MATHEMATICAL STATISTICS MODE TEST PAPER

Q.1 Let $f: \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = x^7 + 5x^3 + 11x + 15$, $x \in \mathbb{R}$.
Then, which of the following statements is TRUE?
(A) f is both one-one and onto
(B) f is neither one-one nor onto
(C) f is one-one but NOT onto
(D) f is onto but NOT one-one

Q.2 Let X be a $U(0, 1)$ random variable and let $Y = X^2$. If ϱ is the correlation coefficient between the random variables X and Y, then $48\varrho^2$ is equal to
(A) 48
(B) 45
C) 35
(D) 30

Q.3 There are three urns, labelled, Urn 1, Urn 2 and Urn 3. Urn 1 contains 2 white balls and 2 black balls, Urn 2 contains 1 white ball and 3 black balls and Urn 3 contains 3 white balls and 1 black ball. Consider two coins with probability of obtaining heads in their single trials as 0.2 and 0.3. The two coins are tossed independently once, and an urn is selected according to the following scheme: Urn 1 is selected if 2 heads are obtained; Urn 3 is selected if 2 tails are obtained; otherwise Urn 2 is selected. A ball is then drawn at random from the selected urn. Then P(Urn 1 is selected | the ball drawn is white) is equal to
(A) 6 / 109
(B) 12 / 109
(C) 1 / 18
(D) 1 / 9

Q.4 Consider the following system of linear equations

$$ax + 2y + z = 0$$
$$y + 5z$$
$$by - 5z = -1$$

Which one of the following statements is TRUE?
(A) The system has unique solution for a=1, b= -1
(B) The system has unique solution for a= -1, b= 1
(C) The system has no solution for a= 1, b= 0
(D) The system has infinitely many solutions for a= 0, b= 0

Q.5 The mean and the standard deviation of weights of ponies in a large animal shelter are 20 kg and 3 kg, respectively. A pony is selected at random from the shelter. Using Chebyshev's

inequality, the value of the lower bound of the probability that the weight of the selected pony is between 14 kg and 26 kg is
(A) 3/4
(B) 1/4
(C) 0
(D) 1

Q.6 A possible value of $b \in \mathbb{R}$ for which the equation $x^4 + bx^3 + 1 = 0$ has no real root is (A) $-11 / 5$
(B) $-3 / 2$
(C) 2
(D) $5 / 2$

Q.7 The volume of the solid generated by revolving the region bounded by the parabola $x = 2y^2 + 4$ and the line $x = 6$ about the line $x = 6$ is
(A) $78\pi / 15$
(B) $91\pi / 15$
(C) $64\pi / 15$
(D) $117\pi / 15$

Q.8 Let P be a 3×3 non-null real matrix. If there exist a 3×2 real matrix Q and a 2×3 real matrix R such that $P = QR$, then
(A) $P\boldsymbol{x} = \mathbf{0}$ has a unique solution, where $\mathbf{0} \in \mathbb{R}^3$
(B) there exists $\boldsymbol{b} \in \mathbb{R}^3$ such that $P\boldsymbol{x} = \boldsymbol{b}$ has no solution
(C) there exists a non-zero $\boldsymbol{b} \in \mathbb{R}^3$ such that $P\boldsymbol{x} = \boldsymbol{b}$ has a unique solution
(D) there exists a non-zero $\boldsymbol{b} \in \mathbb{R}^3$ such that $P\ T\boldsymbol{x} = \boldsymbol{b}$ has a unique solution

Q.9 Let E, F and G be any three events with $P(E) = 0.3$, $P(F|E) = 0.2$, $P(G|E) = 0.1$ and $P(F \cap G|E) = 0.05$. Then $P(E - (F \cup G))$ equals
(A) 0.155
(B) 0.175
(C) 0.225
(D) 0.255

Q.10 Let E and F be any two independent events with $0 < P(E) < 1$ and $0 < P(F) < 1$. Which one of the following statements is NOT TRUE?
(A) P(Neither E nor F occurs) = $(P(E) - 1)(P(F) - 1)$
(B) P(Exactly one of E and F occurs) = $P(E) + P(F) - P(E)P(F)$
(C) P(E occurs but F does not occur) = $P(E) - P(E \cap F)$
(D) P(E occurs given that F does not occur) = $P(E)$

