COMMON P.G. ENTRANCE TEST–2022 (CPET-2022)

Subject Code : **60**

Test Booklet No.:

Entrance Subject : **Mathematics**

Hall Ticket No.:

TEST BOOKLET

Time Allowed : **90** Minutes

Full Marks : **70**

INSTRUCTIONS TO CANDIDATES

1. **Please do not open this Question Booklet until asked to do so.**
2. Check the completeness of the Question Booklet immediately after opening.
3. Enter your **Hall Ticket No.** on the Test Booklet in the box provided alongside. **Do not** write anything else on the Test Booklet.
4. Fill up & darken Hall Ticket No. & Test Booklet No. in the OMR Answer Sheet as well as fill up Test Booklet Serial No. & OMR Answer Sheet Serial No. in the Attendance Sheet carefully. Wrongly filled up OMR Answer Sheets are liable for rejection.
5. Each question has four answer options marked (A), (B), (C) & (D).
6. Answers are to be marked on the Answer Sheet, which is provided separately.
7. Choose the most appropriate answer option and darken the oval completely, corresponding to (A), (B), (C) or (D) against the relevant question number.
8. Use only **Blue/Black Ball Point Pen** to darken the oval for answering.
9. Please do not darken more than one oval against any question, as scanner will read such markings as wrong answer.
10. **Each question carries equal marks. There will be no negative marking for wrong answer.**
11. **Electronic items such as calculator, mobile, etc., are not permitted inside the examination hall.**
12. Don't leave the examination hall until the test is over and permitted by the invigilator.
13. **The candidate is required to handover the original OMR sheet to the invigilator and take the question booklet along with the candidate's copy of OMR sheet after completion of the test.**
14. Sheet for rough work is appended in the Test Booklet at the end.

1. The set of cluster points of the sequence $x_n = 1 + (-1)^n$ is

(A) $\{1\}$ (B) $\{-1, 1\}$ (C) $\{0, 1\}$ (D) $\{0, 2\}$

2. If $x_n = \sin\frac{n\pi}{3}$, then $\lim \sup x_n = ---$

(A) 0 (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) Does not exist

3. The value of $\sum_{n=0}^{\infty} \frac{1}{(n+1)(n+2)}$ is

(A) 1 (B) 2 (C) 3 (D) ∞

4. The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2 \log n}$ is

(A) Divergent (B) conditionally convergent

(C) absolutely convergent (D) oscillatory

5. A sequence $\{x_n\}$ is given by $x_{n+1} = 2 - \frac{1}{x_n}$ be convergent. What is its limit ?

(A) 0 (B) 1 (C) 2 (D) ∞

6. $x_1 = 1, \quad x_{n+1} = \sqrt{2 + x_n}$, then the sequence $\{x_n\}$is

(A)monotonically increasing (B) monotonically decreasing

(C) oscillates between finite limits (D) oscillates infinitely

7. The power series expansion of $(1 + x)\log(1 + x)$is

(A) $x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$ (B) $1 - \frac{x^2}{1.2} + \frac{x^4}{2.3} - \frac{x^6}{3.4} + \ldots$

(C) $x - \frac{x^2}{1.2} + \frac{x^3}{2.3} - \frac{x^4}{3.4} + \ldots$ (D) $x + \frac{x^2}{1.2} - \frac{x^3}{2.3} + \frac{x^4}{3.4} - \frac{x^5}{4.5} + \ldots$

8. What is the radius of convergence of the power series $\sum a_k x^k$, where $a_k = \frac{2^k + 1}{4^k - 2^k}$?

(A) 1 (B) 0 (C) 2 (D) 4

9. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{x + n}$ is

(A) uniformly convergent in $[-1, 1]$

(B) uniformly convergent in $[0, 2)$

(C) uniformly convergent in $[0, \infty)$

(D) not uniformly convergent

10. For what value of k is the integral $\int_1^\infty x^k \frac{x + \sin x}{x - \sin x} dx$ is convergent ?

(A) $k < -1$ (B) $k > 1$

(C) $k > -1$ (D) Divergent for all k

11. $\alpha = (1\ 3)(2\ 7)(4\ 5\ 6)(8)$ and $\beta = (1\ 2\ 3\ 7)(6\ 4\ 8)(5)$ be permutations, then $\alpha\beta = - - -$

(A) $(1\ 2\ 3)(4\ 5\ 6\ 7\ 8)$ (B) $(1\ 2)(3\ 4)(5\ 6)(7\ 8)$

(C) $(1\ 8)(2\ 7)(3\ 5)(4\ 6)$ (D) $(1\ 7\ 3\ 2)(4\ 8)(5\ 6)$

12. In $GL(2, Z_7)$, what is the inverse of the matrix $\begin{bmatrix} 4 & 5 \\ 6 & 3 \end{bmatrix}$?

(A) $\begin{bmatrix} 1 & 3 \\ 5 & 6 \end{bmatrix}$ (B) $\begin{bmatrix} -\frac{1}{6} & -\frac{5}{18} \\ \frac{1}{3} & \frac{2}{9} \end{bmatrix}$

