Section A: Q.1 – Q.10 Carry ONE mark each.

Q.1
Let
$$M = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$
. If a non-zero vector $X = (x, y, z)^T \in \mathbb{R}^3$ satisfies

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 $M^6X = X$, then a subspace of \mathbb{R}^3 that contains the vector X is

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(A)
$$\{(x, y, z)^T \in \mathbb{R}^3 : x = 0, y + z = 0\}$$

(B) $\{(x, y, z)^T \in \mathbb{R}^3 : y = 0, x \neq z = 0\}$

(B) {
$$(x, y, z)^T \in \mathbb{R}^3 : y = 0, x + z = 0$$
}

(C) {
$$(x, y, z)^T \in \mathbb{R}^3 : z = 0, x + y = 0$$
}

(D) {
$$(x, y, z)^T \in \mathbb{R}^3 : x = 0, y - z = 0$$
}

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Q.2 Let $M = M_1M_2$, where M_1 and M_2 are two 3×3 distinct matrices. Consider the following two statements:

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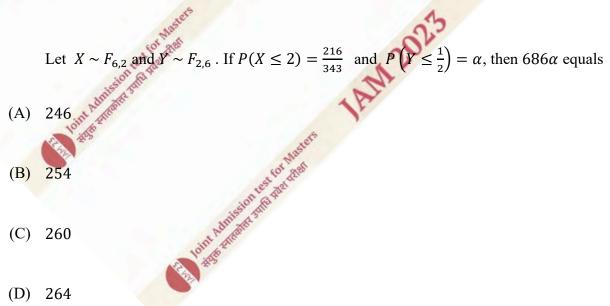
- (I) The rows of *M* are linear combinations of rows of M_2 .
- (II) The columns of M are linear combinations of columns of M_1 .

Then,

- (A) only (I) is TRUE
- (B) only (II) is TRUE
- (C) both (I) and (II) are TRUE
- (D) neither (I) nor (II) is TRUE



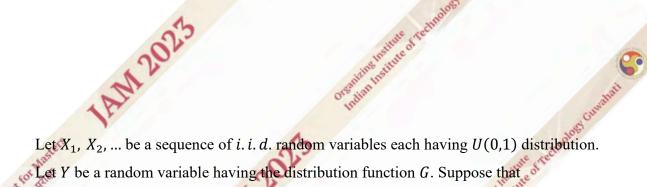
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- (B) 0.62
- (C) 0.64
- (D) 0.66



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 $\frac{-X_n}{-X_n} \le x = G(x), \text{ for all } x \in \mathbb{R}$ $\lim_{n\to\infty} P$

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Then, Var(Y) equals

(A) 1 coint Admit 12

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(C) 1 48

(D)
$$\frac{1}{64}$$

Q.6 Let X_1, X_2, X_3 be a random sample from an $N(\theta, 1)$ distribution, where $\theta \in \mathbb{R}$ is an unknown parameter. Then, which one of the following conditional expectations does NOT depend on θ ?

(A)
$$E(X_1 + X_2 - X_3 | X_1 + X_2)$$

- (B) $E(X_1 + X_2 X_3 | X_2 + X_3)$
- (C) $E(X_1 + X_2 X_3 | X_1 X_3)$
- (D) $E(X_1 + X_2 X_3 | X_1 + X_2 + X_3)$

For the function $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined by

 $f(x,y) = 2x^2 - xy - 3y^2 - 3x + 7y$

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the point (1, 1) is

- (A) a point of local maximum
- (B) a point of local minimum
- (C) a saddle point
- (D) NOT a critical point

Let E_1 , E_2 and E_3 be three events such that

$$P(E_1 \cap E_2) = \frac{1}{4}$$
, $P(E_1 \cap E_3) = P(E_2 \cap E_3) = \frac{1}{5}$ and $P(E_1 \cap E_2 \cap E_3) = \frac{1}{6}$.

Then, among the events E_1 , E_2 and E_3 , the probability that at least two events occur, equals

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Q.8

Let X be a continuous random variable such that $P(X \ge 0) = 1$ and $Var(X) < \infty$. Q.9 Then, $E(X^2)$ is

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otherwise,

- (A) $2 \int_0^\infty x^2 P(X > x) dx$
- (B) $\int_0^\infty x^2 P(X > x) dx$
- (C) $2\int_0^\infty x P(X > x) dx$
- (D) $\int_0^\infty x P(X > x) dx$

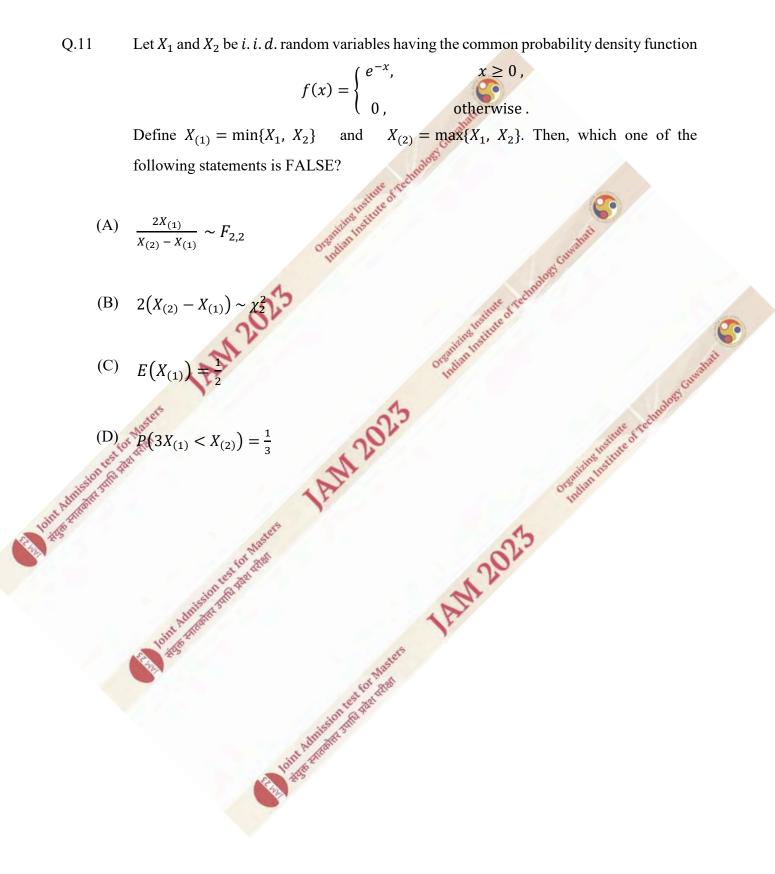
Q.100n restor wat Let X be a random variable having a probability density function if 0 < x < 1 $f(x;\theta) = \begin{cases} (3-\theta) x^{2-\theta}, \\ 0, \end{cases}$

