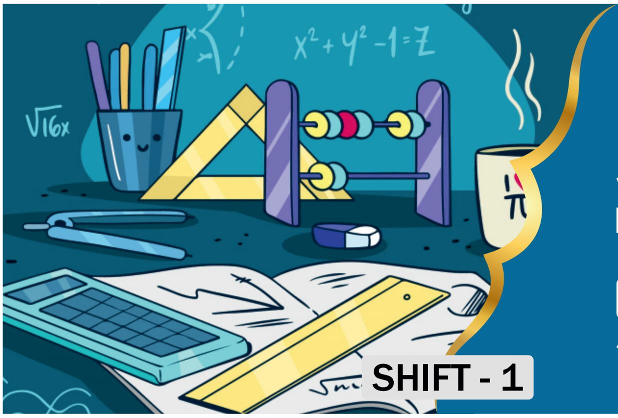
JEE MAIN 2023

APRIL ATTEMPT

PAPER-1 (B.Tech / B.E.)



QUESTIONS & SOLUTIONS

Reproduced from Memory Retention

15 APRIL, 2023

9:00 AM to 12:00 Noon

Duration : 3 Hours | Maximum Marks : 300

SUBJECT - MATHEMATICS

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MATHEMATICS

1. Find three digits number divisible by 3 made from digits 1, 3, 5, 8

(1) 28 (2) 22 (3) 18 (4) 36

Ans. **(2)**

Sol. Case

$1, 1, 1 \rightarrow (1)$ way

$3, 3, 3 \rightarrow (1)$ way

$1, 3, 5 \rightarrow 3! = 6$ way

$1, 3, 8 \rightarrow 3! = 6$ way

$5, 5, 5 \rightarrow (1)$ way

$5, 5, 8 \rightarrow (3)$ way

$8, 8, 5 \rightarrow (3)$ way

$8, 8, 8 \rightarrow (1)$ way

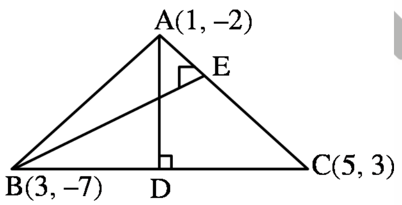
total way = 22

2. Vertices of triangle are A(1, –2), B(3, –7), C(5, 3) and orthocentre is (α, β), then $9\alpha - 6\beta + 40$ is equal to

(1) $\frac{8}{5}$ (2) $\frac{16}{5}$ (3) $\frac{9}{5}$ (4) $\frac{11}{5}$

Ans. **(2)**

Sol. $m_{BC} = 5$, $m_{AB} = -\frac{5}{2}$, $m_{AC} = \frac{5}{4}$. It is not a right angle Δ



Equation of AD is $y + 2 = -\frac{1}{5}(x - 1) \Rightarrow 5y + 10 = -x + 1 \Rightarrow x + 5y = -9$ ......(i)

Equation of BE is $y + 7 = -\frac{4}{5}(x - 3) \Rightarrow 5y + 35 = -4x + 12 \Rightarrow 4x + 5y = -23$ ......(ii)

$\alpha = -\frac{14}{3}, \beta = \frac{1}{5}\left(\frac{14}{3} - 9\right) = \frac{1}{5}\left(-\frac{13}{3}\right) = -\frac{13}{15}$

Now $9\alpha - 6\beta + 40 = -42 + \frac{13 \times 6}{15} + 40 = -2 + \frac{26}{5} = \frac{16}{5}$

3. A bag contain 6 white balls and 4 black balls. A dice is rolled and number comes on dice is equal to number of ball drawn then the probability that all balls are white.

