IIT JAM 2024 MSQ Model Questions

Subject - Mathematics (MS)

Q.1 Let M be a 3×3 real matrix. If $P = M + MT$ and $Q = M - M^T$, then which of the following statements is/are always TRUE?
(A) $\det(P^2 Q^3) = 0$
(B) $\text{trace}(Q + Q^2) = 0$
(C) $X^T Q^2 X = 0$, for all $X \in \mathbb{R}^3$
(D) $X^T PX = 2X^T MX$, for all $X \in \mathbb{R}^3$

Q.2 Let P be a 3×3 matrix having the eigenvalues 1, 1 and 2. Let $(1, -1, 2)^T$ be the only linearly independent eigenvector corresponding to the eigenvalue 1. If the adjoint of the matrix $2P$ is denoted by Q, then which of the following statements is/are TRUE?
(A) $\text{trace}(Q) = 20$
(B) $\det(Q) = 64$
(C) $(2, -2, 4)^T$ is an eigenvector of the matrix Q
(D) $Q^3 = 20Q^2 - 124Q + 256I3$

Q.3 Let X and Y be i.i.d. random variables each having the $N(0, 1)$ distribution. Let $U = X / Y$ and $Z = |U|$. Then, which of the following statements is/are TRUE?
(A) U has a Cauchy distribution
(B) $E(Z^p) < \infty$, for some $p \geq 1$
(C) $E(e^{tZ})$ does not exist for all $t \in (-\infty, 0)$
(D) $Z^2 \sim F_{1,1}$

Q.4 Consider the linear system $A x = b$, where A is an $m \times n$ matrix, x is an $n \times 1$ vector of unknowns and b is an $m \times 1$ vector. Further, suppose there exists an $m \times 1$ vector c such that the linear system $Ax = c$ has NO solution. Then, which of the following statements is/are necessarily TRUE?
(A) If $m \leq n$ and d is the first column of A, then the linear system $Ax = d$ has a unique solution
(B) If $m \geq n$, then $\text{Rank}(A) < n$
(C) $\text{Rank}(A) < m$
(D) If $m > n$, then the linear system $Ax = 0$ has a solution other than $x = 0$

Q.5 Let A be a 3×3 real matrix such that $A \neq I3$ and the sum of the entries in each row of A is 1. Then, which of the following statements is/are necessarily TRUE?
(A) $A - I3$ is an invertible matrix
(B) The set $\{ x \in \mathbb{R}3 : (A - I3)x = 0 \}$ has at least two elements (x is a column vector)
(C) The characteristic polynomial, $p(\lambda)$, of $A + 2 A 2 + A 3$ has $(\lambda - 4)$ as a factor
(D) A cannot be an orthogonal matrix.

Q.6 Consider the function

$f(x, y) = 3x^2 + 4xy + y^2$, $(x, y) \in \mathbb{R}^2$.
If $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$, then which of the following statements is/are TRUE?
(A) The maximum value of f on S is $3 + \sqrt{5}$
(B) The minimum value of f on S is $3 - \sqrt{5}$
(C) The maximum value of f on S is $2 + \sqrt{5}$
(D) The minimum value of f on S is $2 - \sqrt{5}$

Q.7 Let $f: \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. Then, which of the following statements is/are necessarily TRUE?
(A) f'' is continuous
(B) If $f'(0) = f'(1)$, then $f''(x) = 0$ has a solution in $(0, 1)$
(C) f' is bounded on $[8, 10]$
(D) f'' is bounded on $(0, 1)$

Q.8 Let $f: \mathbb{R} \to \mathbb{R}$ be continuous on $\mathbb{R}$ and differentiable on $(-\infty, 0) \cup (0, \infty)$. Which of the following statements is (are) always TRUE?
(A) If f is differentiable at 0 and $f'(0) = 0$, then f has a local maximum or a local minimum at 0
(B) If f has a local minimum at 0, then f is differentiable at 0 and $f'(0) = 0$
(C) If $f'(x) < 0$ for all $x < 0$ and $f'(x) > 0$ for all $x > 0$, then f has a global maximum at 0
(D) If $f'(x) > 0$ for all $x < 0$ and $f'(x) < 0$ for all $x > 0$, then f has a global maximum at 0

