JEE Main Maths Coordinate Geometry Previous Year Questions With Solutions

Question 1: A-line through A $(-5, -4)$ meets the lines $x + 3y + 2 = 0$, $2x + y + 4 = 0$ and $x - y - 5 = 0$ at B, C and D, respectively. If $(15 / AB)^2 + (10 / AC)^2 = (6 / AD)^2$, then the equation of the line is _________.

Solution:

$[x + 5] / [\cos \theta] = [y + 4] / [\sin \theta] = r_1 / AB = r_2 / AC = r_3 / AD$

$(r_1 \cos \theta - 5, r_1 \sin \theta - 4)$ lies on $x + 3y + 2 = 0$

$r_1 = 15 / [\cos \theta + 3 \sin \theta]$

Similarly, $10 / AC = 2 \cos \theta + \sin \theta$ and $6 / AD = \cos \theta - \sin \theta$

Putting in the given relation, we get $(2 \cos \theta + 3 \sin \theta)^2 = 0$

$\tan \theta = -2 / 3$

$\Rightarrow y + 4 = [-2 / 3] (x + 5)$

$2x + 3y + 22 = 0$

Question 2: The equations of two equal sides of an isosceles triangle are $7x - y + 3 = 0$ and $x + y - 3 = 0$, and the third side passes through the point (1, -10). The equation of the third side is ___________.

Solution:

Any line through (1, -10) is given by $y + 10 = m (x - 1)$

Since it makes equal angle say α with the given lines $7x - y + 3 = 0$ and $x + y - 3 = 0$.

Therefore, $\tan \alpha = [m - 7] / [1 + 7m]$

$= [m - (-1)] / [1 + m (-1)]$

$\Rightarrow m = [1 / 3]$ or 3

Hence, the two possible equations of the third side are $3x + y + 7 = 0$ and $x - 3y - 31 = 0$.

Question 3: The graph of the function $\cos x \cos (x + 2) - \cos^2 (x + 1)$ is

A) A straight line passing through $(0, -\sin^2 1)$ with slope 2

B) A straight line passing through (0, 0)

C) A parabola with vertex 75°

D) A straight line passing through the point ($\pi/2$, $-\sin^2 1$) and parallel to the x-axis

Solution:

Let $y = \cos x \cos (x + 2) - \cos^2 (x + 1)$

$= \cos (x + 1 - 1) \cos (x + 1 + 1) - \cos^2 (x + 1)$

$= \cos^2 (x + 1) - \sin^2 1 - \cos^2 (x + 1)$

$= -\sin^2 1$, which represents a straight line parallel to x-axis with $y = -\sin^2 1$ for all x and so also for $x = \pi/2$.

Question 4: In what direction can a line be drawn through the point (1, 2) so that its points of intersection with the line x + y = 4 is at a distance $\sqrt{6} / 3$ from the given point?

Solution:

Let the required line through the point (1, 2) be inclined at an angle θ to the x-axis.

Then its equation is $[x - 1] / [\cos \theta] = [y - 2] / [\sin \theta] = r$ .....(i)

where r is the distance of any point (x, y) on the line from the point (1, 2).

The coordinates of any point on the line (i) are $(1 + r \cos \theta, 2 + r \sin \theta)$.

If this point is at a distance $\sqrt{6} / 3$ form (1, 2), then $r = \sqrt{6} / 3$.

Therefore, the point is $(1 + [\sqrt{6} / 3] \cos \theta, 2 + [\sqrt{6} / 3] \sin \theta)$.

But this point lies on the line x + y = 4.

$\sqrt{6} / 3 (\cos \theta + \sin \theta) = 1$ or

$\sin \theta + \cos \theta = 3 / \sqrt{6}$

$[1 / \sqrt{2}] \sin \theta + [1 / \sqrt{2}] \cos \theta = \sqrt{3} / 2$, {Dividing both sides by $\sqrt{2}$}

$\sin (\theta + 45^\circ) = \sin 60^\circ$ or $\sin 120^\circ$

$\theta = 15^\circ$ or 75°

Question 5: A variable line passes through a fixed point P. The algebraic sum of the perpendicular drawn from (2, 0), (0, 2) and (1, 1) on the line is zero, then what are the coordinates of the P?