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{4}{15}$ (D) $\frac{2}{15}$

Ans. **(1)**

Sol. Required probability

$$\frac{1}{6}\left(\frac{^6C_1}{^{10}C_1}+\frac{^6C_2}{^{10}C_2}+\frac{^6C_3}{^{10}C_3}+.........+\frac{^6C_6}{^{10}C_6}\right)$$

$$\frac{1}{6}\left(\frac{3}{5}+\frac{1}{3}+\frac{1}{6}+\frac{1}{14}+\frac{1}{42}+\frac{1}{210}\right)$$

$$=\frac{1}{6}\left(\frac{126+70+35+15+5+1}{210}\right)$$

$$=\frac{1}{6}\left(\frac{252}{210}\right)=\frac{42}{210}=\frac{1}{5}$$

4. Number of common tangents of the circle $C_1 : (x-3)^2+(y-3)^2=\left(\frac{25}{4}\right)$ and

$C_2 : (x-9)^2+\left(y-\frac{15}{2}\right)^2=25$ is

(1) 1 (2) 2 (3) 3 (4) 4

Ans. **(3)**

Sol. Distance between centre = 7.5

$r_1 + r_2 = 7.5$

$C_1\ C_2 = r_1 + r_2$ so 3 common tangents

5. If $(a+bx+cx^2)^{10}=\sum_{i=0}^{20}p_ix^i$, $p_1 = 20$ and $p_2 = 210$, a, b, c $\in$ N, then the value of 2(a + b + c) is equal to

(1) 12 (2) 15 (3) 18 (4) 22

Ans. **(1)**

Sol. General term = $\frac{10!}{r_1!r_2!r_3!}(a)^{r_1}(bx)^{r_2}(cx^2)^{r_3}$

$r_1 + r_2 + r_3 = 10$

for coefficient of x^1, $r_2 + 2r_3 = 1$

$r_2 = 1, r_3 = 0, r_2 = 9$

$p_1=\frac{10!}{1!0!9!}(a)^9b^1c^0$

$\Rightarrow 10a^9b = 20$

$a^9b = 2$

$\because$ a, b $\in$ N so a = 1, b = 2

for coefficient of x^2

$r_2 + 2r_3 = 2 \Rightarrow r_2 = 2, r_3 = 0, r_1 = 8$

$r_2 = 0, r_3 = 1, r_1 = 9$

$p_2 = \frac{10!}{2!8!}(a^8.b^2) + \frac{10!}{1!9!}(a)^9(b^0)c^1 = 210$

$\frac{10 \times 9}{2} \times 4 + 10c = 210$

$18 + c = 21 \Rightarrow c = 3$

a = 1, b = 2, c = 3

6. Maximum digit is 7 and sum of first two digit is equal to sum of last two digit. Find number of 4 digit code.

Ans. **(344)**

Sol. 4 digit code

Sum of Ist two digit can be $0 = 1.1 = 1^2$

Sum of Ist two digit can be $1 = 2.2 = 2^2$

Sum of Ist two digit can be $2 = (2! + 1) = 3^2$

Sum of Ist two digit can be $3 = (1 + 2 + 1) = 4.4 = 4^2$

Sum of Ist two digit can be $4 = (1 + 2 + 1 + 1) = 5.5 = 5^2$

Sum of Ist two digit can be $5 = 6^2$

Sum of Ist two digit can be $6 = 7^2$

Sum of Ist two digit can be $7 = 8^2$

Sum of Ist two digit can be $8 = 7^2$

Sum of Ist two digit can be $9 = 6^2$

Sum of Ist two digit can be $10 = 5^2$

Sum of Ist two digit can be $11 = 4^2$

Sum of Ist two digit can be $12 = 3^2$

Sum of Ist two digit can be $13 = 2^2$

Sum of Ist two digit can be $14 = 1^2$

Total $= 2(1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2) + 8^2$

$= 2.\frac{7.8.15}{6} + 64 = 280 + 64 = 344$

7. A ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a > b) has length of latus rectum is 4. If minor axis subtends angle of 60º on one of it's focus the sum of squares of length of major axis and minor axis is

Ans. **(320)**

Sol. $\frac{2b^2}{a} = 4 \Rightarrow b^2 = 2a$ ......(i)

$\tan 30º = \frac{b}{ae} = \frac{1}{\sqrt{3}}$

$\sqrt{3}\,b = ae$

$3b^2 = a^2e^2$ ......(ii)

$3(a^2(1 - e^2)) = a^2 e^2$

$3 - 2e^2 = e^2$

$4e^2 = 3 \Rightarrow e = \frac{\sqrt{3}}{2}$

Using (i) and (ii)