(C) $\begin{bmatrix} 3 & 5 \\ 6 & 4 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

13. Let $\langle a \rangle$ be a cyclic group of order 30. A subgroup of order 6 of $\langle a \rangle$ is

(A) $\langle a^2 \rangle$ (B) $\langle a^3 \rangle$ (C) $\langle a^5 \rangle$ (D) $\langle a^6 \rangle$

14. In $U(10)$, which of the following is true ?

(A) $3^{-1} = 3$ (B) $3^{-1} = 7$ (C) $3^{-1} = 1$ (D) $3^{-1} = 9$

15. If A_n is an alternating group, then what is $|A_5|/|A_4|$?

(A) 5 (B) $5/4$ (C) 30 (D) 15

16. If $a = .75632 \times 10^2$ and $b = .235472 \times 10^{-1}$, what is $a + b$ using 6 decimal arithmetic ?

(A) 7.56555×10^3 (B) 7.56555×10^2

(C) $.756555 \times 10^2$ (D) $.756556 \times 10^3$

17. What is the order of approximation of e^x at $x = 1$ obtained from its Maclaurin series correct to 8 decimal places ?

(A) 8 (B) 7 (C) 10 (D) 11

18. Suppose a root of the equation $f(x) = 0$ lies in $[0,\ 1]$. How many steps are required by the method of Bisection to approximate the root with absolute error less than or equal to 0.1 ?

(A) 2 (B) 3 (C) 4 (D) 5

19. The iteration scheme of Secant method is

(A) $x_{k\ +\ 1} = x_k + \frac{x_k - x_{k\ -\ 1}}{f_k - f_{k\ -\ 1}} f_k$ (B) $x_{k\ +\ 1} = x_k - \frac{f_k - f_{k\ -\ 1}}{x_k - x_{k\ -\ 1}} f_k$

(C) $x_{k\ +\ 1} = x_k + \frac{f_k - f_{k\ -\ 1}}{x_k - x_{k\ -\ 1}} f_k$ (D) $x_{k\ +\ 1} = x_k - \frac{x_k - x_{k\ -\ 1}}{f_k - f_{k\ -\ 1}} f_k$

20. The iteration scheme $x_{n\ +\ 1} = \frac{1}{2} x_n \left(1 + \frac{2}{x_n{}^2}\right)$ can be used to obtain successive approximation to

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) $\sqrt{5}$ (D) $\sqrt{7}$

21. Which of the following is not a metric in R ?

(A) $|x - y|$ (B) $|x - y|^{\frac{1}{2}}$ (C) $|x - y|^{\frac{1}{3}}$ (D) $|x - y|^2$

22. If (X, d) is a metric space, then which of the following is not a metric on X ?

(A) $d\ +\ 5$ (B) $\frac{d}{1\ +\ d}$ (C) $1\ +\ \frac{1}{d}$ (D) $5d$

23. Which of the following sequence is having infinite range ?

(A) $\frac{1}{(-1)^n}$ (B) $(-1)^n + 10^{10}$ (C) $\frac{1}{(-2)^n}$ (D) All of these

24. Which of the following function space is not complete ?

(A) $C[a,\ b], d(x,\ y) = \max_{t \in [a,b]} |x(t) - y(t)|$

(B) X, the set of all polynomials in J = $[0,\ 1]$ with $d(x,\ y) = \max_{t\ \in\ J} |x(t) - y(t)|$

(C) The space l^p with usual metric

(D) R^n with usual metric

25. $(X,\ d)$ is discrete if

(A) $d(x,\ y) = |x - y|$ (B) $d(x,\ y) = |x - y|^{\frac{1}{2}}$

(C) $d(x,\ y) = \begin{cases} 0 \ if \ x = y \\ 1 \ if \ x \neq y \end{cases}$ (D) $d(x,\ y) = 2|x - y|$

26. $S = \{(1,\ 0,\ 2),\ (-1,\ 1,\ 1)\}$, which of the following is true ?

(A) S is L.D. (B) $[S] = R^3$

(C) $[S] = R^2$ (D) $(2,\ -1,\ 1) \in [S]$

27. What is the dimension of the vector space of polynomials of degree ≤ 100 ?

(A) 99 (B) 100 (C) 101 (D) Does not exist

28. If $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, what is the number of solutions of $AX = b$?

(A) Unique solution (B) Infinite number of solutions

(C) No solution (D) Exactly three solutions

29. If $A = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 6 & 4 \\ 3 & -1 & 6 \end{bmatrix}$, what is the dimension of column space of A ?