Q.11 Let P be a probability function that assigns the same weight to each of the points of the sample space $\Omega = \{1,2,3,4\}$. Consider the events $E = \{1,2\}$, $F = \{1,3\}$ and $G = \{3,4\}$. Then which of the following statement(s) is (are) true?
(A) E and F are independent
(B) E and G are independent
(C) F and G are independent
(D) E, F and G are independent

Q.12 Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(\theta, \theta + 1)$, where $\theta \in \mathbb{R}$ is the unknown parameter. Let $U = \max\{X1, X2, \ldots, X_n\}$ and $V = \min\{X_1, X_2, \ldots, X_n\}$. Then which of the following statement(s) is (are) true?
(A) U is a consistent estimator of θ
(B) V is a consistent estimator of θ
(C) $2U - V - 2$ is a consistent estimator of θ
(D) $2V - U + 1$ is a consistent estimator of θ

Q.13 Let M be a 3×3 real matrix. If $P = M + M^T$ and $Q = M - M^T$, then which of the following statements is/are always TRUE?
(A) $\det(P^2 Q^3) = 0$
(B) $\text{trace}(Q + Q^2) = 0$
(C) $X^T Q^2 X = 0$, for all $X \in \mathbb{R}^3$
(D) $X^T P X = 2X^T M X$, for all $X \in \mathbb{R}^3$

Q.14 Let P be a 3×3 matrix having the eigenvalues 1, 1 and 2. Let $(1, -1, 2)^T$ be the only linearly independent eigenvector corresponding to the eigenvalue 1. If the adjoint of the matrix $2P$ is denoted by Q, then which of the following statements is/are TRUE?
(A) $\text{trace}(Q) = 20$
(B) $\det(Q) = 64$
(C) $(2, -2, 4)^T$ is an eigenvector of the matrix Q
(D) $Q\,3 = 20Q\,2 - 124Q + 256I_3$

Q.15 Let X and Y be i.i.d. random variables each having the $N(0, 1)$ distribution. Let $U = X/ Y$ and $Z = |U|$. Then, which of the following statements is/are TRUE?
(A) U has a Cauchy distribution
(B) $E(Z^p) < \infty$, for some $p \geq 1$
(C) $E(e^{tZ})$ does not exist for all $t \in (-\infty, 0)$
(D) $Z^2 \sim F1,1$

Q.16 Consider the linear system $A\,x = b$, where A is an $m \times n$ matrix, x is an $n \times 1$ vector of unknowns and b is an $m \times 1$ vector. Further, suppose there exists an $m \times 1$ vector c such that the linear system $Ax = c$ has NO solution. Then, which of the following statements is/are necessarily TRUE?
(A) If $m \leq n$ and d is the first column of A, then the linear system $Ax = d$ has a unique solution

(B) If $m \geq n$, then Rank(A) < n
(C) Rank(A) < m
(D) If $m > n$, then the linear system $Ax = 0$ has a solution other than $x = 0$

Q.17 Let A be a 3 × 3 real matrix such that $A \neq I_3$ and the sum of the entries in each row of A is 1. Then, which of the following statements is/are necessarily TRUE?
(A) $A - I_3$ is an invertible matrix
(B) The set $\{ x \in \mathbb{R}^3 : (A - I_3)x = 0 \}$ has at least two elements (x is a column vector)
(C) The characteristic polynomial, $p(\lambda)$, of $A + 2A^2 + A^3$ has $(\lambda - 4)$ as a factor
(D) A cannot be an orthogonal matrix

Q.18 Consider the function $f(x, y) = 3x^2 + 4xy + y^2$, $(x, y) \in \mathbb{R}^2$.
If $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$, then which of the following statements is/are TRUE?
(A) The maximum value of f on S is $3 + \sqrt{5}$
(B) The minimum value of f on S is $3 - \sqrt{5}$
(C) The maximum value of f on S is $2 + \sqrt{5}$
(D) The minimum value of f on S is $2 - \sqrt{5}$

Q.19 Let $f: \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. Then, which of the following statements is/are necessarily TRUE?
(A) f'' is continuous
(B) If $f'(0) = f'(1)$, then $f''(x) = 0$ has a solution in (0, 1)
(C) f' is bounded on [8, 10]
(D) f'' is bounded on (0, 1)