$3(2a) = a^2\left(\frac{3}{4}\right) \Rightarrow a = 8, b = 4$

$(2a)^2 + (2b)^2 = 4(64 + 16) = 320$

8. Negation of $(p \wedge q) \wedge (\sim (p \wedge q))$ is

(1) $p \vee q$ (2) $\sim(p \vee q)$ (3) $p \vee \sim p$ (4) $(p \vee q) \vee p$

Ans. **(3)**

Sol. We know that $A \wedge (\sim A) \neq \phi$ so negation of this is tautology so by option (3) is correct

9. $\int_0^1 \frac{1}{(2+x-x^2)(1+e^{1-2x})}dx$ is equal to

(1) $\frac{1}{3}\ell n2$ (2) $\frac{1}{3}\ell n4$ (3) $\frac{4}{3}\ell n2$ (4) None of these

Ans. **(1)**

Sol. $I = \int_0^1 \frac{1}{(2+x-x^2)(1+e^{1-2x})}dx$

$I = \int_0^1 \frac{dx}{(2+(1-x)-(1-x)^2)(1+e^{1-2(1-x)})}$

$2I = \int_0^1 \frac{dx}{2+x-x^2}$

$I = -\frac{1}{2}\int_0^1 \frac{dx}{x^2-x-2} = -\frac{1}{6}\int_0^1 \left(\frac{1}{x-2} - \frac{1}{x+1}\right)dx = -\frac{1}{6}\left[\ell n|x-2| - \ell n|x+1|\right]_0^1$

$= -\frac{1}{6}\left[(\ell n1 - \ell n2) - (\ell n2 - \ell n1)\right] = \frac{1}{3}\ell n\,2$

so $\alpha = 3, \beta = 2$

$\Rightarrow \alpha^4 - \beta^4 = 81 - 16 = 65$

10. If $x|x| + 5|x + 2| + 6 = 0$ then number of real roots are

(1) 1 (2) 2 (3) 3 (4) 4

Ans. (1)

Sol. $x < -2$

$-x^2 - 5x - 10 + 6 = 0$

$x^2 + 5x + 4 = 0$

$x = -1, -4$

$x = -4$

$x \in [-2, 0]$

$-x^2 + 5x + 10 + 6 = 0$

$x^2 - 5x - 16 = 0$

$\frac{5 \pm \sqrt{89}}{2}$

no solution

$x > 0$

$x^2 + 5x + 10 + 6 = 0$

$x^2 + 5x + 16 = 0$

no solution

so one solution

11. The mean and standard deviation of 10 observations are 20 and 8 if one number out of 10 is 40 and it is replaced by 50 then find new variance

Ans. (113)

Sol. $\frac{x_1 + x_2 + ...x_9 + 40}{10} = 20 \Rightarrow x + ... x_9 + 40 = 200 \Rightarrow \sum_{i=1}^{9} x_i = 160$

variance $= \frac{\sum (x_i)^2}{10} - (20)^2 = 64 \Rightarrow \sum_{i=1}^{9} x_i^2 = 4640 - 1600 = 3040$

so new variance $= \frac{3040 + 50^2}{10} - \left(\frac{160 + 50}{10}\right)^2 = 149$

12. If $S = \left(\frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{2^2} + \frac{1}{2.3} + \frac{1}{3^2}\right) + \left(\frac{1}{2^3} + \frac{1}{2^2.3} + \frac{1}{2.3^2} + \frac{1}{3^3}\right) + ... \infty$ and $S = \frac{\alpha}{\beta}$ (where α, β are relatively prime numbers the $\alpha + 3\beta$ is equal to

Ans. (5)

Sol. $x = \frac{1}{2}, y = \frac{1}{3}$

$S = (x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + ... \infty$

$(x - y) S = \frac{x^2}{1-x} - \frac{y^2}{1-y}$

$(x - y) S = \frac{x^2 - x^2y - y^2 + y^2x}{(1-x)(1-y)} = \frac{(x^2 - y^2) - xy(x-y)}{(1-x)(1-y)}$