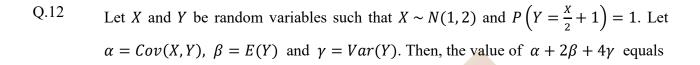
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where $\theta \in \{0,1\}$. For testing the null hypothesis $H_0: \theta = 0$ against $H_1: \theta = 1$, the power of the most powerful test, at the level of significance $\alpha = 0.125$, equals

- (A)
- (B) 0.25
- (C) 0.35
- (D) 0.45

Section A: Q.11 – Q.30 Carry TWO marks each.





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- (A) 5
- **(B)** 6
- (C) 7
- (D) 8
- Teelmolog Curstan rest for Masters A point (a, b) is chosen at random from the rectangular region $[0, 2] \times [0, 4]$. Then, the probability that the area of the probability that the probability the probability that the probability the probability the probability that the probability the probability that the probability the p Q.13 Joint Admission San Stand Stand Suffy probability that the area of the region

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 $x, y \ge 0$ $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} : bx + ay \le ab,$ 1AM 2025

will be less than 2, equals

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- 1 + ln 2 (A) 4
- + ln 2 (B)
- $\frac{2 + \ln 2}{4}$ (C)
- $\frac{1+2 \ln 2}{4}$ (D)

Q.14 Let X_1, X_2, \dots be a sequence of independent random variables such that

$$P(X_{l} = l) = \frac{1}{4} \quad \text{and} \quad P(X_{l} = 2l) = \frac{3}{4}, \quad l = 1, 2, \dots.$$
For some real constants c_{1} and c_{2} , suppose that
$$\frac{c_{1}}{\sqrt{n}} \sum_{l=1}^{n} \frac{X_{l}}{l} + c_{2}\sqrt{n} \quad \stackrel{d}{\longrightarrow} Z^{(2)} N(0,1), \quad \text{as } n \to \infty.$$
Then, the value of $\sqrt{3} (3c_{1} + c_{2})$ equals
$$(A) \quad 2$$

$$(B) \quad 3$$

$$(C) \quad 4_{n}$$

$$(D) \quad D^{(2)} D^{(2)} D^{(2)}$$

$$(C) \quad 4_{n}$$

$$(D) \quad D^{(2)} D^{(2)} D^{(2)}$$

$$(C) \quad 4_{n}$$

$$(D) \quad D^{(2)} D^{(2)} D^{(2)}$$

$$(D) \quad D^{(2)} D^{(2)} D^{(2)} D^{(2)} D^{(2)}$$

$$(D) \quad D^{(2)} D$$

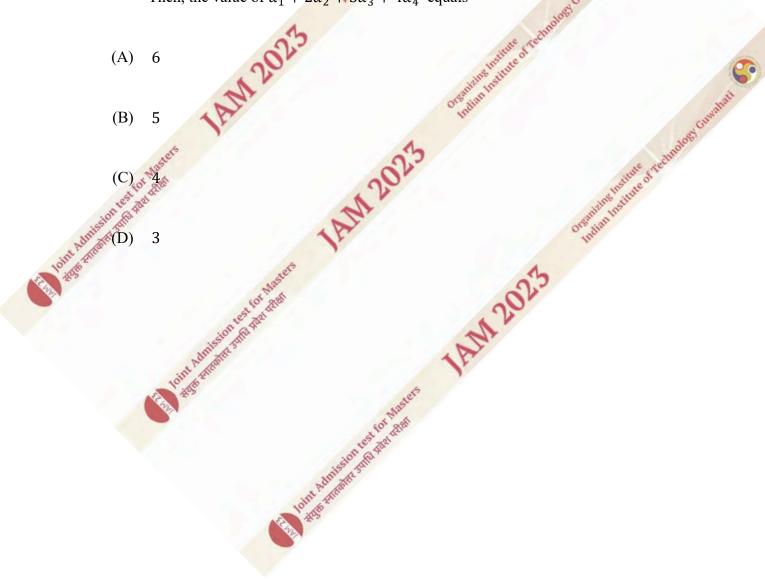
Q.15 Let $X_1, X_2, ...$ be a sequence of *i*. *i*. *d*. random variables such that

$$P(X_1 = 0) = P(X_1 = 1) = P(X_1 = 2) = \frac{1}{3}.$$

Let $S_n = \frac{1}{n} \sum_{i=1}^n X_i$ and $T_n = \frac{1}{n} \sum_{i=1}^n X_i^2$, $n = 1, 2, ...$ Suppose that

$$\alpha_1 = \lim_{n \to \infty} P\left(\left|S_n - \frac{1}{2}\right| < \frac{3}{4}\right), \qquad \alpha_2 = \lim_{n \to \infty} P\left(\left|S_n - \frac{1}{3}\right| < 1\right),$$
$$\alpha_3 = \lim_{n \to \infty} P\left(\left|T_n - \frac{1}{3}\right| < \frac{3}{2}\right) \text{ for a restriction of } \alpha_4 = \lim_{n \to \infty} P\left(\left|T_n - \frac{2}{3}\right| < \frac{1}{2}\right).$$