Solution:

Let P (x_1, y_1), then the equation of the line passing through P and whose gradient is m, is $y - y_1 = m (x - x_1)$.

Now according to the condition

$[\{-2m + (mx_1 - y_1)\} / \{\sqrt{1 + m^2}\}] + [\{2 + (mx_1 - y_1)\} / \{\sqrt{1 + m^2}\}] + [\{1 - m + (mx_1 - y_1)\} / \{\sqrt{1 + m^2}\}] = 0$

$3 - 3m + 3mx_1 - 3y_1 = 0$

$\Rightarrow y_1 - 1 = m (x_1 - 1)$

Since it is a variable line, hold for every value of m.

Therefore, $y_1 = 1$, $x_1 = 1$

$\Rightarrow$ P(1, 1)

Question 6: The area enclosed within the curve $|x|+|y|= 1$ is ____________.

Solution:

The given lines are $\pm x \pm y = 1$

i.e., $x + y = 1$, $x - y = 1$, $x + y = -1$ and $x - y = -1$.

These lines form a quadrilateral whose vertices are A (−1, 0), B (0, −1), C (1, 0) and D (0, 1).

Obviously, ABCD is a square.

The length of each side of this square is $\sqrt{1^2 + 1^2} = \sqrt{2}$

Hence, the area of the square is $\sqrt{2} * \sqrt{2} = 2$ sq. units

Trick: Required area $= 2c^2 / |ab| = [2 * 1^2] / [|1 * 1|] = 2$.

Question 7: The locus of a point P, which divides the line joining (1, 0) and (2 cos θ, 2 sin θ) internally in the ratio 2 : 3 for all θ, is a ________.

Solution:

Let the coordinates of the point P, which divides the line joining (1, 0) and (2 cos θ, 2 sin θ) in the ratio 2 : 3 be (h, k). Then,

$h = [4 \cos \theta + 3] / [5]$ and $k = [4 \sin \theta] / [5]$

$\cos\theta = [5h - 3] / [4]$ and $\sin\theta = [5k] / [4]$

$([5h - 3] / [4])^2 + ([5k] / [4])^2 = 1$ {since $\cos^2\theta + \sin^2\theta = 1$}

$(5h - 3)^2 + (5k^2) = 16$

Therefore, locus of (h, k) is $(5x - 3)^2 + (5y)^2 = 16$, which is a circle.

Question 8: The area of a parallelogram formed by the lines $ax \pm by \pm c = 0$, is __________.

Solution:

$ax \pm by \pm c = 0$

$\Rightarrow x / [\pm c / a] + y/ [\pm c / b] = 1$ which meets on axes at A (ca, 0), C (−ca, 0), B (0, cb), D (0, −cb).

Therefore, the diagonals AC and BD of quadrilateral ABCD are perpendicular.

Hence, it is a rhombus.

So, the area = $(1/2) \times AC \times BD$

$= (1/2) \times (2c/a) \times (2c/b)$

$= 2c^2 / ab$.

Question 9: If the sum of the distances of a point from two perpendicular lines in a plane is 1, then its locus is ________.

Solution:

Required locus of the point (x, y) is the curve$|x| + |y| = 1$.

If the point lies in the first quadrant, then $x > 0$, $y > 0$ and so $|x| + |y| = 1$

$\Rightarrow x + y = 1$, which is straight line.

If the point (x, y) lies in the second quadrant then $x < 0$, $y > 0$ and so $|x| + |y| = 1$

$\Rightarrow -x + y = 1$.

Similarly, for the third and fourth quadrants, the equations are $-x -y = 1$ and $x - y = 1$.

