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I

Total No. of Questions—24

Total No. of Printed Pages—4 **Regd. No.** | | | | | | | | | | |

Part III

MATHEMATICS

Paper II(A)

(English Version)

Time : 3 Hours ***Max. Marks : 75***

Note :—This question paper consists of THREE sections A, B and C.

SECTION A 10×2=20

I. Very Short Answer Type Questions :

(*i*) Answer ALL questions.

(*ii*) Each question carries TWO marks.

1. Find the multiplicative inverse of :

$$7 + 24i.$$

2. Simplify $i^2 + i^4 + i^6 + \dots\dots\dots$ up to $(2n + 1)$ terms.

3. If $x = \text{cis}\ \theta$, then find the value of $x^6 + \frac{1}{x^6}$.

4. Form the quadratic equation whose roots are :

$$\frac{p-q}{p+q} \text{ and } \frac{-(p+q)}{p-q}\ (p \neq \pm q).$$

5. Find the algebraic equation whose roots are 2 times the roots of $x^5 - 2x^4 + 3x^3 - 2x^2 + 4x + 3 = 0$.

6. Find the number of functions from a set A containing 5 elements into a set B containing 4 elements.

7. If ${}^{15}C_{2r-1} = {}^{15}C_{2r+4}$, find r.

8. If ${}^{22}C_r$ is the largest binomial coefficient in the expansion of $(1 + x)^{22}$, find the value of ${}^{13}C_r$.

9. Find the mean deviation from the mean of the following discrete data :

6, 7, 10, 12, 13, 4, 12, 16.

10. For a binomial distribution with mean 6 and variance 2, find the first two terms of the distribution.

SECTION B

5×4=20

II. Short Answer Type Questions :

(*i*) Answer ANY FIVE questions.

(*ii*) Each question carries FOUR marks.

11. If the real part of $\left(\frac{z+1}{z+i}\right)$ is 1, then find the locus of z where $z = x + iy$.

12. Prove that :

$$\frac{1}{3x+1} + \frac{1}{x+1} - \frac{1}{(3x+1)(x+1)}$$

does not lie between 1 and 4, if x is real.

13. Find the number of 4-letter words that can be formed using the letters of the word MIRACLE. How many of them :

(*i*) Begin with a vowel

(*ii*) Begin and end with vowels

(*iii*) End with a consonant.

14. Prove that :

$$\frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{1.3.5 \ldots\ldots\ldots (4n-1)}{\{1.3.5 \ldots\ldots\ldots\ldots (2n-1)\}^2}.$$

15. Resolve $\frac{3x^3 - 2x^2 - 1}{x^4 + x^2 + 1}$ into partial fractions.

16. The probabilities of three mutually exclusive events are respectively given as $\frac{1+3P}{3}, \frac{1-P}{4}, \frac{1-2P}{2}$. Prove that $\frac{1}{3} \le P \le \frac{1}{2}$.

17. If A and B are independent events of a random experiment, show that A^C and B^C are also independent.

SECTION C

5×7=35

III. Long Answer Type Questions :

(i) Answer ANY FIVE questions.

(ii) Each question carries SEVEN marks.

18. If α, β are the roots of the equation $x^2 - 2x + 4 = 0$, then for any $n \in N$ show that :

$$\alpha^n + \beta^n = 2^{n+1} \cos\left(\frac{n\pi}{3}\right).$$

19. Find the polynomial equation whose roots are the translates of those of the equation :

$$x^4 - 5x^3 + 7x^2 - 17x + 11 = 0 \text{ by } -2.$$

20. If the coefficients of x^9, x^{10}, x^{11} in the expansion of $(1 + x)^n$ are in A.P., then prove that :

$$n^2 - 41n + 398 = 0.$$

21. If

$$x = \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \frac{1.3.5.7}{3.6.9.12} + ...,$$

then prove that :

$$9x^2 + 24x = 11.$$

22. Find the mean deviation about median for the following continuous distribution :

Marks Obtained	**No. of Boys**
0—10	6
10—20	8
20—30	14
30—40	16
40—50	4
50—60	2

23. Suppose that an urn B_1 contains 2 white and 3 black balls and another urn B_2 contains 3 white and 4 black balls. One urn is selected at random and a ball is drawn from it. If the ball drawn is found black, find the probability that the urn chosen was B_1.

24. A random variable X has the following probability distribution :

$X = x$	$p(X = x)$
0	0
1	k
2	$2k$
3	$2k$
4	$3k$
5	k^2
6	$2k^2$
7	$7k^2 + k$

Find :

(i) k

(ii) the mean and

(iii) $p(0 < X < 5)$.