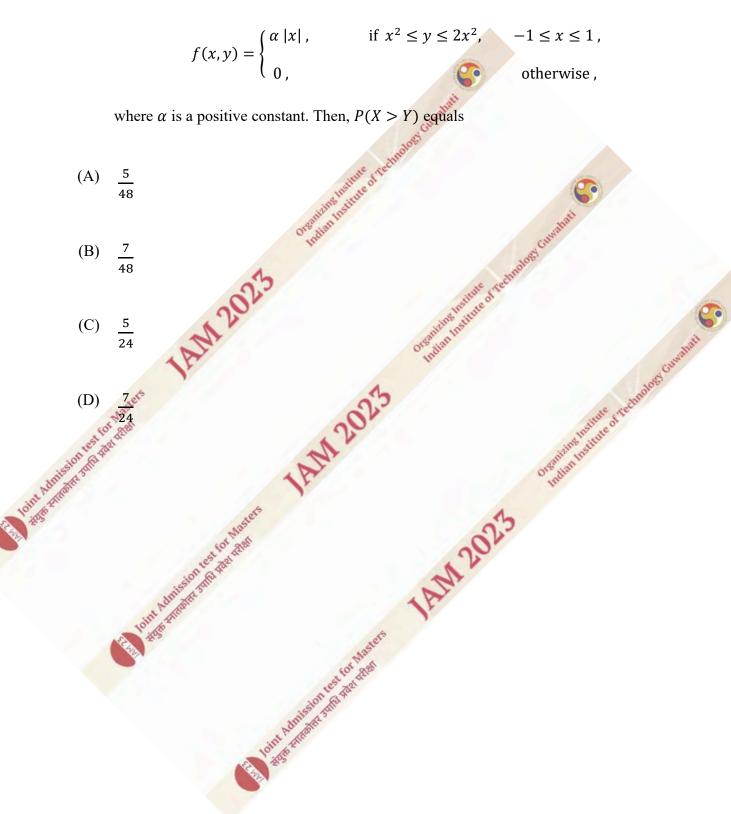
Then, the value of $\alpha_1 + 2\alpha_2 + 3\alpha_3 + 4\alpha_4$ equals



Q.16 For $x \in \mathbb{R}$, the curve $y = x^2$ intersects the curve $y = x \sin x + \cos x$ at exactly *n* points. Then, *n* equals



Q.17 Let (X, Y) be a random vector having the joint probability density function



Q.18 Let X_1 , X_2 , X_3 , X_4 be a random sample of size 4 from an $N(\theta, 1)$ distribution, where $\theta \in \mathbb{R}$ is an unknown parameter. Let $\overline{X} = \frac{1}{4} \sum_{i=1}^{4} X_i$, $g(\theta) = \theta^2 + 2\theta$ and $L(\theta)$ be the Cramer-Rao lower bound on variance of unbiased estimators of $g(\theta)$. Then, which one of the following statements is FALSE?

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- (A) $L(\theta) = (1+\theta)^2$
- (B) $\bar{X} + e^{\bar{X}}$ is a sufficient statistic for θ
- (C) $(1 + \bar{X})^2$ is the uniformly minimum variance unbiased estimator of $g(\theta)$

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(D) $Var((1+\bar{X})^2) \ge \frac{(1+\theta)^2}{2}$

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Q.19 Let $X_1, X_2, ..., X_n$ be a random sample from a population having the probability density function

$$f(x;\mu) = \begin{cases} \frac{1}{2}e^{-\left(\frac{x-2\mu}{2}\right)}, & \text{if } x > 2\mu, \\ 0, & \text{otherwise}, \end{cases}$$

where $-\infty < \mu < \infty$. For estimating μ , consider estimators

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$$T_1 = \frac{\bar{X} - 2}{2}$$
 and $T_2 = \frac{nX_{(1)} - 2}{2n}$,

where $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$ and $X_{(1)} = \min\{X_1, X_2, \dots, X_n\}$. Then, which one of the following statements is TRUE?

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- (A) T_1 is consistent but T_2 is NOT consistent
- (B) T_2 is consistent but T_1 is NOT consistent

(C) Both T_1 and T_2 are consistent

(D) Neither T_1 nor T_2 is consistent

Let $X_1, X_2, ..., X_n$ be a random sample from a $U\left(\theta + \frac{\sigma}{\sqrt{3}}, \theta + \sqrt{3}\sigma\right)$ distribution, where Q.20 $\sigma > 0$ are unknown parameters. Let $\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$ $\theta \in \mathbb{R}$ and and $S = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2}$. Let $\hat{\theta}$ and $\hat{\sigma}$ be the method of moment estimators of θ and σ , respectively. Then, which one of the following statements is FALSE?