Hence, the required locus is the curve consisting of the sides of the square.

Question 10: The line $2x + 3y = 12$ meets the x-axis at A and y-axis at B. The line through (5, 5) perpendicular to AB meets the x-axis, y-axis and the AB at C, D and E, respectively. If O is the origin of coordinates, then the area of OCEB is _______.

Solution:

Coordinates of the origin: O(0, 0)

The line $2x + 3y = 12$ meets the y-axis at B.

That means B = (0, 4).

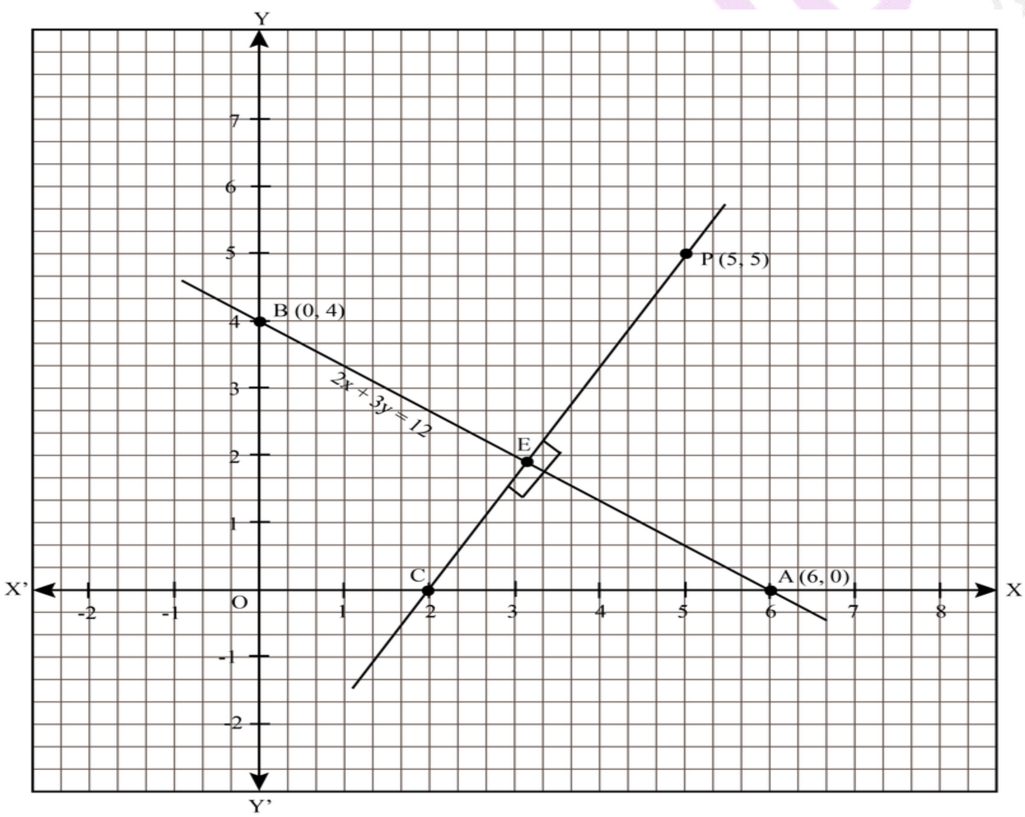
The equation of any line perpendicular to the line $2x + 3y = 12$ and passes through (5, 5) is $3x - 2y = 5$ ......(i)

The line (i) meets the x-axis at C.

Therefore, the coordinates of C are (5/3, 0).

Similarly, by solving the line AB and (i), we get; the coordinates of E as (3, 2).

Thus, O (0, 0), C (5 / 3, 0), E (3, 2) and B (0, 4).



Now,

Area of OCEB) = ar(ΔOAB) - ar(ΔCAE)

$= (1/2) \times 6 \times 4 = (1/2) \times (2\sqrt{13}/3) \times \sqrt{13}$

= 23/3 sq.units.

