MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The value of $\sum_{k=0}^{6} {}^{51-k}C_3$ is

(1) ${}^{52}C_4 - {}^{46}C_4$ (2) ${}^{52}C_4 - {}^{45}C_4$

(3) ${}^{51}C_4 - {}^{45}C_4$ (4) ${}^{51}C_4 - {}^{46}C_4$

Answer (2)

Sol. ${}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 .... \underbrace{{}^{45}C_3 + {}^{45}C_4}_{{}^{46}C_4} - {}^{45}C_4$

${}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + \underbrace{{}^{46}C_3 + {}^{46}C_4}_{{}^{47}C_4} - {}^{45}C_4$

$\vdots$

$\Rightarrow$ ${}^{52}C_4 - {}^{45}C_4$

2. If $f(x) = 2x^n + \lambda$ and $f(4) = 133$, $f(5) = 255$, then sum of positive integral divisors of $f(3) - f(2)$ is

(1) 60 (2) 22

(3) 40 (4) 6

Answer (1)

Sol. $2 \cdot 4^n + \lambda = 133$ ...(i)

$2 \cdot 5^n + \lambda = 255$ ...(ii)

$2(5^n - 4^n) = 122$

$n = 3$

$f(x) = 2x^3 + \lambda$

$f(3) - f(2) = 2(3^3 - 2^3)$

$= 38$

Divisors = 1, 2, 19, 38

3. If $\left|\frac{z+2i}{z-i}\right| = 2$ is a circle, then centre of the circle is

(1) (0, 0) (2) (0, 2)

(3) (2, 0) (4) (–2, 0)

Answer (2)

Sol. $(z+2i)(\bar{z}-2i) = 4(z-i)(\bar{z}+i)$

$\Rightarrow z\bar{z} + 2i\bar{z} - 2iz + 4 = 4z\bar{z} - 4i\bar{z} + 4iz + 4$

$\Rightarrow 3z\bar{z} - 6i\bar{z} + 6iz = 0$

$\Rightarrow z\bar{z} - 2i\bar{z} + 2iz = 0$

$\therefore$ Centre = $2i$ *i.e.*, (0, 2)

4. If $\frac{dy}{dt} + \alpha \cdot y = \gamma \cdot e^{-\beta t}$, then $\lim_{t\to\infty} y(t)$,

where $\alpha > 0$, $\beta > 0$, $\gamma > 0$, $\alpha \neq \beta$, is equal to

(1) 0 (2) 1

(3) Does not exist (4) $\alpha\beta$

Answer (1)

Sol. $\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$

Integrating factor (I.F.) = $e^{\alpha t}$

Solution of D.E.

$ye^{\alpha t} = \gamma \int e^{(\alpha-\beta)t} dt$

$\Rightarrow ye^{\alpha t} = \frac{\gamma}{\alpha-\beta} \cdot e^{(\alpha-\beta)t} + C$

$\Rightarrow y(t) = \frac{\gamma}{\alpha-\beta} \cdot \alpha^{-\beta t} + C \cdot e^{-\alpha t}$

$\lim_{t\to\infty} y(t) = 0$

5. If $(p \to q) \nabla (p \Delta q)$ is tautology, then operator ∇, Δ denotes

(1) $\Delta \to$ OR, $\nabla \to$ AND (2) $\Delta \to$ AND, $\nabla \to$ OR

(3) $\Delta \to$ AND, $\nabla \to$ AND (4) $\Delta \to$ OR, $\nabla \to$ OR

Answer (4)

Sol. $(p \to q) \nabla (p \Delta q) \equiv T$

Only if ∇ is OR and Δ is OR

6. Number of numbers between 5000 and 10000 by using the digits 1,3,5,7,9 without repetition is equal to

(1) 120 (2) 72

(3) 12 (4) 6

Answer (2)

Sol. The left most digit can be chosen in 3 ways i.e., 5, 7, 9.

Now, the digits can be chosen from remaining digits for remaining places in 4 ways, 3 ways, 2 ways and 1 way.

$\therefore$ $3 \times 4 \times 3 \times 2 \times 1 = 72$

7. If $f(x) = \log_{\sqrt{m}}\left(\sqrt{2}(\sin x - \cos x) + m - 2\right)$, the range of $f(x)$ is [0, 2], then the value of m is

(1) 3 (2) 4

(3) 5 (4) None

Answer (2)

Sol. We know that $\sin x - \cos x \in \left[-\sqrt{2}, \sqrt{2}\right]$

$\therefore$ $\log_{\sqrt{m}}\left((\sin x - \cos x) + m - 2\right)$

$\in \left[\log_{\sqrt{m}}(m-4), \log_{\sqrt{m}} m\right]$

$\therefore$ $\log_{\sqrt{m}}(m-4) = 0$ and $\log_{\sqrt{m}} m = 2$

$\Rightarrow$ $m = 5$

8. If A be a symmetric matrix and B and C are skew symmetric matrices of same order, then

(1) $A^{13} \cdot B^{26} - B^{26} \cdot A^{13}$ is symmetric

(2) $AC - A$ is symmetric

(3) $A^{13}B^{26} - B^{26} \cdot A^{13}$ is skew symmetric

(4) $AC - A$ is skew symmetric

Answer (3)