(A) 1 (B) 2 (C) 3 (D) 4

30. Given $A = \begin{bmatrix} 4 & 2 & 1 \\ 1 & 4 & 2 \\ 2 & 1 & 4 \end{bmatrix}$, what is the product of the eigen values of A ?

(A) 16 (B) 7 (C) 49 (D) 56

31. What is the value of $\lim_{Z \to 0} f(z)$,where $f(z) = \frac{x^2 y}{x^4 + y^2}$, $z \neq 0$

(A) 1 (B) $\frac{1}{2}$ (C) 0 (D) Does not exist

32. $f(z) = iz + 2$ is

(A) not continuous at $(0, 0)$ (B) no where differentiable

(C) differentiable at every point (D) continuous but not differentiable

33. If $\cos(1 + i) = u + iv$,then what is v ?

(A) $\cos 1 . \cosh 1$ (B) $-\cos 1 . \cosh 1$

(C) $\sin 1 . \sinh 1$ (D) $-\sin 1 . \sinh 1$

34. The value of $\int_C \frac{dz}{z-3i}$, where Cis the circle $|z| = \pi$is

(A) 1 (B) πi (C) 2πi (D) π

35. The statement of Liouville's theorem is

(A) If $f$is constant, then f is entire

(B) If f is entire and bounded, then$f$is constant

(C) Every non-constant polynomial of degree n has at most n roots

(D) Every continuous function is bounded

36. A typist type 2 letters erroneously for every 100 letters. What is the probability that the tenth letter typed is the first erroneous letter ?

(A) 0.167 (B) 0.0157 (C) 0.157 (D) 0.0167

37. A fair die is thrown several times. What is the probability that a 3 will appear before 4 ?

(A) 0.33 (B) 0.66 (C) 0.5 (D) 0.45

38. If density function of a continuous random variable is $f(x) = \frac{c}{1+x^2}$, $-\infty < x < \infty$, what is the value of c ?

(A) 1 (B) π (C) $\frac{1}{\pi}$ (D) $\frac{\pi}{2}$

39. If X is uniformly distributed over $(0, 10)$, what is the probability that $X < 2$?

(A) $\frac{1}{10}$ (B) $\frac{1}{2}$ (C) $\frac{1}{20}$ (D) $\frac{1}{5}$

40. The probability density function of the length of time that a person speaks over phone is $f(x) = \begin{cases} B\, e^{-\frac{x}{6}}, x > 0 \\ 0 \;\; otherwise \end{cases}$, what is the probability that the person will talk less than 4 minutes ?

(A) 0.2432 (B) 0.4865 (C) 0.8125 (D) 0.333

41. What is the remainder of the polynomial $x^5 + x^4 + x^3 + x + 1$ on division by $x + 2$?

(A) $x - 21$ (B) 24 (C) -5 (D) -25

42. Which of the following rings is a field ?

(A)Z_4 (B) Z (C) Z_3 (D) Z_6

43. What is the unity of $\{0, 2, 4\}$ as a subring of Z_6 ?

(A)0 (B) 2 (C) 4 (D) Does not exist

44. What is centre of the ring Z ?

(A) {0} (B){1} (C) Z (D) {0, 1}

45. For what value of integer $n > 1$, $a^n = a \ \forall a \in Z_6$?

(A) 1 (B) 2 (C) 3 (D) 4

46. For a plane curve which of the following is appropriate ?

(A) Curvature is zero (B) Torsion is non-zero

(C) Curvature is non-zero constant (D) Torsion is zero

47. A necessary and sufficient condition for a curve to be a helix is

(A) curvature=torsion

(B) curvature varies directly as torsion

(C) curvature varies inversely as torsion

(D) both curvature and torsion are constants

48. If $\vec{r}$ is position vector of any point on a space curve then $[\vec{r}', \vec{r}'', \vec{r}'''] = ?$

(A) κ (B) τ (C) $\kappa\tau^2$ (D) $\kappa^2\tau$

49. The number of non-basic variables in the balanced transportation problem with 4 rows and 5 columns is

(A) 12 (B) 20 (C) 10 (D) 9

50. The value of game of the payoff matrix

$$\begin{array}{c} \\ \text{Player A} \end{array} \begin{array}{c} \text{Player B} \\ \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \end{array} \begin{bmatrix} 1 & 1 \\ 4 & -3 \end{bmatrix} \end{array} \text{ is}$$