$(x - y) S = \frac{(x+y-xy)(x-y)}{(1-x)(1-y)}$

$$S = \frac{\frac{1}{2}+\frac{1}{3}-\frac{1}{6}}{\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)} = \frac{\frac{3+2-1}{6}}{\frac{1}{2}\cdot\frac{2}{3}} = \frac{\frac{4}{6}}{\frac{1}{3}} = 2$$

13. Let $f(x) = \ln(4x^2 + 11x + 6) + \sin^{-1}(4x + 3) + \cos^{-1}\left(\frac{10x+6}{3}\right)$. Let domain of f(x) is $(\alpha, \beta]$ find

$|10(\alpha - \beta)| =$

Ans. **(2.5)**

Sol. $-1 \le 4x + 3 \le 1 \Rightarrow -4 \le 4x \le -2 \Rightarrow -1 \le x \le \frac{-1}{2}$ ...(i)

and $-1 \le \frac{10x+6}{3} \le 1 \Rightarrow -3 \le 10x + 6 \le 3 \Rightarrow -9 \le 10x \le -3 \Rightarrow \frac{-9}{10} \le x \le \frac{-3}{10}$ ...(ii)

and $4x^2 + 11x + 6 > 0 \Rightarrow 4x^2 + 8x + 3x + 6 > 0 \Rightarrow 4x(x + 2) + 3(x + 2) > 0$

$\Rightarrow (x + 2)(4x + 3) > 0 \Rightarrow x \in (-\infty, -2) \cup \left(\frac{-3}{4}, \infty\right)$ ...(iii)

Intersection of (i), (ii), (iii) is $\left(\frac{-3}{4}, \frac{-1}{2}\right]$

$|10(\alpha - \beta)| = \left|10\left(\frac{-3}{4} + \frac{1}{2}\right)\right| = \frac{5}{2}$

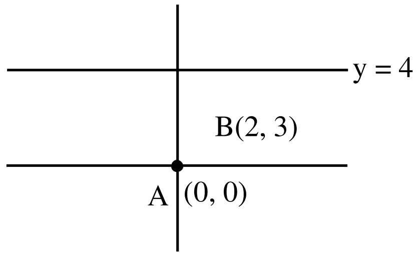
14. Let A(0, 0), B(2, 3), C(t, 4) are vertices of triangle $t = \alpha$, $t = \beta$ are two values of 't' corresponding to the maximum and minimum value of perimeter of triangle the value of $6\alpha + 5\beta$ is (Where $t \in (0, 4]$)

(1) 16 (2) 24 (3) 32 (4) 36

Ans. **(3)**

Sol. Image of A is line y = 4 is A(0, 8)

Equation of A′B is $(y-8) = \frac{3-8}{2-0}(x-0)$



$y - 8 = -\frac{5}{2}x$

at $y = 4 \Rightarrow -4 = -\frac{5}{2}\beta \Rightarrow \beta = \frac{8}{5}$

for maximum determine values at boundary points

$PA + PB = \sqrt{(t^2+16)} + \sqrt{(t-2)^2+1}$

maximum at t = 4, $PA + PB = \sqrt{32} + \sqrt{5}$

$\alpha = 4,\ \beta = \frac{8}{5} \Rightarrow 6\alpha + 5\beta = 32$

15. Let f(x) = max (1 + x + [x], 2 + x, x + 2[x]) for x ∈ [0, 2], m is number of points where f(x) is discontinuous in [0, 2] and n is number of points where f(x) is non-differentiable in (0, 2) then $(m+n)^2 + 2$ is equal to

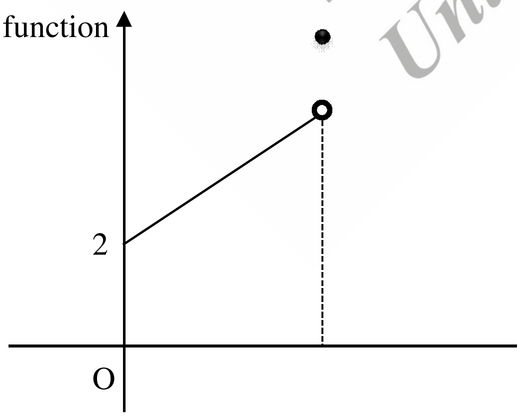
(1) 2 (2) 3 (3) 6 (4) 8

Ans. **(2)**

Sol. $1 + x + [x] = \begin{cases} 1+x & x \in [0,1) \\ 2+x & x \in [1,2) \\ 5 & x = 2 \end{cases}$

