1. Let f be the function on $\mathbb{R}$ defined by $f(x) = x^3 - 3x^2 + ax - 1$, where $a \in \mathbb{R}$. Then the set of all possible values of a for which f is strictly increasing is

   (a) $[3, \infty)$
   (b) $(-\infty, 3]$
   (c) $[-3, 0]$
   (d) $[0, 3]$

2. If $\dfrac{1}{(1+i)^{2023}} = te^{i\theta}$, where $t \in \mathbb{R}$ and $0 \leq \theta < 2\pi$, then the value of θ is

   (a) $\dfrac{\pi}{4}$
   (b) $\dfrac{3\pi}{4}$
   (c) $\dfrac{5\pi}{4}$
   (d) $\dfrac{7\pi}{4}$

3. Let S be the set of all 4-digit natural numbers with the following properties:

   (i) every digit of any element of S belongs to the set $\{0, 1, 3, 5, 7, 9\}$,
   (ii) every element of S is divisible by 5, and
   (iii) no element of S is divisible by 2.

   Then the number of elements in S is

   (a) 180
   (b) 216
   (c) 360
   (d) 250

4. If a teacher assigns homework on the nth day, the probability that she will assign homework on the $(n+1)$th day is $\frac{1}{3}$. If she does not assign homework on the nth day, the probability that she will assign homework on the $(n+1)$th day is $\frac{2}{3}$. If she assigned homework on a Monday then the probability that she will assign homework on the Thursday of the week is

   (a) $\dfrac{1}{3}$

   (b) $\dfrac{7}{27}$

   (c) $\dfrac{13}{27}$

   (d) $\dfrac{2}{3}$

5. The value of $\sin\dfrac{2\pi}{23} + \sin\dfrac{4\pi}{23} + \cdots + \sin\dfrac{42\pi}{23} + \sin\dfrac{44\pi}{23}$ is

   (a) -1

   (b) 0

   (c) 1

   (d) 2

6. Let $P = (a, b)$ be a point in the Euclidean plane, with a and b nonzero. For any point S on the x-axis, let T be the point of intersection of the line PS with the y-axis. Let M be the midpoint of the segment ST. Then the locus of M, as S varies on the x-axis, is given by

   (a) $xy = ab$

   (b) $xy = \dfrac{ab}{4}$

   (c) $xy = ay + bx$

   (d) $2xy = ay + bx$

7. Let f be a differentiable function on $\mathbb{R}$ satisfying the conditions

   (i) $f(x) = \int\limits_0^x (f(t))^{\frac{1}{3}} dt$ for all $x \in \mathbb{R}$, and

   (ii) $f(x) > 0$ for all $x > 0$.

   Then the value of $f(3)$ is

   (a) $2\sqrt{2}$

   (b) $3\sqrt{3}$

   (c) $\frac{1}{2}$

   (d) $\frac{1}{3}$

8. Let $f(x) = \ln x - 2023x + 2023$ for all $x \in (0, \infty)$. Then the number of points at which the graph of f cuts the x axis is

   (a) 0

   (b) 2

   (c) 3

   (d) 1

9. Let N be the number of integers n such that

   (i) $n = 2^a 3^b 5^c$ where a, b, c are non-negative integers ≤ 10, and

   (ii) n is neither a square nor a cube of a natural number.

   Then N is equal to

   (a) 848

   (b) 849

   (c) 1051

   (d) 1059

10. Let ABC be a triangle and let a, b and c denote the lengths of the sides BC, CA and AB respectively. Let α and β be positive real numbers such that

$$\alpha(\angle A) + \beta(\angle B) = (\alpha + \beta)(\angle C).$$

Then

(a) $\alpha a + \beta b = (\alpha + \beta)c$

(b) $\alpha a + \beta b = (\alpha + \beta)c$ implies $a = b$

(c) $\alpha a + \beta b > (\alpha + \beta)c$

(d) $\alpha a + \beta b = (\alpha + \beta)c$ implies $\alpha a = \beta b$

11. For $a, b \in \mathbb{R}$, with $a > 0$, let $N(a, b)$ denote the number of elements in the set $\{x \in \mathbb{R} \mid x + a \sin x = b\}$. Then

(a) $N(a, b) = 1$ for all a, b.