(A) -3 (B) 4 (C) 1 (D) 2

51. What is the period of $f(x) = |\sin x| + |\cos x|$?

(A) $\frac{\pi}{2}$ (B) 2π (C) π (D) 4π

52. $\lim_{x\to 0} \frac{e^{\tan x} - e^{x}}{\tan x - x}$ is equal to

(A) 0 (B)e (C) e^{-1} (D) 1

53. If $2^{x} + 2^{y} = 2^{x+y}$, then the value of $\frac{dy}{dx}$ at $x = y = 1$is

(A) 1 (B) 2 (C) 0 (D) -1

54. The angle of intersection of the curves $r = a\cos\theta$, $2r = a$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) 0

55. The maximum value of $\frac{\log x}{x}$ is

(A) e (B) $\frac{1}{e}$ (C) 0 (D) 1

56. The function $f(x) = x(x + 3)e^{-x/2}$ satisfy all conditions of Rolle's theorem in the interval $[-3, 0]$. Then the value of c is

(A) 0 (B) 1 (C) 2 (D) -2

57. The value of $\lim_{x\to 0} \frac{e^{ax} - e^{-ax}}{\log(1 + bx)}$ is

(A) $\frac{2a}{b}$ (B) $\frac{b}{2a}$ (C) $\frac{e^{a} - e^{-a}}{\log(1 + b)}$ (D) 0

58. If $z = \tan^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$, then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ =-------

(A) $\sin z \cos z$ (B) 0 (C) $\tan z$ (D) None of these

59. A curve is convex upwards if

(A) $\frac{d^2y}{dx^2} > 0$ (B) $\frac{d^2y}{dx^2} < 0$

(C) $\frac{d^2y}{dx^2}=0$ and $\frac{d^3y}{dx^3} \neq 0$ (D) $\frac{dy}{dx}$ becomes infinite

60. The asymptotes of the curve $x^2y^2 - a^2(x^2 + y^2) = 0$ form a

(A) circle (B) triangle (C) square (D) pentagon

61. The folium of Descartes is given by the equation

(A) $x^2 + y^2 = 3ax^2y^2$ (B) $x^3 + y^3 = 3axy$

(C) $x^3 + y^3 = 3ax^2y^2$ (D) $x^4 + y^4 = 3axy$

62. The particular integral of $(D^2 + 4) = x \sin x$ is

(A) $\frac{1}{9}(3x \sin x - 2\cos x)$ (B) $\frac{1}{9}(3x \cos x - \sin x)$

(C) $3x \sin x - 2\cos x$ (D) $3x \cos x - \sin x$

63. The roots of the system $\begin{cases} \frac{dx}{dt} + 4x + 3y = 0 \\ \frac{dy}{dt} + 2x + 5y = 0 \end{cases}$ is

(A) -2, 7 (B) 2, -7 (C) -2, -7 (D) 2, 7

64. Area of a loop of $r^2 = a^2 \sin 2\theta$ is

(A) a^2 (B) $\frac{1}{2}a^2$ (C) $\frac{1}{4}a^2$ (D) $\frac{1}{8}a^2$

65. The perimeter of the cardioid $r = a(1 - \cos\theta)$ is

(A) 2a (B) 4a (C) 6a (D) 8a

66. The value of $\int_0^1 \int_y^{\sqrt{y}} (x^2 + y^2)dy\,dx$ is

(A) $\frac{1}{35}$ (B) $\frac{2}{35}$ (C) $\frac{3}{35}$ (D) $\frac{4}{35}$

67. The directional derivative of $f(x,y) = e^{x^2y^2}$ at the point $P(1, -1)$ in the direction towards $Q(2, 3)$ is

(A) $\frac{-6e}{\sqrt{17}}$ (B) $\frac{6e}{\sqrt{17}}$ (C) $\frac{e}{\sqrt{17}}$ (D) $\frac{-e}{\sqrt{17}}$

68. The value of $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2 + y^2 + z^2)dx\,dy\,dz$ is

(A) 0 (B) $\frac{2}{3}abc\,(a^2 + b^2 + c^2)$

(C) $\frac{8}{3}abc\,(a^2 + b^2 + c^2)$ (D) none of these

69. The sufficient condition that the surface $z = f(x, y)$ should represent a developable surface $\left(p = \frac{\partial f}{\partial x},\ q = \frac{\partial f}{\partial y}\right)$ is

(A) $p = 0$ (B) $q = 0$ (C) $\frac{\partial p}{\partial x}=0$ (D) $\frac{\partial(p,q)}{\partial(x,y)}=0$

70. What is the condition that the differential equation $P\,du^2 + 2Qdu\,dv + Rdv^2 = 0$ represents orthogonal family of curves is

(A) $ER - 2FQ + GP = 0$ (B) $EQ - 2FR + GP = 0$

(C) $EG - 2FP + RQ = 0$ (D) $EF - 2GP + FR = 0$

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ROUGH WORK

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