Sol. A is symmetric $\Rightarrow A^{13}$ is symmetric

B is skew symmetric $\Rightarrow B^{26}$ is symmetric

Now,

Let $A^{13} = P$ and B^{26} is Q

$A^{13} \cdot B^{26} - B^{26} \cdot A^{13}$

$\Rightarrow$ $PQ - QP$

Now, $(PQ - QP)^T = (PQ)^T - (QP)^T$

$= Q^T \cdot P^T - P^T \cdot Q^T$

$= QP - PQ$

$= -(PQ - QP)$

$= -\left(A^{13}.B^{26} - B^{26}.A^{13}\right)$

$\Rightarrow$ $A^{13}.B^{26} - B^{26}.A^{13}$ is skew-symmetric matrix.

9. Consider the function

$$f(x) = \begin{cases} (1 + |\cos x|)^{\frac{\lambda}{|\cos x|}} & x < \frac{\pi}{2} \\ \mu & x = \frac{\pi}{2} \\ e^{\frac{\cot 6x}{\cot 4x}} & x > \frac{\pi}{2} \end{cases}$$

(1) $\lambda = \frac{2}{3}, \mu = e^{\frac{2}{3}}$ (2) $\lambda = e^{\frac{2}{3}}, \mu = \frac{2}{3}$

(3) $\lambda = \frac{3}{2}, \mu = e^{\frac{3}{2}}$ (4) $\lambda = e^{\frac{3}{2}}, \mu = \frac{3}{2}$

Answer (1)

Sol. $\lim\limits_{x \to \frac{\pi}{2}^-} f(x) = e^{\lim\limits_{x \to \frac{\pi}{2}^-} |\cos x| \cdot \frac{\lambda}{|\cos x|}} = e^{\lambda}$

$\boxed{\mu = e^{\lambda}}$

$\lim\limits_{x \to \frac{\pi}{2}^+} f(x) = e^{\left(\lim\limits_{x \to \frac{\pi}{2}^+} \frac{\cot 6x}{\cot 4x}\right)}$

$= e^{\left(\lim\limits_{h \to 0^+} \frac{\cot 6h}{\cot 4h}\right)}$

$= e^{\frac{2}{3}}$

$\mu = e^{\frac{2}{3}}, \ \lambda = \frac{2}{3}$

10. Two dice are rolled. If the probability of the sum of the number on dice is n, where $\sqrt{n-2}, \sqrt{3n}, n+2$ are in geometric progression, is $\frac{x}{48}$, then the value of x is

(1) 4 (2) 12

(3) 7 (4) 3

Answer (1)

Sol. As given, $\left(\sqrt{3n}\right)^2 = (n-2)(n+2)$

$\Rightarrow$ $3n = n^2 - 4$

$\Rightarrow$ $n = 4$ $\quad n = -1$ (not possible)

Favourable outcomes (1, 2) (2, 1), (2, 2)

Total outcomes = 6 × 6 = 36

Given $\frac{3}{36} = \frac{x}{48}$

$\Rightarrow$ $x = 4$

11. Let $\vec{a} = -\hat{i} - \hat{j} + \hat{k}$ such that $\vec{a}\cdot\vec{b} = 1$ and $\vec{a}\times\vec{b} = \hat{i} - \hat{j}$ then $\vec{a} - 6\vec{b}$ equals

(1) $3(\hat{i}+\hat{j}+\hat{k})$

(2) $\hat{i}+\hat{j}+\hat{k}$

(3) $2(\hat{i}+\hat{j}+\hat{k})$

(4) $4(\hat{i}+\hat{j}+\hat{k})$

Answer (1)

Sol. $(\vec{a}\times\vec{b}) = \hat{i} - \hat{j}$

$\Rightarrow \vec{a}\times(\vec{a}\times\vec{b}) = (-\hat{i}-\hat{j}+\hat{k})\times(\hat{i}-\hat{j})$

$\Rightarrow (\vec{a}\cdot\vec{b})\vec{a} - (\vec{a}\cdot\vec{a})\vec{b} = \hat{i}+\hat{j}+2\hat{k}$

$\Rightarrow 3\vec{b} = -2\hat{i} - 2\hat{j} - \hat{k}$

$\therefore \vec{a} - 6\vec{b} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$= 3(\hat{i}+\hat{j}+\hat{k})$

12. $16\int_1^2 \frac{dx}{x^3(x^2+2)^2}$ is equal to

(1) $\frac{11}{12} + \ln 4$

(2) $\frac{11}{12} - \ln 4$

(3) $\frac{11}{6} - \ln 4$

(4) $\frac{11}{6} + \ln 4$

Answer (3)

Sol. $I = \int \frac{dx}{x^3(x^2+2)^2}$

$= \frac{1}{4}\int \frac{x}{x^2+2}dx + \frac{1}{4}\int \frac{x}{(x^2+2)^2}dx - \frac{1}{4}\int \frac{1}{x}dx + \frac{1}{4}\int \frac{1}{x^3}dx$

