## IIT JAM 2024 NAT Model Questions

## Subject - Mathematics (MA)

Q. 1 The number of distinct subgroups of $Z_{999}$ is $\qquad$ .
Q. 2 The number of elements of order 12 in the symmetric group S 7 is equal to $\qquad$ .
Q. 3 Consider the region $G=\left\{(x, y, z) \in \mathbb{R}^{3}: 0<z<x^{2}-y^{2}, x^{2}+y^{2}<1\right\}$. Then the volume of $G$ is equal to $\qquad$ . (Rounded off to two decimal places)
Q. 4 Let $f(x, y)=e^{x} \sin y, x=t^{3}+1$ and $y=t^{4}+t$. Then $d f / d t$ at $t=0$ is . (rounded off to two decimal places)
Q. 5 Let $f(x, y)=0$ be a solution of the homogeneous differential equation $(2 x+5 y) d x-(x+3 y) d y=0$. If $f(x+\alpha, y-3)=0$
is a solution of the differential equation
$(2 x+5 y-1) d x+(2-x-3 y) d y=0$
then the value of $\alpha$ is .
Q. 6 The sum of the series $1 / 2\left(2^{2}-1\right)+1 / 3\left(3^{2}-1\right)+1 / 4\left(4^{2}-1\right)+\cdots$ is .
Q. 7 Consider the expansion of the function $f(x)=3 /(1-x)(1+2 x)$ in powers of $x$, that is valid in $|x|<12$. Then the coefficient of $x^{4}$ is .
Q. 8 The minimum value of the function $f(x, y)=x^{2}+x y+y^{2}-3 x-6 y+11$ is .
Q. 9 Let $f(x)=\sqrt{ } x+\alpha x, x>0$ and $g(x)=a 0+a 1(x-1)+a 2(x-1) 2$ be the sum of the first three terms of the Taylor series of $f(x)$ around $x=1$. If $g(3)=3$, then $\alpha$ is.
Q. 10 If $x^{2}+x y^{2}=c$, where $c \in R$, is the general solution of the exact differential equation $M(x$, y) $d x+2 x y d y=0$, then $M(1,1)$ is .
Q. 11 The volume of the solid of revolution of the loop of the curve $y^{2}=x^{4}(x+2)$ about the $x$-axis (round off to 2 decimal places) is $\qquad$
Q. 12 Let $G=\{n \in \mathbb{N}: n \leq 55, \operatorname{gcd}(n, 55)=1\}$ be the group under multiplication modulo 55 . Let $x$ $\in G$ be such that $x 2=26$ and $x>30$. Then $x$ is equal to $\qquad$
Q. 13 The number of critical points of the function $f(x, y)=\left(x^{2}+3 y^{2}\right) e^{-(x 2+y 2)}$ is $\qquad$
Q. 14 The number of elements in the set $\left\{x \in S_{3}: x^{4}=e\right\}$, where $e$ is the identity element of the permutation group $S_{3}$, is $\qquad$
Q. 15 Let $M$ be a $3 \times 3$ matrix with real entries such that $M^{2}=M+2 I$, where $I$ denotes the $3 \times 3$ identity matrix. If $\alpha, \beta$ and $\gamma$ are eigenvalues of $M$ such that $\alpha \beta \gamma=-4$, then $\alpha+\beta+\gamma$ is equal to $\qquad$ .

## ANSWER KEY

| Question No. | Question Type (QT) | Subject Name (SN) | Key/Range (KY) | Mark (MK) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | NAT | MA | 8 to 8 | 1 |
| 2 | NAT | MA | 420 to 420 | 1 |
| 3 | NAT | MA | 0.49 to 0.51 | 1 |
| 4 | NAT | MA | 2.70 to 2.72 | 1 |
| 5 | NAT | MA | 7 to 7 | 1 |
| 6 | NAT | MA | 0.25 to 0.25 | 2 |
| 7 | NAT | MA | 33 to 33 | 2 |
| 8 | NAT | MA | 2 to 2 | 2 |
| 9 | NAT | MA | 0.5 to 0.5 | 2 |
| 10 | NAT | MA | 3 to 3 | 2 |
| 11 | NAT | MA | 6.60 to 6.80 | 2 |
| 12 | NAT | MA | 31 to 31 or 46 to 46 | 2 |
| 13 | NAT | MA | 5 to 5 | 2 |
| 14 | NAT | MA | 4 to 4 | 2 |
| 15 | NAT | MA | 3 TO 3 | 2 |