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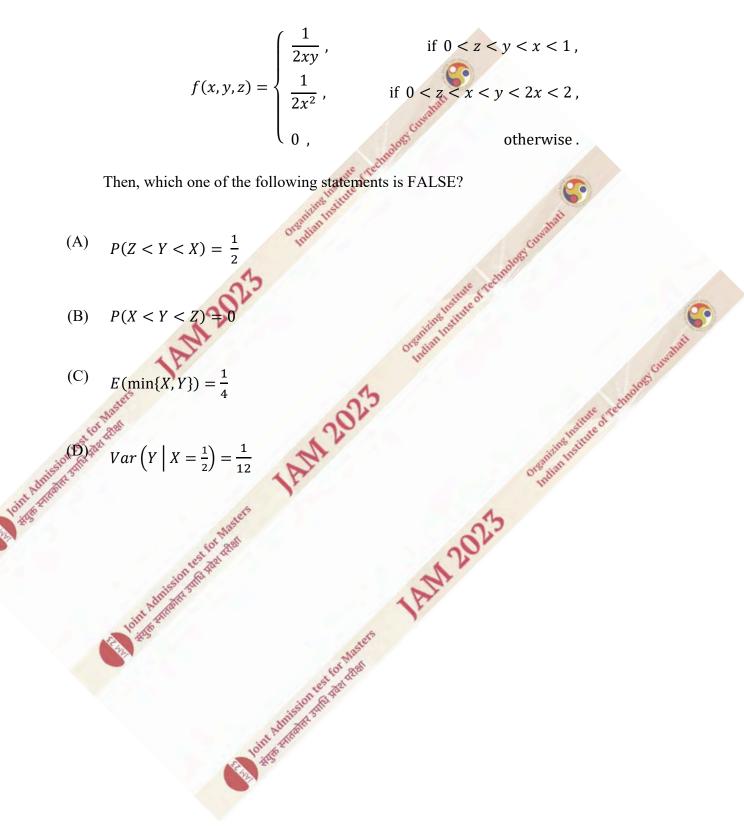
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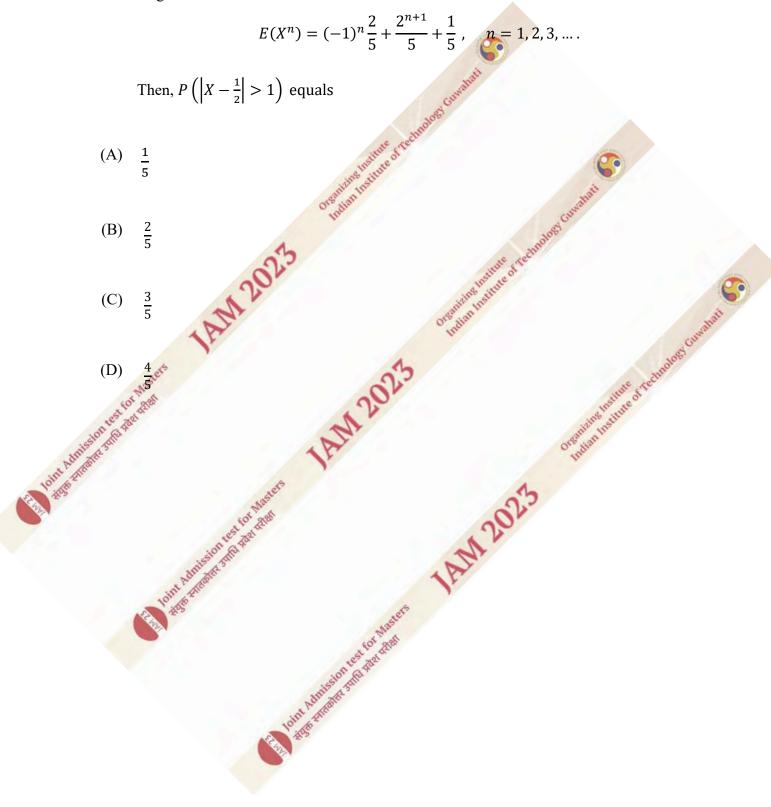
- (A) $\hat{\sigma} + \sqrt{3} \hat{\theta} = \sqrt{3} \overline{X} 3S$
- (B) $2\sqrt{3}\,\hat{\sigma} + \hat{\theta} = \overline{X} 4\sqrt{3}\,S$
- (C) $\sqrt{3} \hat{\sigma} + \hat{\theta} = \bar{X} + \sqrt{3}S$ (D) $\hat{\sigma} \sqrt{3} \hat{\theta} = 9S \sqrt{3} \bar{X}$

pint Admission les on some and and and and and Let (X, Y, Z) be a random vector having the joint probability density function



Q.21

Q.22 Let *X* be a random variable such that the moment generating function of *X* exists in a neighborhood of zero and



Q.23 Let X be a random variable having a probability mass function p(x) which is positive only for non-negative integers. If

$$p(x+1) = \left(\frac{\ln 3}{x+1}\right)p(x), \qquad x = 0, 1, 2, \dots,$$

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then Var(X) equals

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- (A) ln 3
- **(B)** ln 6
- ln 9 (C)
- ln 18 (D)

Indian Institute Let $\{a_n\}_{n \ge 1}$ be a sequence such that $a_1 = 1$ and $4a_{n+1} = \sqrt{45 + 16a_n}$, n = 1, 2, 3, ...Then, which one of the following statements is TRUE?

 $\{a_n\}_{n\geq 1}$ is monotonically increasing and converges to 17 8 (A)

(B) $\{a_n\}_{n\geq 1}$ is monotonically increasing and converges to $\frac{9}{4}$

- (C) $\{a_n\}_{n\geq 1}$ is bounded above by 8
- $\sum_{n=1}^{\infty} a_n$ is convergent (D)

Q.25 Let the series S and T be defined by

$$\sum_{n=0}^{\infty} \frac{2 \cdot 5 \cdot 8 \cdots (3n+2)}{1 \cdot 5 \cdot 9 \cdots (4n+1)} \quad \text{and} \quad \sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{-n^2}$$

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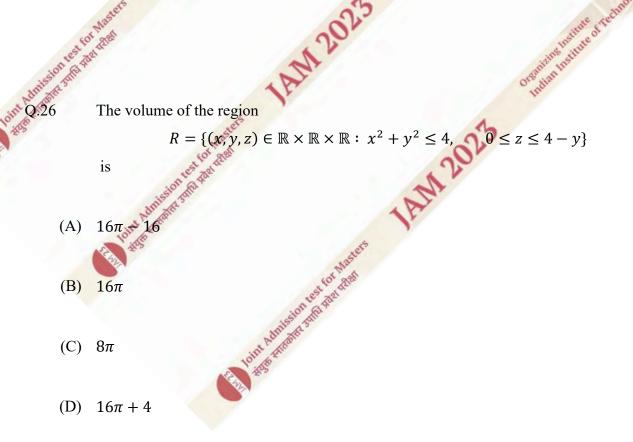
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respectively. Then, which one of the following statements is TRUE?