$= \frac{\ln(x^2+2)}{8} - \frac{\ln x}{4} - \frac{1}{8(x^2+2)} - \frac{1}{8x^3}$

$16\int_1^2 \frac{dx}{x^3(x+2)^2} = 2\ln 6 - 2\ln 3 - 4\ln 2 + \frac{11}{6}$

13. If $A = \begin{bmatrix} \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \\ \frac{-1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$

If $M = A^TBA$, then the matrix $AM^{2023}A^T$ is

(1) $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

(2) $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & -2023i \\ 0 & -1 \end{bmatrix}$

(4) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Answer (1)

Sol. $AA^T = I$

$M = A^TBA$

$AM^{2023}A^T$

$= A\underbrace{(A^TBA)(A^TBA)(A^TBA).....(A^TBA)}_{2023\text{ times}}A^T$

$= B^{2023}$

$B^2 = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix}$

$B^3 = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3i \\ 0 & 1 \end{bmatrix}$

$B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

14. ?

15. ?

16. ?

17. ?

18. ?

19. ?

20. ?

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Find the remainder when $(2023)^{2023}$ is divided by 35.

Answer (7)

Sol. $2023 \equiv -7 \pmod{35}$

$(2023)^2 \equiv 14 \pmod{35}$

$(2023)^4 \equiv -14 \pmod{35}$

$(2023)^{16} \equiv -14 \pmod{35}$

$(2023)2020 \equiv -14 \pmod{35}$

and $(2023)^3 \equiv 7 \pmod{35}$

$\therefore (2023)^{2023} \equiv 7 \pmod{35}$

$\therefore$ remainder = 7

22. $\int_{1/3}^{3} |\ln x|\, dx = \frac{m}{n}\ln\left(\frac{n^2}{e}\right)$ then value of $m^2 + n^2 - 5$

Answer (20)

Sol. $\int_{1/3}^{1} -\ln x dx + \int_{1}^{3} \ln x dx$

$= -\left(x\ln x - x\right)\Big|_{1/3}^{1} + \left(x\ln x - x\right)\Big|_{1}^{3}$

$= \frac{2}{3} + \frac{1}{3}\ln\frac{1}{3} + 3\ln 3 - 2$

$= \frac{4}{3}\left(\ln 9 - \ln e\right)$

$= \frac{4}{3}\ln\left(\frac{3^2}{e}\right)$

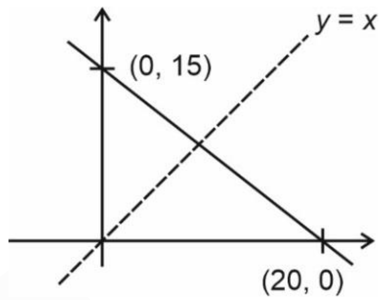
$m = 4, n = 3$

$m^2 + n^2 - 5 = 20$

23. A triangle is formed with x-axis, y-axis & line $3x + 4y = 60$. A point $P(a, b)$ lies strictly inside the triangle such that a is a positive integer and b is a multiple of 'a'. Find such number of points (a, b).

Answer (31)

Sol.



as b is multiple of a the required points will lie on line $y = kx$ ($k \in Z$)

$\therefore \quad 3x + 4kx = 60$

$\Rightarrow \quad x = \frac{60}{3+4k}$

If $k = 1$	8 integral values
$k = 2$	5 integral values
$k = 3$	3 integral values
$k = 4$	3 integral values
$k = 5$	2 integral values
$k = 6$	2 integral values
$k = 7$	1 integral values
$k = 8$	1 integral values
⋮	⋮
$k = 14$	1 integral values
$\therefore$	Total 31 points

24. If $a, b, \frac{1}{18}$ are in G.P. and $\frac{1}{10}, \frac{1}{a}, \frac{1}{b}$ are in A.P. then find value of $a + 180b$

Answer (20.00)

Sol. $b^2 = \frac{a}{18}, \frac{2}{a} = \frac{1}{10} + \frac{1}{b}$

$\Rightarrow \quad a = \frac{20b}{10+b}$

OR $18b^2 = \frac{20b}{10+b}$

$b = 0$ (rejcted)

OR $9b = \frac{10}{10+b}$

$90b + 9b^2 = 10$

$\Rightarrow \quad 9b^2 + 90b - 10 = 0$

$a + 180b = 18b^2 + 180b$

$= 20$

25. In a city, 25% of the population is smoker and a smoker has 27 times more than chance of being diagnosed with lung cancer. A person is selected at random and found to be diagnosed with lung cancer. If the probability of him being smoker is $\frac{K}{40}$, find the value of K.

Answer (36.00)

Sol. Probability of a person being smoker $= \frac{1}{4}$

Probability of a person being non smoker $= \frac{3}{4}$

$$P\left(\frac{\text{Person is smoker}}{\text{Person diagnosed with cancer}}\right) = \frac{\frac{1}{4}\cdot 27P}{\frac{1}{4}\cdot 27P + \frac{3}{4}P}$$

(P is probability that a non-smoker is diagnosed with cancer)

$= \frac{27}{30} = \frac{9}{10} = \frac{36}{40}$

26.

27.

28.

29.

30.

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