Question 11: Line L has intercepts a and b on the co-ordinate axes. When the axes are rotated through a given angle keeping the origin fixed, the same line L has intercepts p and q, find the relation between a,b and p,q.

Solution:

Suppose we rotate the coordinate axes in the anti-clockwise direction through an angle α.

The equation of the line L with respect to old axes is [x / a] + [y / b] = 1.

In this question replacing x by x cosα − y sinα and y by x sinα + y cosα, the equation of the line with respect to new axes is [x cosα − y sinα] / [a] + [x sinα + y cosα] / [b] = 1

x * (cosα / a + sinα / b) + y * (cosα / b − sinα / a) = 1 .....(i)

The intercepts made by (i) on the co-ordinate axes are given as p and q.

Therefore, [1 / p] = [cosα / a] + [sinα / b] and [1 / q] = [cosα / b] − [sinα / a]

Squaring and adding, we get $[1 / p^2] + [1 / q^2] = [1 / a^2] + [1 / b^2]$.

Question 12: The equation of the line which bisects the obtuse angle between the lines x − 2y + 4 = 0 and 4x − 3y + 2 = 0, is _________.

Solution:

The equations of the bisectors of the angles between the lines are

[x − 2y + 4] / √[1 + 4] = ± [4x − 3y + 2] / √[16 + 9]

Taking a positive sign, we have;

(4 − √5) x − (3 − 2√5) y − (4√5 − 2) = 0 .....(i)

and negative sign gives (4 + √5) x − (2√5 + 3) y + (4√5 + 2) = 0 .....(ii)

Let θ be the angle between the line (i) and one of the given lines, then

$$\tan\theta = \left|\frac{\frac{1}{2} - \frac{4-\sqrt{5}}{3-2\sqrt{5}}}{1 + \frac{1}{2}\cdot\frac{4-\sqrt{5}}{3-2\sqrt{5}}}\right|$$

= √[5 + 2] > 1

Hence, line (i), i.e., $(4 - \sqrt{5})x - (3 - 2\sqrt{5})y - (4\sqrt{5} - 2) = 0$, bisects the obtuse angle between the given lines.

Question 13: The points (1, 3) and (5, 1) are the opposite vertices of a rectangle. The other two vertices lie on the line $y = 2x + c$, then the value of c will be __________.

Solution:

Let ABCD be a rectangle.

Given A (1, 3) and C (5, 1).

We know that the intersecting point of the diagonal of a rectangle is the same or at the midpoint.

So, the midpoint of AC is (3, 2).

Hence, $y = 2x + c$ passes through (3, 2).

$2 = 6 + c$

Therefore, $c = -4$.

Question 14: The equation of the lines, which passes through the point (3, - 2) and are inclined at 60° to the line $\sqrt{3}x + y = 1$ ____________.

Solution:

The equation of any straight line passing through (3, 2) is $y + 2 = m(x - 3)$ …....(i)

The slope of the given line is $-\sqrt{3}$.

So, $\tan 60^{\circ} = \pm [m - (-\sqrt{3})] / [1 + m(-\sqrt{3})]$

On solving, we get $m = 0$ or $\sqrt{3}$

Putting the values of m in (i), the required equation of lines are

$y + 2 = 0$ and $\sqrt{3}x - y = 2 + 3\sqrt{3}$.

Question 15: If the slope of a line passing through point A (3, 2) is 3/4, then the points on the line which are 5 units away from A are ________.

Solution:

The equation of line passes through (3, 2) and of slope 3/4 is $3x - 4y - 1 = 0$

Let the point be (h, k), then

$3h - 4k - 1 = 0$ …….(i) and

$(h - 3)^2 + (y - 2)^2 = 5^2$ (ii)

On solving the equations, we get $h = -1, 7$ and $k = -1, 5$.

Hence, points are (-1, -1) and (7, 5).