MATHEMATICS MODEL TEST PAPER

Q.1 Let V be the real vector space consisting of all polynomials in one variable with real coefficients and having degree at most 6, together with the zero polynomial. Then which one of the following is true?
(A) $\{f \in V : f(1/2) \notin \mathbb{Q}\}$ is a subspace of V.
(B) $\{f \in V : f(1/2) = 1\}$ is a subspace of V.
(C) $\{f \in V : f(1/2) = f(1)\}$ is a subspace of V.
(D) $\{f \in V : f'(1/2) = 1\}$ is a subspace of V.

Q.2 Let G be a group of order 2022. Let H and K be subgroups of G of order 337 and 674, respectively. If $H \cup K$ is also a subgroup of G, then which one of the following is FALSE?
(A) H is a normal subgroup of $H \cup K$.
(B) The order of $H \cup K$ is 1011.
(C) The order of $H \cup K$ is 674.
(D) K is a normal subgroup of $H \cup K$

Q.3 Let $0 < \alpha < 1$ be a real number. The number of differentiable functions $y : [0, 1] \to [0, \infty)$, having continuous derivative on [0, 1] and satisfying $y'(t) = (y(t))^{\alpha}$, $t \in [0, 1]$, $y(0) = 0$, is
(A) exactly one.
(B) exactly two.
(C) finite but more than two.
(D) infinite.

Q.4 Let $P : R \to R$ be a continuous function such that $P(x) > 0$ for all $x \in R$. Let y be a twice differentiable function on R satisfying $y''(x) + P(x)y\ 0\ (x) - y(x) = 0$ for all $x \in R$. Suppose that there exist two real numbers a, b ($a < b$) such that $y(a) = y(b) = 0$. Then
(A) $y(x) = 0$ for all $x \in [a, b]$.
(B) $y(x) > 0$ for all $x \in (a, b)$.
(C) $y(x) < 0$ for all $x \in (a, b)$.
(D) $y(x)$ changes sign on (a, b).

Q.5 Let $f : R \to R$ be a continuous function satisfying $f(x) = f(x + 1)$ for all $x \in R$. Then
(A) f is not necessarily bounded above.
(B) there exists a unique $x_0 \in R$ such that $f(x_0 + \pi) = f(x_0)$.
(C) there is no $x_0 \in R$ such that $f(x_0 + \pi) = f(x_0)$.
(D) there exist infinitely many $x_0 \in R$ such that $f(x_0 + \pi) = f(x_0)$.

Q.6 Let $M_n(R)$ be the real vector space of all $n \times n$ matrices with real entries, $n \geq 2$. Let $A \in M_n(R)$. Consider the subspace W of $M_n(R)$ spanned by $\{I_n, A, A^2, \ldots\}$. Then the dimension of W over R is necessarily (A) ∞
(B) n^2

(C) n
(D) at most n

Q.7 Consider the following statements. I. The group (Q, +) has no proper subgroup of finite index. II. The group (C \ {0}, ·) has no proper subgroup of finite index. Which one of the following statements is true?
(A) Both I and II are TRUE.
(B) I is TRUE but II is FALSE.
(C) II is TRUE but I is FALSE.
(D) Neither I nor II is TRUE.

Q.8 Consider the family of curves $x^2 - y^2 = ky$ with parameter $k \in R$. The equation of the orthogonal trajectory to this family passing through (1, 1) is given by
(A) $x^3 + 3xy^2 = 4$.
(B) $x^2 + 2xy = 3$.
(C) $y^2 + 2x^2 y = 3$.
(D) $x . + 2xy^2 = 3$.

Q.9 Which one of the following statements is true?
(A) Exactly half of the elements in any even order subgroup of S5 must be even permutations.
(B) Any abelian subgroup of S5 is trivial.
(C) There exists a cyclic subgroup of S5 of order 6.
(D) There exists a normal subgroup of S5 of index 7.