$x + 2[x] = \begin{cases} x & x \in [0,1) \\ x+2 & x \in [1,2) \\ 5 & x = 2 \end{cases}$

Graph of function



m =1, n = 1

$(m+n)^2 + 2 = (1+0)^2 + 2 = 3$

16. Two arithmetic means A_1 & A_2 and 3 geometric means G_1, G_2, G_3 are inserted between two positive numbers a & b, then the value of $G_1^4 + G_2^4 + G_3^4 + G_1^2 . G_3^2$ is

(1) $(A_1 + A_2) G_1G_3$

(2) $(A_1 + A_2)^2 G_1G_3$

(3) $(A_1 + A_2) (G_1 + G_3)$

(4) $(A_1 + A_2)^2 (G_1+G_3)^2$

Ans. **(2)**

Sol. a, G_1, G_2, G_3, b

$G_1 = ar, G_2 = ar^2, G_3 = ar^3, b = ar^4$

$a_1, A_1, A_2, b \Rightarrow A_1 + A_2 = a + b$

$G_1^4 + G_2^4 + G_3^4 + G_1^2 G_3^2 = (ar)^4 + (ar^2)^4 + (ar^3)^4 + (ar^2)^2(ar^3)^2$

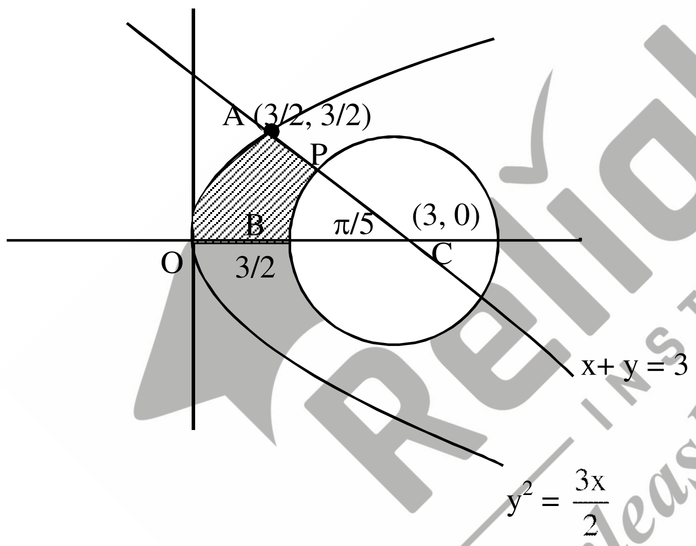
$= a^4[r^4 + r^8 + r^{12} + r^8] = a^4r^4(1 + 2r^4 + r^8) = a^4r^4 (1 + r^4)^2 = a^2r^4(a + ar^4)^2$

$= G_1(G_3) (a + b)^2 = G_1G_3(A_1 + A_2)^2$

17. Find area bounded curve $2y^2 = 3x$, $y = 0$, $(x - 3)^2 + y^2 = 2$ and $x + y = 3$ and above x-axis is A Find $4(\pi + 4A)$.

Ans. **(42)**

Sol.



$2y^2 = 3x, y = 3 - x$

$2(3 - x)^2 = 3x$

$2(9 - 6x + x^2) = 3x$

$2x^2 - 15x + 18 = 0$

$2x^2 - 12x - 3x + 18 = 0$

$2x^2 - 12x - 3x + 18 = 0$

$2x (x - 6) - 3(x - 6) = 0$

$(x - 6) (2x - 3) = 0$

$x = 6, 3/2$

Area of A & C $= \frac{\pi r^2}{4} = \frac{\pi \times 2}{4} = \frac{9}{8}$

Area of ARC = $\frac{1}{2}r^2\theta = \frac{\pi}{4}$

Area = $\int_0^{3/2} \frac{\sqrt{3}}{\sqrt{2}}\sqrt{x}\,dx + = \frac{\sqrt{3}}{\sqrt{2}} \times \left(\frac{x^{3/2}}{3/2}\right)_0^{3/2} = \frac{\sqrt{3}}{\sqrt{2}\times 3}\times 2\left(\left(3/2\right)^{3/2} - 0\right)$