(b) there does not exist any a such that $N(a, b) = 1$ for all b.

(c) $N(a, b)$ is finite for all a, b.

(d) there exist a, b such that $N(a, b)$ is infinite.

12. Let N be the number of solutions of the equation

$$x_0 + 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 = 6,$$

with x_0, x_1, x_2, x_3, x_4 and x_5 taking non-negative integer values. Then

(a) $N < 50$

(b) $50 \leq N < 100$

(c) $100 \leq N < 1000$

(d) $1000 \leq N$

13. For $a, b > 0$ let $F(a, b) = \int\limits_a^b |\sin 2\pi x| dx$. Then

(a) $F(10, 11) = 2F(0, \frac{1}{2})$

(b) $F(\frac{41}{4}, \frac{43}{4}) = \frac{1}{2}F(\frac{1}{2}, 1)$

(c) $F(\frac{1}{8}, \frac{1}{4}) = F(1, 2)$

(d) $F(\frac{41}{4}, \frac{43}{4}) = \frac{2}{3}F(0, \frac{3}{4})$

14. Let $S = \{x, y, z\}$ and $f : S \to \mathbb{N}$ be a function. Let A be a subset of $\mathbb{N}$ such that the following conditions are satisfied:

   (i) if $f(x) \in A$ then $f(y) \in A$, and
   (ii) if $f(z) \notin A$ then $f(y) \notin A$.

   Then it follows that

   (a) whenever $f(x) \in A$, $f(z) \in A$.
   (b) whenever $f(x) \notin A$, $f(z) \notin A$.
   (c) whenever $f(z) \in A$, $f(x) \in A$.
   (d) whenever $f(z) \notin A$, $f(x) \notin A$.

15. Let $\mathbf{a}$ and $\mathbf{b}$ be non-zero vectors. Let S be the set of vectors $\mathbf{v}$ such that $\mathbf{a} \times \mathbf{v} = \mathbf{b}$. Then

   (a) there exists a positive real number r such that $||\mathbf{v}|| < r$ for all $\mathbf{v} \in S$.
   (b) S is non-empty if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.
   (c) S is contained in a plane.
   (d) if $\mathbf{v}_1$ and $\mathbf{v}_2$ are in S, then there exists $\lambda \in \mathbb{R}$ such that $\mathbf{v}_1 - \mathbf{v}_2 = \lambda\mathbf{a}$.

16. Let C_1, C_2 and C_3, be three circles having the same radius r, which touch each other externally. Then

   (a) for any circle C which is touched internally by C_1 and C_2, C_3 lies within C.
   (b) there is no circle C touched internally by C_1, C_2 and C_3.
   (c) a circle C touched internally by C_1, C_2 and C_3 has radius $\left(1 + \frac{2}{\sqrt{3}}\right) r$.
   (d) the radius of any circle C touched internally by C_1 and C_2 is at least $2r$.

17. Let $S = \{(a, b) \mid a, b \in \mathbb{Z}\}$. Let R be the equivalence relation on S defined by $(a, b)R(c, d)$ if $a^2 + b^2 = c^2 + d^2$. For $(a, b) \in S$ let $F(a, b)$ denote the equivalence class $\{(c, d) \in S \mid (a, b)R(c, d)\}$ of (a, b). Then

   (a) there exists $(a, b) \in S$ such that $F(a, b)$ has only one element.
   (b) there exists $(a, b) \in S$ such that $F(a, b)$ has exactly 4 elements.
   (c) there exists $(a, b) \in S$ such that $F(a, b)$ has exactly 6 elements.
   (d) there exists $(a, b) \in S$ such that $F(a, b)$ has infinitely many elements.