Q.10 Which one of the following statements is true?
(A) (Z, +) is isomorphic to (R, +).
(B) (Z, +) is isomorphic to (Q, +).
(C) (Q/Z, +) is isomorphic to (Q/2Z, +).
(D) (Q/Z, +) is isomorphic to (Q, +).

Q.11 Let $a, b, c \in R$ such that $a < b < c$. Which of the following is/are true for any continuous function $f : R \to R$ satisfying $f(a) = b$, $f(b) = c$ and $f(c) = a$?
(A) There exists $\alpha \in (a, c)$ such that $f(\alpha) = \alpha$
(B) There exists $\beta \in (a, b)$ such that $f(\beta) = \beta$
(C) There exists $\gamma \in (a, b)$ such that $(f \circ f)(\gamma) = \gamma$
(D) There exists $\delta \in (a, c)$ such that $(f \circ f \circ f)(\delta) = \delta$

Q.12 Let $a, b \in R$ and $a < b$. Which of the following statement(s) is/are true?
(A) There exists a continuous function $f : [a, b] \to (a, b)$ such that f is one-one
(B) There exists a continuous function $f : [a, b] \to (a, b)$ such that f is onto
(C) There exists a continuous function $f : (a, b) \to [a, b]$ such that f is one-one
(D) There exists a continuous function $f : (a, b) \to [a, b]$ such that f is onto

Q.13 Let V be a non-zero vector space over a field F. Let $S \subset V$ be a non-empty set. Consider the following properties of S:

(I) For any vector space W over F, any map $f : S \to W$ extends to a linear map from V to W.

(II) For any vector space W over F and any two linear maps $f, g : V \to W$ satisfying $f(s) = g(s)$ for all $s \in S$, we have $f(v) = g(v)$ for all $v \in V$.

(III) S is linearly independent.

(IV) The span of S is V.

Which of the following statement(s) is /are true?

(A) (I) implies (IV)

(B) (I) implies (III)

(C) (II) implies (III)

(D) (II) implies (IV)

Q.14 Consider the following system of linear equations $x + y + 5z = 3$, $x + 2y + mz = 5$ and $x + 2y + 4z = k$. The system is consistent if

(A) $m \neq 4$

(B) $k \neq 5$

(C) $m = 4$

(D) $k = 5$

Q.15 Let G be a group with identity e. Let H be an abelian non-trivial proper subgroup of G with the property that $H \cap gHg^{-1} = \{e\}$ for all $g \notin H$. If K = $g \in G : gh = hg$ for all $h \notin H$, then

(A) K is a proper subgroup of H

(B) H is a proper subgroup of K

(C) K = H

(D) there exists no abelian subgroup $L \subseteq G$ such that K is a proper subgroup of L

Q.16 Let G be a noncyclic group of order 4. Consider the statements I and II:

I. There is NO injective (one-one) homomorphism from G to $\mathbb{Z}_8$

II. There is NO surjective (onto) homomorphism from $\mathbb{Z}_8$ to G Then

(A) I is true

(B) I is false

(C) II is true

(D) II is false

Q.17 Let G be a nonabelian group, $y \in G$, and let the maps f, g, h from G to itself be defined by $f(x) = yxy^{-1}$, $g(x) = x^{-1}$ and $h = g \circ g$. Then

(A) g and h are homomorphisms and f is not a homomorphism

(B) h is a homomorphism and g is not a homomorphism

(C) f is a homomorphism and g is not a homomorphism

(D) f, g and h are homomorphisms

Q.18 Let S and T be linear transformations from a finite dimensional vector space V to itself such that $S(T(v)) = 0$ for all $v \in V$. Then
(A) rank(T) ≥ nullity(S)
(B) rank(S) ≥ nullity(T)
(C) rank(T) ≤ nullity(S)
(D) rank(S) ≤ nullity(T)

Q.19 Consider the intervals $S = (0, 2]$ and $T = [1, 3)$. Let $S°$ and $T°$ be the sets of interior points of S and T, respectively. Then the set of interior points of $S \setminus T$ is equal to
(A) $S \setminus T°$
(B) $S \setminus T$
(C) $S° \setminus T°$
(D) $S° \setminus T$

