $C_1\left(\frac{13}{2}, \frac{15}{2}\right) r_1 = \sqrt{\frac{169}{4} + \frac{225}{4} - 13} = \sqrt{\frac{169 + 225 - 52}{4}} = \sqrt{\frac{342}{4}} \approx 9.2$

$C_2(3, 3) r_2 = \sqrt{9 + 9 + 7} = \sqrt{25} = 5$

$C_1C_2 = \sqrt{\left(\frac{13}{2} - 3\right)^2 + \left(\frac{15}{2} - 3\right)^2} = \sqrt{\frac{49}{4} + \frac{81}{4}} = \sqrt{\frac{130}{4}} = \sqrt{\frac{342}{2}} = 5.7$

Hence $r_1 - r_2 < C_1C_2 < r_1 + r_2$

So number of common tangents

18. If $(a + bx + cx^2)^{10} = \sum_{i=0}^{20} p_i x^i$ and $p_1 = 20$, $p_2 = 240$ then value of $2(a+b+c)$ is equal to (where $a, b, c \in N$)

(1) 18 (2) 12 (3) 16 (4) 10

Ans. (1)

Sol general term $= \frac{10!}{r_1! r_2! r_3!} a^{r_1} (bx)^{r_2} (cx^2)^{r_3}$

Coefficient of $x^1 = p_1 = \frac{10!}{9! 1! 0!} a^9 b = 20$

$a^9 b = 2 \ldots\ldots\ldots(1)$

Coefficient of $x^2 = p_2 = \frac{10!}{8! 2! 0!} a^8 b^2 + \frac{10!}{9! 0! 1!} a^9 c = 240$

$45 a^8 b^2 + 10 a^9 c = 240$

$9 a^8 b^2 + 2 a^9 c = 48 \ldots\ldots(2)$

By (1) $a, b \in N \Rightarrow a = 1, b = 2$

By (2) $c = 6$

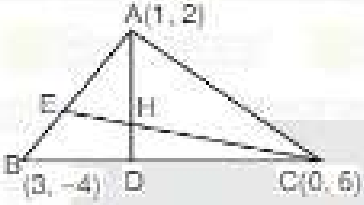
Hence $2(a+b+c) = 18$

19. Orthgocentre of a triangle having vertices as A (1, 2), B(3, –4) and C(0, 6) is

(1) (–129, –37) (2) (7, 3) (3) (129, 37) (4) (–37, –129)

Ans. (1)

Sol.



$m_{BC} = \frac{6+4}{0-3} = -\frac{10}{3}$

Equation of altitude AD is

$y - 2 = \frac{3}{10}(x - 1)$

$3x - 10y + 17 = 0$ ..... (1)

$m_{AB} = \frac{2+4}{1-3} = -3$

Equation of altitude CE is

$y - 6 = \frac{1}{3}(x - 0)$

$x - 3y + 18 = 0$ ..... (2)

Point of intersection of both altitudes is

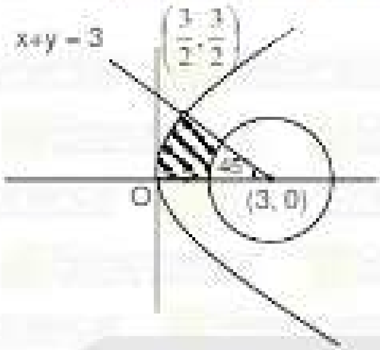
H (–129, –37)

20. Area of region bounded by curve $2y^2 = 3x$, line $x + y = 3$ and $y = 0$, excluded area of circle $(x - 3)^2 + y^2 = 2$ and above x-axis is A then the value of $4(\pi + 4A)$ is equal to

(1) 21 (2) 28 (3) 42 (4) 49

Ans. (3)

Sol.



By solving

$x + y = 3$ with $2y^2 = 3x$

We have $x = \frac{3}{2}$ & $y = \frac{3}{2}$

So required area

$= \int_0^{3/2}\sqrt{\frac{3}{2}x}\,dx + \int_{\frac{3}{2}}^{3}(3-x)dx$

–area of sector of circle

$= \sqrt{\frac{3}{2}}\left.\frac{x^{3/2}}{3/2}\right|_0^{3/2} + \frac{1}{2}\left(\frac{3}{2}\right)\left(3-\frac{3}{2}\right) - \frac{\pi}{4}\left(\frac{(\sqrt{2})^2}{2}\right)$

$A = \frac{3}{2} + \frac{9}{8} - \frac{\pi}{4} = \frac{21-2\pi}{8}$

$\Rightarrow 4(\pi + 4A) = 42$