Q.9 Let P be a 2×2 real matrix such that every non-zero vector in $\mathbb{R}^2$ is an eigenvector of P. Suppose that λ_1 and λ_2 denote the eigenvalues of P and $P \begin{bmatrix} \sqrt{2} \\ \sqrt{3} \end{bmatrix} = \begin{bmatrix} 2 \\ t \end{bmatrix}$ for some $t \in \mathbb{R}$.
Which of the following statements is (are) TRUE?
(A) $\lambda_1 \neq \lambda_2$
(B) $\lambda_1 \lambda_2 = 2$
(C) $\sqrt{2}$ is an eigenvalue of P
(D) $\sqrt{3}$ is an eigenvalue of �

Q.10 Let P be an $n \times n$ non-null real skew-symmetric matrix, where n is even. Which of the following statements is (are) always TRUE?
(A) $P\mathbf{x} = \mathbf{0}$ has infinitely many solutions, where $\mathbf{0} \in \mathbb{R}^n$
(B) $P\mathbf{x} = \lambda\mathbf{x}$ has a unique solution for every non-zero $\lambda \in \mathbb{R}$
(C) If $Q = (I_n + P)(I_n - P)^{-1}$, then $Q^TQ = I_n$
(D) The sum of all the eigenvalues of P is zero

Q.11 Let $X1, X2, \dots, Xn$ be a random sample from a $U(\theta, 0)$ distribution, where $\theta < 0$. If $Tn = \min\{X1, X2, \dots, Xn\}$, then which of the following sequences of estimators is (are) consistent for θ?
(A) T_n
(B) $T_n - 1$
(C) $T_n + 1/n$
(D) $T_n - 1 - 1/n^2$

Q.12 Let P be a probability function that assigns the same weight to each of the points of the sample space $\Omega = \{1,2,3,4\}$. Consider the events $E = \{1,2\}$, $F = \{1,3\}$ and $G = \{3,4\}$. Then which of the following statement(s) is (are) true?

(A) E and F are independent

(B) E and G are independent

(C) F and G are independent

(D) E, F and G are independent

Q.13 Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(\theta, \theta + 1)$, where $\theta \in \mathbb{R}$ is the unknown parameter. Let $U = \max\{X_1,X_2, \ldots, X_n\}$ and $V = \min\{X_1,X_2, \ldots, X_n\}$. Then which of the following statement(s) is (are) true?

(A) U is a consistent estimator of θ

(B) V is a consistent estimator of θ

(C) $2U - V - 2$ is a consistent estimator of θ

(D) $2V - U + 1$ is a consistent estimator of θ

Q.2 Consider the ordinary differential equation dy x dy / dx + y = x for $0<x<1$. Which of the following is (are) solution(s) to the above?

(A) y(x) = x / 2

(B) y(x) = x / 2 + 2 / x

(C) y(x) = x / 2 - 2 / x

(D) y (x) = 0

Q.3 Let f :[0,1]   be a continuous function such that

f (0) = -1, f(1 / 2) = 1, f (1) = -1

Then

(A) f attains the value 0 at least twice in [0,1]

(B) f attains the value 0 exactly twice in [0,1]

(C) f attains the value 0 exactly once in [0,1]

(D) the range of f is [1,1]

ANSWER KEY

Question No.	Question Type (QT)	Subject Name (SN)	Key/Range (KY)	MSrk (MK)
1	MSQ	MS	A, D	2
2	MSQ	MS	A, C	2

3	MSQ	MS	A, D	2
4	MSQ	MS	C	2
5	MSQ	MS	B, C	2
6	MSQ	MS	C, D	2
7	MSQ	MS	B, C	2
8	MSQ	MS	D	2
9	MSQ	MS	B, C	2
10	MSQ	MS	B, C, D	2
11	MSQ	MS	A, C	2
12	MSQ	MS	A, C	2
13	MSQ	MS	B, C, D	2
14	MSQ	MS	A, B, C	2
15	MSQ	MS	A	2