$= \frac{\sqrt{2}}{\sqrt{3}}\left(\frac{3\sqrt{3}}{2\sqrt{2}}\right) = \frac{3}{2}$

Total area (A) = $\frac{3}{2} + \frac{9}{8} - \frac{\pi}{4}$

$4A = \frac{21}{2} - \pi$

$= 4(\pi + 4A) = 42$

18. Let A be a square matrix of order m and m, n related by $m^2 + n^2 - 6m - 4n + 13 = 0$, $|A| = 10$, then $|nAdj(Adj(mA)| = 3^a \cdot 5^b \cdot 6^c$. Find a + b + c

Ans. **(16)**

Sol. $m^2 - 6m + n^2 - 4n + 13 = 0$

$(m-3)^2 + (n-2)^2 = 0$

m = 2, n = 2

$|2Adj\ (Adj\ (3A))|$

$2^3\ |Adj\ (Adj\ 3A)|$

$8|Adj\ (3A)|^2 = 4|3A|^4 = 8 \times 3^{12}\ |A|^4 = 2^3 \times 3^{12} \times 10^4 = 6^7 \times 3^5 \times 5^4$

19. If a relation R is given by (a, b) R(c, d) $\Rightarrow$ 3a + 2b = 4c + 5d where a, b, c, d $\in$ {1, 2, 3, 4}, then total number of elements in this relation are

(1) 4 (2) 5 (3) 6 (4) 7

Ans. **(3)**

Sol.

3a	2b	4c	5d
3	2	4	5
6	4	8	10
9	6	12	15
12	8	16	20

3a + 2b can be = 5, 7, 9, 11, 8, 10, 12, 14, 11, 13, 15, 17, 14, 16, 18, 20

4c + 5d can be = 9, 14, 19, 24, 13, 18, 23, 28, 17, 22, 27, 32, 21, 26, 31, 36

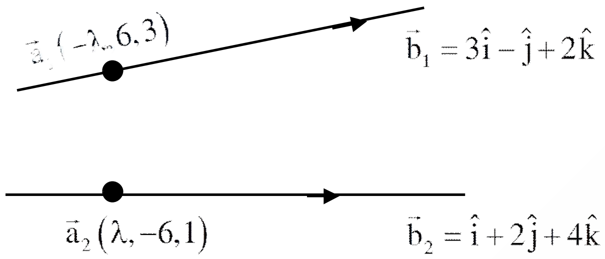
Equality holds at 9, 14, 13, 17, 14, 18

Total elements are 6

20. $\frac{x+\lambda}{3}=\frac{y-6}{-1}=\frac{z-3}{2}$, $\frac{x-\lambda}{1}=\frac{y-6}{2}=\frac{z-1}{3}$ if shortest distance between two line is $\sqrt{3}$. Find $\sum 80\lambda$.

Ans. **(800)**

Sol.



$\vec{a}_2-\vec{a}_1 = 2\lambda\hat{i}-12\hat{j}-2\hat{k}$

$$\vec{b}_1\times\vec{b}_2 = \begin{vmatrix} i & j & k \\ 3 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i}(-7)-\hat{j}(7)+\hat{k}(7)$$

$$= \left|\frac{\left(\vec{a}_2-\vec{a}_1,\left(\vec{b}_1\times\vec{b}_2\right)\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right| = \left|\frac{-14\lambda+84-14}{7\sqrt{3}}\right| = \sqrt{3}$$

$|-14\lambda + 70| = 21$

$-14\lambda + 70 = \pm 21$

$-14\lambda = -91$

$\lambda = \frac{91}{14}$

$-14\lambda = -49$

$\lambda = \frac{49}{14}$

$80\left(\frac{91}{14}+\frac{49}{14}\right) = 80(10) = 800$

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