266

(I)

Total No. of Questions—**24**

Total No. of Printed Pages—**4**

Regd. No. | | | | | | | | | | |

Part III

MATHEMATICS

Paper II-A

(English Version)

Time : 3 Hours ***Max. Marks : 75***

***Note* :—** This question paper consists of THREE Sections A, B and C.

SECTION A

10×2=20

(I) Very short answer type questions :

(*i*) Answer ALL questions.

(*ii*) Each question carries TWO marks.

1. Find the complex conjugate of $(3 + 4i)(2 - 3i)$.

2. Write $z = -\sqrt{3} + i$ in modulus-amplitude form.

3. If α, β are the roots of the equation $x^2 + x + 1 = 0$, then prove that $\alpha^4 + \beta^4 + \alpha^{-1}\beta^{-1} = 0$.

4. Find the quadratic equation, the sum of whose roots is 7 and the sum of the squares of the roots is 25.

5. If the product of the roots of $4x^3 + 16x^2 - 9x - a = 0$ is 9, then find 'a'.

P.T.O.

6. If ${}^{n}P_7 = 42\ {}^{n}P_5$, find n.

7. Find the number of positive divisors of 1080.

8. If ${}^{22}C_r$ is the largest binomial co-efficient in the expansion of $(1 + x)^{22}$, find the value of ${}^{13}C_r$.

9. Find the mean deviation from the mean of the following discrete data :

3, 6, 10, 4, 9, 10.

10. For a binomial distribution with mean 6 and variance 2, find the first two terms of the distribution.

SECTION B 5×4=20

(II) Short answer type questions :

(*i*) Answer ANY FIVE questions.

(*ii*) Each question carries FOUR marks.

11. Show that the points in the Argand plane represented by the complex numbers $-2 + 7i$, $\frac{-3}{2} + \frac{1}{2}i$, $4 - 3i$, $\frac{7}{2}(1 + i)$ are the vertices of a rhombus.

12. Find the range of the expression $\frac{x^2 + x + 1}{x^2 - x + 1}$.

13. If the letters of the word MASTER are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the rank of the word MASTER.

14. Prove that for $3 \leq r \leq n$,

$$^{(n-3)}C_r + 3\,^{(n-3)}C_{(r-1)} + 3\,^{(n-3)}C_{(r-2)} + ^{(n-3)}C_{(r-3)} = ^{n}C_r.$$

15. Resolve $\dfrac{x^2 - 3}{(x+2)\left(x^2+1\right)}$ into partial fractions.

16. The probability for a contractor to get a road contract is $\frac{2}{3}$ and to get a building contract is $\frac{5}{9}$. The probability to get at least one contract is $\frac{4}{5}$. Find the probability that he gets both the contracts.

17. (*a*) Define conditional probability.

(*b*) If A and B are independent events with P(A) = 0.2, P(B) = 0.5, then find :

(*i*) P(A/B)

(*ii*) P(B/A)

(*iii*) $P(A \cap B)$.

SECTION C

5×7=35

(III) Long answer type questions :

(*i*) Answer ANY FIVE questions.

(*ii*) Each question carries SEVEN marks.

18. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, prove that :

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{3}{2} = \sin^2\alpha + \sin^2\beta + \sin^2\gamma.$$

19. Solve $4x^3 - 24x^2 + 23x + 18 = 0$, given that the roots of this equation are in arithmetic progression.

20. If I, n are positive integers, $0 < f < 1$ and if $\left(7 + 4\sqrt{3}\right)^n = \text{I} + f$, then show that :

(*i*) I is an odd integer

(*ii*) $(\text{I} + f)(1 - f) = 1$.

21. Find the sum to infinite terms of the series :

$$1 + \frac{1}{3} + \frac{1 \cdot 3}{3 \cdot 6} + \frac{1 \cdot 3 \cdot 5}{3 \cdot 6 \cdot 9} + \ldots\ldots\ldots \infty.$$

22. Calculate the variance and standard deviation for the following discrete frequency distribution :

x_i	f_i
4	3
8	5
11	9
17	5
20	4
24	3
32	1

23. State and prove Bayes' theorem.

24. The Range of a random variable X is {0, 1, 2}. Given that $P(X = 0) = 3C^3$, $P(X = 1) = 4C - 10C^2$, $P(X = 2) = 5C - 1$:

(*i*) Find the value of C

(*ii*) $P(X < 1)$, $P(1 < X \leq 2)$ and $P(0 < X \leq 3)$.