- (A) *S* is convergent and *T* is divergent
- Fine of Te (B) S is divergent and T is convergent
- (C) Both S and T are convergent
- Both S and T are divergent (D)



Q.27 For real constants α and β , suppose that the system of linear equations

$$x + 2y + 3z = 6;$$
 $x + y + \alpha z = 3;$ $2y + z = \beta,$

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has infinitely many solutions. Then, the value of $4\alpha + 3\beta$ equals

- (A) 18
- (B) 23
- (C) 28
- (D) 32

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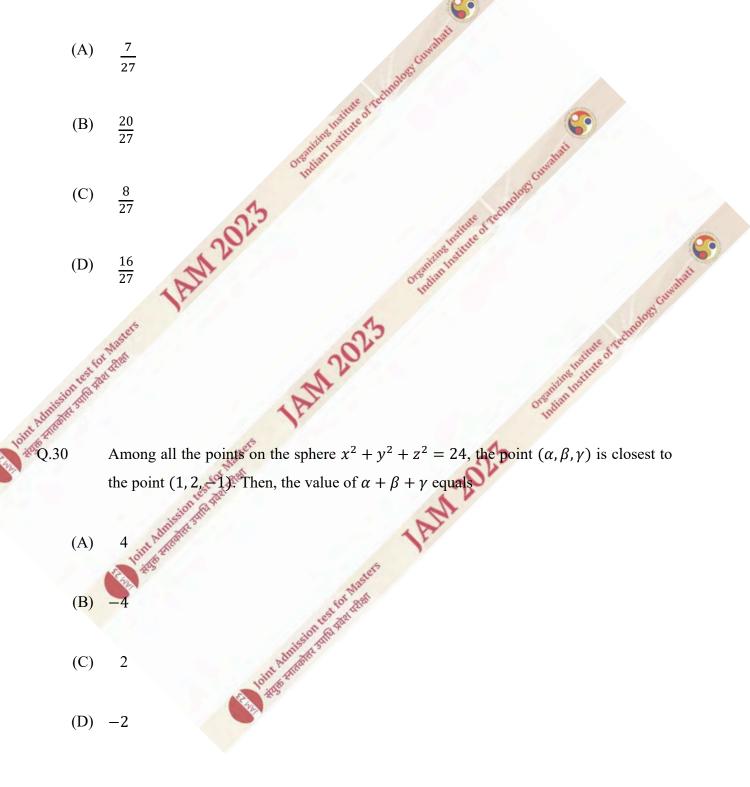
toint Q.2

Let x_1 , x_2 , x_3 and x_4 be observed values of a random sample from an $N(\theta, \sigma^2)$ distribution, where $\theta \in \mathbb{R}$ and $\sigma > 0$ are unknown parameters. Suppose that $\bar{x} = \frac{1}{4} \sum_{i=1}^{4} x_i = 3.6$ and $\frac{1}{3} \sum_{i=1}^{4} (x_i - \bar{x})^2 = 20.25$. For testing the null hypothesis $H_0: \theta = 0$ against $H_1: \theta \neq 0$, the *p*-value of the likelihood ratio test equals

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- (A) 0.712
- (B) 0.208
- (C) 0.104
- (D) 0.052

Q.29 Let *X* and *Y* be jointly distributed random variables such that, for every fixed $\lambda > 0$, the conditional distribution of *X* given $Y = \lambda$ is the Poisson distribution with mean λ . If the distribution of *Y* is *Gamma* $\left(2, \frac{1}{2}\right)$, then the value of P(X = 0) + P(X = 1) equals



Section B: Q.31 – Q.40 Carry TWO marks each.

Q.31 Let *M* be a 3 × 3 real matrix. If $P = M + M^T$ and $Q = M - M^T$, then which of the following statements is/are always TRUE?

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- (A) $\det(P^2Q^3) = 0$
- (B) trace $(Q + Q^2) = 0$

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- (C) $X^T Q^2 X = 0$, for all $X \in \mathbb{R}^3$
- (D) $X^T P X = 2X^T M X$, for all $X \in \mathbb{R}^3$

Let X_1 , X_2 , X_3 be *i.i.d.* random variables, each having the N(0, 1) distribution. Then, Q.32 which of the following statements is/are TRUE?

