IIT JAM 2024 NAT Model Questions

Subject - Mathematics Statistics (MS)

Q.1 Let f: $\mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = x^2 - x$, $x \in \mathbb{R}$. Let g: $\mathbb{R} \to \mathbb{R}$ be a twice differentiable function such that $g(x) = 0$ has exactly three distinct roots in the open interval (0, 1). Let $h(x) = f(x)g(x)$, $x \in \mathbb{R}$, and h'' be the second order derivative of the function h. If n is the number of roots of $h''(x) = 0$ in (0, 1), then the minimum possible value of n equals ________

Q.2 Let X_1, X_2, X_3 be $i.\,i.\,d.$ random variables, each having the $N(2, 4)$ distribution. If $P(2X_1 - 3X_2 + 6X_3 > 17) = 1 - \Phi(\beta)$, then β equals ________ .

Q.3 Let the probability mass function of a random variable X be given by $P(X = n) = k / (n - 1)n$, $n = 2, 3, \dots$, where k is a positive constant. Then, $P(X \geq 17 \mid X \geq 5)$ equals ________

Q.4 A box contains a certain number of balls out of which 80% are white, 15% are blue and 5% are red. All the balls of the same colour are indistinguishable. Among all the white balls, α% are marked defective, among all the blue balls, 6% are marked defective and among all the red balls, 9% are marked defective. A ball is chosen at random from the box. If the conditional probability that the chosen ball is white, given that it is defective, is 0.4, then α equals ________

Q.5 Two fair dice are tossed independently and it is found that one face is odd and the other one is even. Then the probability (round off to 2 decimal places) that the sum is less than 6 equals __________

Q.53 The volume (round off to 2 decimal places) of the region in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$) bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 2$ and $y + z = 4$ equals __________

Q.55 In an ethnic group, 30% of the adult male population is known to have heart disease. A test indicates high cholesterol level in 80% of adult males with heart disease. But the test also indicates high cholesterol levels in 10% of the adult males with no heart disease. Then the probability (round off to 2 decimal places), that a randomly selected adult male from this population does not have heart disease given that the test indicates high cholesterol level, equals __________

Q.57 Let X and Y be jointly distributed continuous random variables, where Y is positive valued with $E(Y^2) = 6$. If the conditional distribution of X given $Y = y$ is $U(1 - y, 1 + y)$, then $Var(X)$ equals __________

Q.9 Let $X_1, X_2, \dots, X_{10}$ be i.i.d. $N(0, 1)$ random variables. If $T = X_1^2 + X_2^2 + \cdots + X_{10}^2$, then E($1 / T$) equals __________

Q.10 Let X be a sample observation from $U(\theta, \theta^2)$ distribution, where $\theta \in \Theta = \{2,3\}$ is the unknown parameter. For testing H_0: $\theta = 2$ against H_1: $\theta = 3$, let α and β be the size and power, respectively, of the test that rejects H_0 if and only if $X \geq 3.5$. Then $\alpha + \beta$ equals ____________

Q.11 Let $X_1 \sim$ Gamma(1, 4), $X_2 \sim$ Gamma(2, 2) and $X_3 \sim$ Gamma(3, 4) be three independent random variables. If $Y = X_1 + 2X_2 + X_3$, then $E((Y/4)^4)$ equals ________

Q.12 Let $X1 \sim N(2, 1)$, $X2 \sim N(-1, 4)$ and $X3 \sim N(0, 1)$ be mutually independent random variables. Then, the probability that exactly two of these three random variables are less than 1, equals ________ (round off to two decimal places)

Q.57 Two points are chosen at random on a line segment of length 9 cm. The probability that the distance between these two points is less than 3 cm is ______________________

Q.60 In a colony all families have at least one child. The probability that a randomly chosen family from this colony has exactly k children is $(0.5)^k$, k = 1,2,.... . A child is either a male or a female with equal probability. The probability that such a family consists of at least one male child and at least one female child is _________

Q.51 Let $S \subseteq \mathbb{R}^2$ be the region bounded by the parallelogram with vertices at the points (1, 0), (3, 2), (3, 5) and (1, 3). Then, the value of the integral $\iint_s (x + 2y)\, dx\, dy\, S$ is equal to _____________.

ANSWER KEY

Question No.	Question Type (QT)	Subject Name (SN)	Key/Range (KY)	Mark (MK)
1	NAT	MS	3 to 3	1
2	NAT	MS	0.5 to 0.5	1
3	NAT	MS	0.25 to 0.25	1
4	NAT	MS	1.125 to 1.125	1
5	NAT	MS	0.30 to 0.35	1

6	NAT	MS	3.50 to 3.70	2
7	NAT	MS	0.20 to 0.25	2
8	NAT	MS	2	2
9	NAT	MS	0.12 to 0.13	2
10	NAT	MS	1.10 to 1.20	2
11	NAT	MS	3024 to 3024	2
12	NAT	MS	0.63 to 0.65	2
13	NAT	MS	0.5 to 0.6	2
14	NAT	MS	0.3 to 0.4	2
15	NAT	MS	42	2