Q.20 Let P be an n x n non-null real skew-symmetric matrix, where n is even. Which of the following statements is (are) always TRUE?
(A) $P\mathbf{x} = \mathbf{0}$ has infinitely many solutions, where $\mathbf{0} \in \mathbb{R}^n$
(B) $P\mathbf{x} = \lambda\mathbf{x}$ has a unique solution for every non-zero $\lambda \in \mathbb{R}$
(C) If $Q = (I_n + P)(I_n - P)^{-1}$, then $Q^TQ = I_n$
(D) The sum of all the eigenvalues of P is zero

Q.21 Two fair dice are tossed independently and it is found that one face is odd and the other one is even. Then the probability (round off to 2 decimal places) that the sum is less than 6 equals __________

Q.22 In a production line of a factory, each packet contains four items. Past record shows that 20% of the produced items are defective. A quality manager inspects each item in a packet and approves the packet for shipment if at most one item in the packet is found to be defective. Then the probability (round off to 2 decimal places) that out of the three randomly inspected packets at least two are approved for shipment equals __________ .

Q.23 Let X be the number of heads obtained in a sequence of 10 independent tosses of a fair coin. The fair coin is tossed again X number of times independently, and let Y be the number of heads obtained in these X number of tosses. Then $E(X + 2Y)$ equals ___________.

Q.24 Let 0, 1, 0, 0, 1 be the observed values of a random sample of size five from a discrete distribution with the probability mass function $P(X = 1) = 1 - P(X = 0) = 1 - e^{-\lambda}$, where $\lambda > 0$. The method of moments estimate (round off to 2 decimal places) of λ equals ___________.

Q.25 Let E, F and G be three events such that $P(E \cap F \cap G) = 0.1, P(G|F) = 0.3$ and $P(E|F \cap G) = P(E|F)$. Then $P(G|E \cap F)$ equals _____________.

Q.26 Let X be a sample observation from $U(\theta, \theta^2)$ distribution, where $\theta \in \Theta = \{2,3\}$ is the unknown parameter. For testing
H_0: $\theta = 2$ against H_1: $\theta = 3$,

let α and β be the size and power, respectively, of the test that rejects $H0$ if and only if $X \geq 3.5$. Then $\alpha + \beta$ equals _____________ .

Q.27 Two points are chosen at random on a line segment of length 9 cm. The probability that the distance between these two points is less than 3 cm is _____________ .

Q.28 If X is a U (0,1) random variable, then P{min(X,1-X)≤1/4} = __________________.

Q.29 In a colony all families have at least one child. The probability that a randomly chosen family from this colony has exactly k children is $(0.5)^k$; k = 1, 2,... . A child is either a male or a female with equal probability. The probability that such a family consists of at least one male child and at least one female child is __________ .

Q.30 Let $X_1 \sim N(2, 1)$, $X_2 \sim N(-1, 4)$ and $X_3 \sim N(0, 1)$ be mutually independent random variables. Then, the probability that exactly two of these three random variables are less than 1, equals _________ (round off to two decimal places)

ANSWER KEY

Question No.	Question Type (QT)	Subject Name (SN)	Key/Range (KY)	Mark (MK)
1	MCQ	MS	A	1
2	MCQ	MS	B	1

3	MCQ	MS	A	1
4	MCQ	MS	A	1
5	MCQ	MS	B	1
6	MCQ	MS	B	2
7	MCQ	MS	C	2
8	MCQ	MS	B	2
9	MCQ	MS	C	2
10	MCQ	MS	B	2
11	MSQ	MS	A;C	2
12	MSQ	MS	B;C;D	2
13	MSQ	MS	A;D	2
14	MSQ	MS	A;C	2
15	MSQ	MS	A;D	2
16	MSQ	MS	C	2
17	MSQ	MS	B;C	2
18	MSQ	MS	C;D	2
19	MSQ	MS	B;C	2
20	MSQ	MS	B;C;D	2
21	NAT	MS	0.30 to 0.35	1
22	NAT	MS	0.88 to 0.95	1
23	NAT	MS	10	1

24	NAT	MS	0.45 to 0.55	1
25	NAT	MS	0.25 to 0.35	1
26	NAT	MS	1.10 to 1.20	2
27	NAT	MS	0.5 to 0.6	2
28	NAT	MS	0.49 to 0.51	2
29	NAT	MS	0.3 to 0.4	2
30	NAT	MS	0.63 to 0.65	2