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(A) $\frac{\sqrt{2} (X_1 - X_2)}{\sqrt{(X_1 + X_2)^2 + 2X_3^2}} \sim t_1$

(B)
$$\frac{(X_1+X_2)^2}{(X_1-X_2)^2+2X_3^2} \sim F_{1,2}$$

$$(C) \quad E\left(\frac{X_1}{X_2^2 + X_3^2}\right) = 0$$

(D)
$$P(X_1 < X_2 + X_3) = \frac{1}{2}$$

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Q.33 Let $x_1, x_2, ..., x_{10}$ be the observed values of a random sample of size 10 from an $N(\theta, \sigma^2)$ distribution, where $\theta \in \mathbb{R}$ and $\sigma > 0$ are unknown parameters. If

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$$\bar{x} = \frac{1}{10} \sum_{i=1}^{10} x_i = 0$$
 and $s = \sqrt{\frac{1}{9} \sum_{i=1}^{10} (x_i - \bar{x})^2} = 2$

then based on the values of \bar{x} and s and using Student's *t*-distribution with 9 degrees of freedom, 90% confidence interval(s) for θ is/are

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- (A) (−0.8746, ∞)
- (B) (-0.8746, 0.8746)
- (C) (-1.1587, 1.1587)

(D) (→∞, 0.8746)

Q.34 Let (X_1, X_2) be a random vector having the probability mass function

$$f(x_1, x_2) = \begin{cases} \frac{c}{x_1! x_2! (12 - x_1 - x_2)!}, & \text{if } x_1, x_2 \in \{0, 1, \dots, 12\}, x_1 + x_2 \le 12, \\ 0, & \text{otherwise}, \end{cases}$$

where c is a real constant. Then, which of the following statements is/are TRUE?

(A) $E(X_1 + X_2) = 8$ $Var(X_1 + X_2) = \frac{8}{3}$ (B) $Cov(X_1, X_2)$, chrolog Gunnaral (C) $Var(X_1 + 2X_2) = 8$ AM 202 (D) dian Institute loint Admissi 1AN 202-1011 Admission

Let P be a 3 × 3 matrix having the eigenvalues 1, 1 and 2. Let $(1, -1, 2)^T$ be the only Q.35 linearly independent eigenvector corresponding to the eigenvalue 1. If the adjoint of the matrix 2P is denoted by Q, then which of the following statements is/are TRUE?

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- (A) trace(Q) = 20
- **(B)**

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- det(Q) = 64 $(2, -2, 4)^{T}$ is an eigenvector of the matrix Q (C)
- (D) $Q^3 = 20Q^2 124Q + 256I_3$

Q.36 Let $f: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x,y) = \begin{cases} \frac{xy(x+y)}{x^2+y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

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Then, which of the following statements is/are TRUE?

- (A) f is continuous on $\mathbb{R} \times \mathbb{R}$
- (B) The partial derivative of f with respect to y exists at (0, 0), and is 0
- (C) The partial derivative of f with respect to x is continuous on $\mathbb{R} \times \mathbb{R}$

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(D) f is NOT differentiable at (0, 0)

Q.37

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Let X and Y be *i.i.d.* random variables each having the N(0,1) distribution. Let $U = \frac{x}{Y}$ and Z = |U|. Then, which of the following statements is/are TRUE?

- (A) U has a Cauchy distribution
- (B) $E(Z^p) < \infty$, for some $p \ge 1$
- (C) $E(e^{tZ})$ does not exist for all $t \in (-\infty, 0)$
- (D) $Z^2 \sim F_{1,1}$

Q.38 Which of the following is/are TRUE?

(A)
$$\int_0^1 \int_0^1 e^{\max\{x^2, y^2\}} dx \, dy = e - 1$$

(B)
$$\int_0^1 \int_0^1 e^{\min\{x^2, y^2\}} dx \, dy = \int_0^1 e^{t^2} dt - (e-1)$$

(C) $\int_0^1 \int_0^1 e^{\max\{x^2, y^2\}} dx \, dy = 2 \int_0^1 \int_y^1 e^{x^2} dx \, dy$

(D)
$$\int_0^1 \int_0^1 e^{\min\{x^2, y^2\}} dx \, dy = 2 \int_0^1 \int_1^y e^{y^2} dx \, dy$$

Q.39 Let X be a random variable having the probability density function

 $(x) = \begin{cases} \frac{5}{x^6}, \\ 0 \end{cases}$

if x > 1,

otherwise.

Then, which of the following statements is/are TRUE for the distribution of X?

- (A) The coefficient of variation is $\frac{4}{\sqrt{15}}$
- (B) The first quartile is $\left(\frac{3}{4}\right)^{\frac{1}{5}}$
- (C) The median is $(2)^{\frac{1}{5}}$

(D) The upper bound obtained by Chebyshev's inequality for $P\left(X \ge \frac{5}{2}\right)$ is $\frac{1}{15}$

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Q.40 Based on 10 data points (x_1, y_1) , (x_2, y_2) , ..., (x_{10}, y_{10}) on a variable (X, Y), the simple regression lines of Y on X and X on Y are obtained as 2y - x = 8 and y - x = -3, respectively. Let $\bar{x} = \frac{1}{10} \sum_{i=1}^{10} x_i$ and $\bar{y} = \frac{1}{10} \sum_{i=1}^{10} y_i$. Then, which of the following statements is/are TRUE?

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 $\sum_{i=1}^{10} x_i = 140$ (A)

(B)
$$\sum_{i=1}^{10} y_i = 110$$

(C)
$$\frac{\sum_{i=1}^{10} (x_i - \bar{x}) y_i}{\sqrt{\left(\sum_{i=1}^{10} (x_i - \bar{x})^2\right) \left(\sum_{i=1}^{10} (y_i - \bar{y})^2\right)}} = -$$

(D)
$$\frac{\sum_{i=1}^{10} (x_i - \bar{x})^2}{\sum_{i=1}^{10} (y_i - \bar{y})^2} = 2$$

Section C: Q.41 – Q.50 Carry ONE mark each.

Let $f: \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = x^2 - x$, $x \in \mathbb{R}$. Let $g: \mathbb{R} \to \mathbb{R}$ be a twice Q.41 differentiable function such that g(x) = 0 has exactly three distinct roots in the open interval (0, 1). Let h(x) = f(x)g(x), $x \in \mathbb{R}$, and h'' be the second order derivative of the function h. If n is the number of roots of h''(x) = 0 in (0, 1), then the minimum possible value of *n* equals

Let X_1 , X_2 , be a sequence of *i*. *i*. *d*. random variables, each having the probability Bology Gurs density function

$$f(x) = \begin{cases} \frac{x^2 e^{-x}}{2}, \\ \frac{x^2 e^{-x}}{2$$

if $x \ge 0$,

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otherwise.