Q.20 Let $f(x) = \cos(|\pi - x|) + (x - \pi) \sin |x|$ and $g(x) = x^2$ for $x \in \mathbb{R}$. If $h(x) = f(g(x))$, then
(A) h is not differentiable at $x = 0$
(B) $h'(\sqrt{\pi}) = 0$
(C) $h''(x) = 0$ has a solution in $(-\pi, \pi)$
(D) there exists $x_0 \in (-\pi, \pi)$ such that $h(x_0) = x_0$

Q.21 Let T : $P_2(R) \rightarrow P_4(R)$ be the linear transformation given by $T(p(x)) = p(x^2)$. Then the rank of T is equal to .

Q. 22If y is the solution of
$y'' - 2y' + y = e^x$, $y(0) = 0$,
$y'(0) = -1/2$,
then y(1) is equal to_______. (rounded off to two decimal places)

Q.23 For $\sigma \in S_8$, let $o(\sigma)$ denote the order of σ. Then $\max\{o(\sigma) : \sigma \in S_8\}$ is equal to _______.

Q. 24 For $g \in Z$, let $\bar{g} \in Z_8$ denote the residue class of g modulo 8. Consider the group $Z^{\times}_8 = \{\bar{x} \in Z_8 : 1 \leq x \leq 7, \gcd(x, 8) = 1\}$ with respect to multiplication modulo 8. The number of group isomorphisms from $Z^{\times}_8$ onto itself is equal to .

Q. 25 Let V be the volume of the region $S \subseteq R^3$ defined by $S = \{(x, y, z) \in R^3 : xy \leq z \leq 4, 0 \leq x^2 + y^2 \leq 1\}$. Then V / π is equal to . (rounded off to two decimal places)

Q.26 The number of elements of order two in the group S4 is equal to ______.

Q.27 The least possible value of k, accurate up to two decimal places, for which the following problem $y''(t) + 2y'(t) + ky(t) = 0$, $t \in R$, $y(0) = 0$, $y(1) = 0$, $y(1/2) = 1$, has a solution is ________.

Q.28 Consider those continuous functions f : R → R that have the property that given any x ∈ R, f(x) ∈ Q if and only if f(x + 1) ∈ R \ Q. The number of such functions is ________.

Q.29 The minimum value of the function $f(x, y) = x^2 + xy + y^2 - 3x - 6y + 11$ is _________.

Q.30 If $x^2 + xy^2 = c$, where $c \in \mathbb{R}$, is the general solution of the exact differential equation $M(x, y)\,dx + 2xy\,dy = 0$, then $M(1, 1)$ is _______.

ANSWER KEY

Question No.	Question Type (QT)	Subject Name (SN)	Key/Range (KY)	Mark (MK)
1	MCQ	MA	C	1
2	MCQ	MA	B	1
3	MCQ	MA	B	1
4	MCQ	MA	A	1
5	MCQ	MA	D	1
6	MCQ	MA	D	2
7	MCQ	MA	A	2
8	MCQ	MA	A	2
9	MCQ	MA	C	2
10	MCQ	MA	C	2
11	MSQ	MA	A;C;D	2
12	MSQ	MA	A;C;D	2
13	MSQ	MA	B;D	2
14	MSQ	MA	A;D	2
15	MSQ	MA	C;D	2

16	MSQ	MA	A;B	2
17	MSQ	MA	C	2
18	MSQ	MA	A;B;C	2
19	MSQ	MA	C	2
20	MSQ	MA	A;B;C;D	2
21	NAT	MA	3 to 3	1
22	NAT	MA	-0.01 to 0.01	1
23	NAT	MA	15 to 15	1
24	NAT	MA	6 to 6	1
25	NAT	MA	3.99 to 4.01	1
26	NAT	MA	9	2
27	NAT	MA	10.8 to 10.9	2
28	NAT	MA	0	2
29	NAT	MA	2 to 2	2
30	NAT	MA	3 to 3	2