For some real constants β , γ and k, suppose that

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$$\lim_{n \to \infty} P\left(\frac{1}{n} \sum_{i=1}^{n} X_i \le x\right) = \begin{cases} 0, & \text{if } x < \beta, \\ kx, & \text{if } \beta \le x \le \gamma, \\ k\gamma, & \text{if } x > \gamma. \end{cases}$$

Then, the value of $2\beta + 3\gamma + 6k$ equals

Q.42

Q.43 Let α and β be real constants such that

$$\lim_{x \to 0^+} \frac{\int_0^x \left(\frac{\alpha t^2}{1+t^4}\right) dt}{\beta x - \sin x} = 1.$$

Then, the value of $\alpha + \beta$ equals _

Q.44 Let $X_1, X_2, ..., X_{10}$ be a random sample from an $N(0, \sigma^2)$ distribution, where $\sigma > 0$ is an unknown parameter. For some real constant *c*, let $Y = \frac{c}{10} \sum_{i=1}^{10} |X_i|$ be an unbiased estimator of σ . Then, the value of *c* equals ______ (round off to two decimal places)

Q.45

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Let X be a random variable having the probability density function

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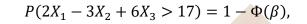
 $f(x) = \begin{cases} \frac{x}{2}, \\ 0, \end{cases}$

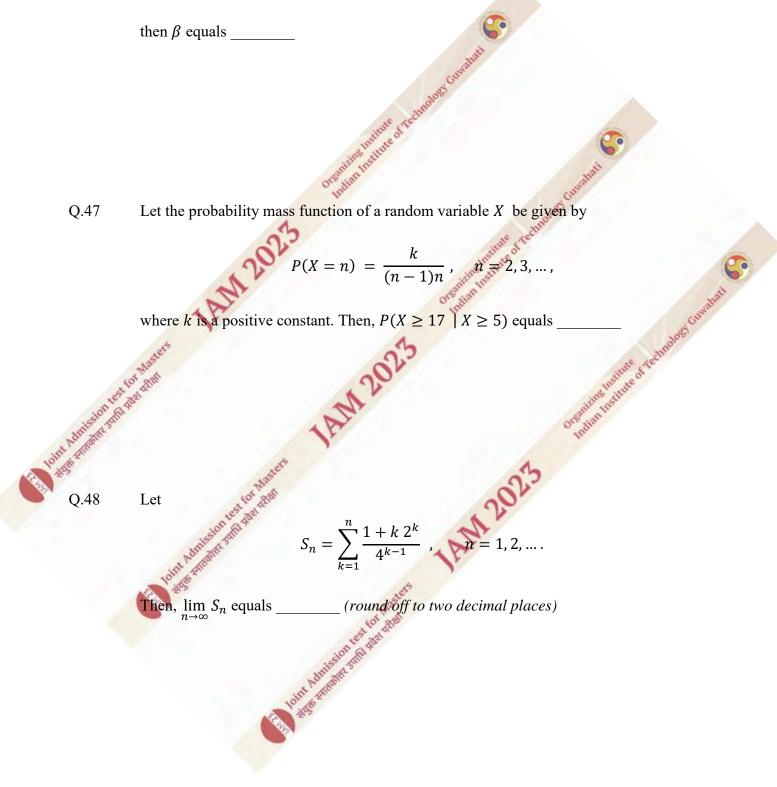
if 0 < x < 2,

otherwise .

Then, $Var\left(\ln\left(\frac{2}{x}\right)\right)$ equals ______

Q.46 Let X_1, X_2, X_3 be *i*. *i*. *d*. random variables, each having the N(2, 4) distribution. If





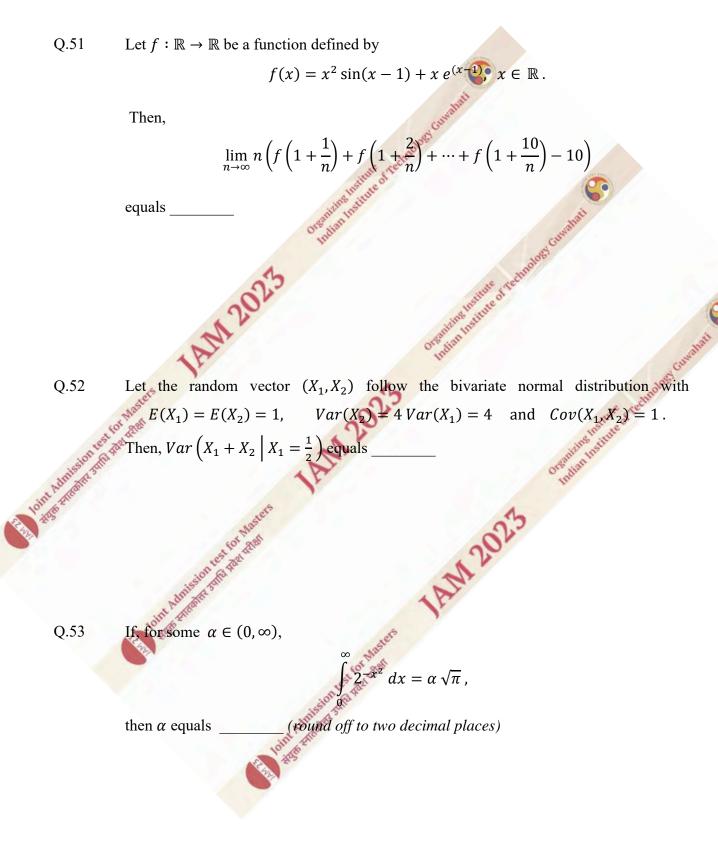
Q.49 A box contains a certain number of balls out of which 80% are white, 15% are blue and 5% are red. All the balls of the same color are indistinguishable. Among all the white balls, α % are marked defective, among all the blue balls, 6% are marked defective and among all the red balls, 9% are marked defective. A ball is chosen at random from the box. If the conditional probability that the chosen ball is white, given that it is defective, is 0.4, then α equals ______

Q.50 Let X_1 , X_2 be a random sample from a distribution having a probability density function

$$f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}}, & \text{otherwise}, \\ 0, & \text{otherwise}, \end{cases}$$

where $\theta \in (0, \infty)$ is an unknown parameter. For testing the null hypothesis $H_0: \theta = 1$ against $H_1: \theta \neq 1$, consider a test that rejects H_0 for small observed values of the statistic $W = \frac{X_1 + X_2}{2}$. If the observed values of X_1 and X_2 are 0.25 and 0.75, respectively, then the *p*-value equals (round off to two decimal places)

Section C: Q.51 – Q.60 Carry TWO marks each.



Q.54 Let $x_1 = 2.1$, $x_2 = 4.2$, $x_3 = 5.8$ and $x_4 = 3.9$ be the observed values of a random sample X_1 , X_2 , X_3 and X_4 from a population having a probability density function

$$f(x;\theta) = \begin{cases} \frac{x}{\theta^2} e^{-\frac{x}{\theta}}, & \text{if } x > 0, \\ 0, & \text{otherwise}, \end{cases}$$

where $\theta \in (0, \infty)$ is an unknown parameter. Then, the maximum likelihood estimate of $Var(X_1)$ equals _____

Q.55 Let $x_1 = 2$, $x_2 = 5$ and $x_3 = 4$ be the observed values of a random sample from a population having a probability mass function

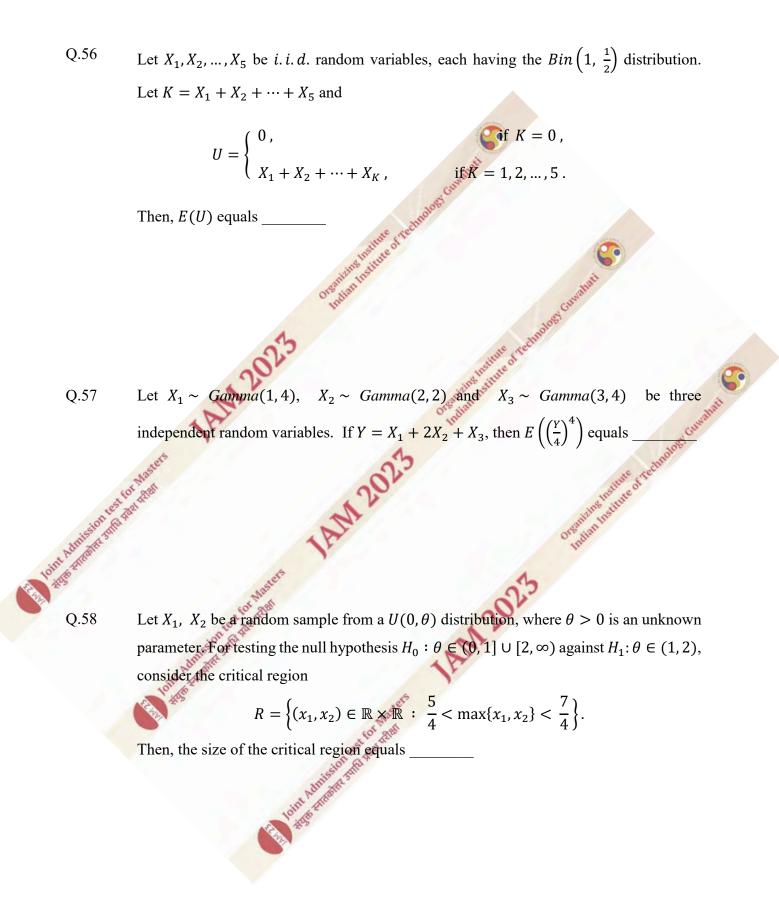
$$f(x;\theta) = \begin{cases} \theta(1-\theta)^x, & \text{if } x \\ 0, & \end{cases}$$

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where $\theta \in (0, 1)$ is an unknown parameter. If $\hat{\tau}$ is the uniformly minimum variance unbiased estimate of θ^2 , then 156 $\hat{\tau}$ equals

if x = 0, 1, 2, ...,

otherwise



- Q.59 Let $X_1, X_2, ..., X_5$ be a random sample from a $Bin(1, \theta)$ distribution, where $\theta \in (0, 1)$ is an unknown parameter. For testing the null hypothesis $H_0: \theta \le 0.5$ against $H_1: \theta > 0.5$, consider the two tests T_1 and T_2 defined as:
 - T_1 : Reject H_0 if, and only if, $\sum_{i=1}^5 X_i = 5$.
 - T_2 : Reject H_0 if, and only if, $\sum_{i=1}^5 X_i \ge 3$.

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Let β_i be the probability of making Type-II error, at $\theta = \frac{2}{3}$, when the test T_i , i = 1, 2, is used. Then, the value of $\beta_1 + \beta_2$ equals _____ (round off to two decimal places)

Q.60

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Let $X_1 \sim N(2, 1)$, $X_2 \sim N(-1, 4)$ and $X_3 \sim N(0, 1)$ be mutually independent random variables. Then, the probability that exactly two of these three random variables are less than 1, equals ______ (round off to two decimal places)

1AM 202.